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MODERN BUSINESS ARITHMETIC

COMPLETE COURSE

REVISED

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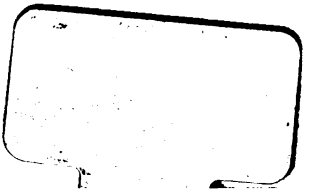

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MODERN BUSINESS ARITHMETIC

COMPLETE COURSE

REVISED EDITION

BY

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PREFACE

THIS book provides a year's work in the arithmetic of modern business. All topics which have no other claim for inclusion than that of tradition have been omitted and no topic is included unless it has a legitimate place in modern business practice. Topics which have acquired importance recently because of changes in the organization and conduct of business are included. Among the topics so included are Contract Purchases and Installment Payments, Depreciation, Advertising, Insolvency and Bankruptcy, Comparative Statistics to Promote Buying and Selling Efficiency, Profit and Loss by Departments, Finding the Profit and Loss on Each Sale, Factory Costs, and Tabulations for the Sales Manager.

The illustrations and model forms were chosen to acquaint the student with actual business conditions. The problems were selected from those which actually arise in important business activities, and the methods of solution are those which are used in business practice. Many of the drill exercises are put in ruled forms such as will be met in the business world; and the student is given practice in the art of presenting business statistics and other material in a neat, concise, and attractive manner. The traditional sequence of topics has been abandoned in some cases in favor of a grouping of topics in which there is a relation of business experience.

The necessity for a high degree of accuracy and facility in arithmetical computation has been fully recognized. Abundant material for practice and drill in the fundamental processes is provided. The student is required frequently to check his results in order to form the habit of checking, so necessary in business, and numerous exercises to be finished within a time limit are included to improve speed in computation. Since business expediency demands that many problems shall be solved orally, a large amount of oral work has been included.

Each chapter of the book is adequately motivated. The material is so presented that the student sees the business need for each process, and numerous practical problems based on actual business conditions and practices of today are found in every chapter. The problems chal-

lenge the interest of the student because they portray actual business situations. The material of the book is not experimental. Its value is vouched for by the enthusiastic testimony of scores of business men who have examined it with care, and by thousands of students who have been prepared in business arithmetic by the use of the earlier editions of this text.

From the point of view of both material and method this book is based on actual business conditions and practices, not on theory. The ruled forms and the tabulated business statistics so extensively used in the book have demonstrated their effectiveness in aiding in the development of rapidity, accuracy, and neatness. Many specialists in various business and industrial activities were consulted, and the statements of business customs, as well as the tabulation of materials and the selection and arrangement of problems, are based on the practices of the day.

The authors wish to acknowledge their indebtedness to the numerous teachers and business men who have aided them by offering valuable suggestions and discriminating criticisms, and by furnishing materials.

HARRY ANSON FINNEY.
JOSEPH CLIFTON BROWN.

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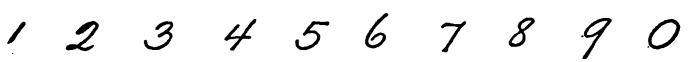
INTRODUCTION

To the Student

If you expect to master the arithmetic of the modern business world, you should strive to develop accuracy and speed in computation and neatness in the appearance of your work. A high degree of accuracy is indispensable. Be very careful about all of your work and form the habit of checking your results.

Work as rapidly as you can without reducing your accuracy. The material in this book has been prepared to aid you in the formation and development of proper study habits and to enable you to master the essentials of the arithmetic of modern business in as short a time as possible.

Take pride in the appearance of your work. Most of your papers should be prepared with ink, because that is the way business records are kept. When ruling is to be done, make the lines fine. Make your figures small and similar to those in the following model:



FUNDAMENTAL PROCESSES

CHAPTER I

ADDITION

1. Drill Tables. The following table contains the forty-five combinations of two numbers, each of which is less than ten. Practice until you can state these sums in any order, without error, in less than twenty-five seconds. Do not repeat the numbers to be added. State *results only*. Try to excel your own record.

ADDITION DRILL CHART

2	6	4	6	1	7	4	7	2	2	3	3	1	2	1
9	8	4	9	2	7	6	9	5	4	5	6	1	8	5
7	1	8	5	4	8	2	3	4	5	2	1	5	9	4
8	4	8	9	5	9	3	4	9	5	7	3	6	9	7
1	6	3	6	1	3	1	5	2	3	5	1	2	4	3
9	7	3	6	7	8	8	7	2	9	8	6	6	8	7

The following group of thirty-six combinations contains the inversions possible, omitting the pairs of equal numbers. Practice until you can state the sums in any order without hesitation.

ADDITION DRILL CHART

4	7	9	2	6	3	6	8	7	4	7	8
3	5	5	1	5	1	1	5	4	1	6	3
7	4	7	8	9	8	9	9	5	6	9	8
1	2	2	4	3	6	7	2	2	4	4	7
9	8	9	3	9	8	6	5	7	5	5	6
8	1	6	2	1	2	3	4	3	1	3	2

NOTE. It is suggested that these charts and those which follow be used for frequent drill.

Following are the eighty-one combinations, including inversions, of two numbers, each of which is less than ten. Practice on this exercise until you can state the sums in less than a minute.

ADDITION DRILL CHART

6	1	3	4	4	5	2	4	6	5	1	3	3	5	9	7	5
1	3	2	4	6	4	7	3	2	9	1	8	3	2	6	8	8
2	1	1	8	4	8	6	4	8	9	5	3	7	8	4	9	3
1	8	9	6	5	5	7	7	9	8	6	9	7	8	8	4	7
7	2	9	1	3	9	8	9	1	6	9	2	4	5	8	6	6
6	9	1	2	4	5	7	3	7	5	2	6	9	7	2	8	9
4	2	7	6	2	8	8	7	3	2	1	2	9	1	7	7	5
1	3	9	6	4	1	3	1	6	2	5	8	7	6	3	4	5
6	8	4	2	5	6	9	5	1	7	3	7	3				
3	4	2	5	1	4	9	3	4	5	5	2	1				

The following exercise contains the nine digits in groups of three. Practice on these combinations until you can state the sums in less than two and a half minutes. Each column should be added both upward and downward.

ADDITION DRILL CHART

8	8	7	5	8	4	5	8	9	6	7	5	4	4	6	8	5	4
6	8	3	5	3	2	5	6	7	5	5	3	3	4	3	2	4	3
2	2	3	2	2	2	3	3	2	3	3	2	3	3	3	2	2	2
3	5	3	6	6	4	5	7	6	9	7	8	6	7	9	8	6	6
2	2	3	3	6	4	4	7	4	8	2	7	2	3	2	7	6	5
2	2	2	2	2	2	4	2	3	2	2	2	2	2	2	1	3	4
7	8	9	8	9	8	7	7	9	9	6	8	7	9	8	9	5	7
7	8	6	5	4	8	4	5	4	3	5	3	6	9	4	8	5	4
3	3	2	3	2	1	2	2	3	2	2	3	2	1	2	3	4	4

NOTE. Certain addition combinations are especially difficult and should therefore receive more emphasis than easy combinations such as 2 and 2, 5 and 5. The most difficult combinations are 8 and 5; 8 and 6; 8 and 7; 8 and 9; 9 and 7; 9 and 8; 7 and 6; and their reverses.

8	8	9	9	9	7	5	9	9	8	9	7	8	8	9	6	7	6
4	6	9	3	6	4	3	6	8	8	6	7	5	7	5	4	7	6
4	1	2	3	1	3	3	3	1	4	4	1	4	6	2	2	4	5

6	6	7	8	9	9	9	8	7	8	9	9	5	7	9	9	8	9
4	6	7	7	7	7	5	5	6	8	7	8	4	6	8	8	5	9
4	4	6	4	5	7	3	5	5	7	6	5	3	3	6	8	2	8

7	9	8	9	9	9	7	7	8	8	9	8	9	9	9	8	7	8
5	6	6	9	5	9	6	5	4	6	9	8	5	7	4	7	6	7
5	5	5	5	5	7	6	4	3	4	4	6	4	1	4	3	4	4

8	9	9	8	9	6	9	9	7	8	9	9
7	9	8	6	7	5	6	9	7	8	8	7
5	6	7	6	4	5	6	3	5	5	4	3

Oral Work

Add the following and check results by adding in reverse order. In this, and in all other work in addition, state *the sums only*. For example, in adding 3, 4, 8, 9, say 7, 15, 24.

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.
a.	3	5	7	2	9	1	6	8	5	4	9	3	5	5	7	9
b.	7	3	9	1	5	3	7	5	6	9	8	9	3	9	3	8
c.	2	8	3	5	7	1	4	9	6	4	6	7	9	4	9	3
d.	6	3	8	1	4	2	7	8	9	6	7	8	7	8	2	7
e.	6	3	7	5	2	9	5	1	7	4	8	4	1	9	8	5
f.	7	8	5	3	9	1	4	7	6	4	3	9	8	7	9	9
g.	8	4	9	5	7	3	5	8	5	9	9	6	9	8	9	8
h.	9	2	5	3	7	4	8	5	9	6	6	1	6	3	5	6

State the sums in each of the following. The first two numbers in each example should be thought of as one number. Thus, in the first example, think 10, 17.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
7	5	9	1	2	5	8	2	4	6	3	7	5	9
2	4	1	6	7	4	1	7	3	2	8	4	3	9
8	4	6	9	2	9	5	7	3	8	5	3	7	5

NOTE. The sum of the numbers in the rows may be found also.

15.	16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.
8	5	3	7	2	5	6	9	6	8	5	7	8	4
6	5	3	8	4	7	4	7	8	5	7	4	8	5
4	6	2	8	3	5	6	5	7	3	6	9	3	6

29.	30.	31.	32.	33.	34.	35.	36.	37.	38.	39.	40.	41.	42.
7	7	4	9	5	7	4	6	8	3	6	5	7	9
6	7	3	6	9	5	8	3	6	7	4	5	7	3
7	4	8	5	3	7	3	8	5	4	2	6	9	4

2. Adding numbers by grouping may increase your speed. Some rapid computers use group addition. For example,

$$\left.\begin{matrix}8\\3\end{matrix}\right\} 11$$
$$\left.\begin{matrix}4\\2\end{matrix}\right\} 6$$
$$17$$

The acquisition of speed and accuracy in group addition requires much practice. Some computers gain in speed by grouping numbers whose sum is ten.

Use group addition to find the sums in the following exercise.

This work should be done orally. Add both upward and downward.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
6	5	8	4	7	4	8	5	2	6	3	5	4	6
3	2	2	3	4	3	6	7	3	7	7	8	9	6
5	7	3	4	8	5	3	7	5	4	9	4	7	3
4	3	6	2	3	5	7	6	4	8	4	7	3	5

15.	16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.
3	7	4	2	5	3	6	2	4	6	8	9	7	5
3	6	4	8	5	3	7	6	4	8	5	7	3	6
7	5	3	6	5	8	4	5	6	4	7	3	8	2
7	6	3	4	1	7	4	7	4	8	9	5	3	7

29.	30.	31.	32.	33.	34.	35.	36.	37.	38.	39.	40.	41.	42.
3	2	6	8	6	4	2	4	6	4	8	5	2	3
3	6	2	5	8	9	5	3	7	5	3	5	7	9
9	5	3	6	8	9	5	2	6	2	8	5	3	9
5	2	7	4	2	7	3	7	9	4	2	7	4	6

43.	44.	45.	46.	47.	48.	49.	50.	51.	52.	53.
4	8	9	5	3	2	6	8	4	3	6
3	6	8	4	2	6	7	4	3	6	8
7	4	6	3	7	8	5	3	6	7	9
4	7	5	2	8	6	4	8	3	7	9
2	6	4	7	9	6	4	2	6	7	5
7	4	8	3	9	5	9	3	6	7	5

54.	55.	56.	57.	58.	59.	60.	61.	62.	63.	64.
7	3	2	8	5	3	7	3	7	5	9
5	3	6	8	7	4	2	6	8	3	7
4	5	6	4	7	3	8	4	2	8	3
5	2	3	5	7	8	5	3	6	7	8
4	7	6	4	8	9	2	6	3	7	5
2	3	6	7	8	4	2	5	6	3	6

Find the sums in examples 1–14 by the following method:

In the illustration we say, 72, 112, 119, 139, 143, 183, 186. That is, we add 72 and 40, getting 112, then add 112 and 7, getting 119; then add 119 and 20, getting 139, then 139 and 4, getting 143; then add 143 and 40, getting 183, then 183 and 3, getting 186.

43	
24	
47	
72	

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
24	46	37	53	58	67	48	39	68	73	66	81	93	65
33	52	44	63	47	59	73	58	42	86	59	78	38	75

3. Recording Addition by Columns. For convenience in checking, the total of each column may be recorded separately. This will able you to resume the work where it was discontinued if you are

interrupted after having added one or more columns. Three methods are in use.

a.	*b.*	*c.*
369	369	369
487	487	487
352	352	352
896	896	896
294	294	294
28	20	28
37	37	39
20	28	23
2,398	2,398	

Illustrations (*a*) and (*b*) show the method of adding each column *without carrying* the tens. The column totals are then added to obtain the final result. Illustration (*c*) shows the method of adding to each column the amount carried from the column at the right.

Written Work

Make all figures of uniform size and easily legible and place figures of the same order in the same column.

Copy and find the sums. Check each sum by adding in the reverse order.

1.	2.	3.	4.	5.
5,823	84,239	387,935	9,328,577	38,421
4,923	42,937	512,937	4,912,374	31,238
9,382	31,629	491,327	5,823,746	312
5,128	49,238	395,825	4,923,748	273,845
3,258	83,153	416,239	7,239,482	3,9547
7,362	42,841	482,423	1,423,849	315,288
8,127	52,384	923,857	8,237,421	4,234
4,239	94,856	412,831	9,423,748	21,582

6.	7.	8.	9.	10.	11.
27	371	419	372	97	996
83	927	82	496	36	37
95	482	92	18	547	321
84	413	412	739	969	493
82	958	892	847	38	782
15	285	9216	96	437	946
81	381	594	382	692	531
48	294	5	75	99	28
79	148	592	989	831	327

Each student should prepare original examples for addition, following the instruction of the teacher. Exchange papers, criticize the form, and find the sums. Check all results. The method explained in article 3 should be used rather frequently in checking.

The ability to give and to take dictation of numbers should be developed. The accountant must acquire the ability to copy and to take from dictation figures for use in computation. Numbers may be dictated by the teacher, or by members of the class.

In preparing certain kinds of reports it is sometimes necessary to add columns horizontally. When additions are made in this way time in recopying of figures is frequently saved. Care must be taken to see that units are added to units, tens to tens, etc.

Add horizontally and vertically. See that the results check.

1.

427 + 823 + 579 =
386 + 547 + 387 =
542 + 786 + 395 =
746 + 408 + 548 =
902 + 875 + 386 = ___
___ + ___ + ___ =

2.

974 + 386 + 277 + 385 =
276 + 843 + 946 + 837 =
478 + 397 + 852 + 478 =
309 + 974 + 706 + 840 =
768 + 897 + 467 + 904 = ___
___ + ___ + ___ + ___ =

3.

9,642 + 8,047 + 9,485 =
8,731 + 9,206 + 3,479 =
4,926 + 7,348 + 7,496 =
8,397 + 9,479 + 8,744 = ___
___ + ___ + ___ =

4.

3,870 + 3,947 + 8,074 =
4,967 + 7,843 + 5,765 =
3,856 + 5,768 + 3,987 =
4,978 + 9,746 + 8,456 = ___
___ + ___ + ___ =

1. The following are the number of men, women, and children who paid admission to a county fair:

Paid Admissions	Mon.	Tues.	Wed.	Th rs.	Fri.	Sat.	Totals
Men.	1,426	2,874	3,947	4,068	3,872	2,596	
Women . . .	897	1,463	1,896	2,947	2,865	1,975	
Children . . .	483	796	1,267	2,796	3,214	847	

Rule a suitable blank form and copy the above statistics.

Find the total number of admissions paid each day and the total number of men, of women, and of children for the week.

Written Work

The following table shows the weekly sales report made by a number of salesmen during a certain week.

DAY	J.C OLSEN	D.R BACON	F.G.BATES	D.O.ESTEY	TOTAL
MONDAY	216 45	538 74	816 98	319 47	
TUESDAY	358 93	619 84	725 48	629 47	
WEDNESDAY	412 57	593 27	639 74	725 84	
THURSDAY	385 13	634 96	744 47	594 72	
FRIDAY	518 92	718 96	639 37	584 38	
SATURDAY	639 47	639 62	729 47	846 94	
TOTAL					

Find the total sales made by each salesman during the week, and enter these totals in the proper places at the bottom of the table.

Find the total sales made each day, and enter these totals in the column at the right.

Find the total sales made by all salesmen during the week.

The grand total should be found in two ways: by adding the totals at the foot of the blank, and those at the right. If these sums agree,

you may assume that the additions are correct; if they do not agree, you should find the error.

4. How to Rule a Blank Form. Ruling the forms for your written work will furnish valuable practice as a preparation for bookkeeping in school and in business. Follow the instructions given below:

a. Make the columns just wide enough for the figures, — about seven figures to an inch. Remember that the total may have one or two more figures than any of the addends (numbers to be added).

b. Make the horizontal lines parallel and about $\frac{1}{4}$ of an inch apart.

c. Make the spaces for headings and totals $\frac{3}{8}$ of an inch deep.

d. Place the form in the middle of the paper.

5. How to Enter Statistics. Much of the clerk's work in many business offices consists of entering statistics on ruled forms.

a. Always write figures on the line.

b. Point off figures in groups of three.

c. Make figures of uniform size and easily legible.

d. Keep the figures close to the right side of the column.

e. Keep the columns vertical: units above units, tens above tens, etc.

f. Write the figures at equal distances from each other.

g. Verify the copying by checking with the original.

h. Be very careful not to omit a figure or to copy figures in the wrong order.

Written Work

1. The following table shows the delivery records of a store running five wagons. A record of daily deliveries is kept and the week's totals are found. If the summary shows that any one wagon is required to make an unreasonably large number of deliveries, a change in the routes may be necessary. By finding the total number of deliveries made each week, the average cost per delivery may be computed.

Rule a blank form similar to this model; copy the statistics, and find:

a. The total number of deliveries made during the week by each wagon.

b. The total number of deliveries made each day.

c. The total number of deliveries for the week.

DAY	WAGON NO 1	WAGON NO 2	WAGON NO 3	WAGON NO 4	WAGON NO 5	TOTAL
MONDAY	243	319	279	143	241	
TUESDAY	216	326	391	178	216	
WEDNESDAY	257	287	294	121	322	
THURSDAY	301	291	341	147	296	
FRIDAY	294	304	303	218	213	
SATURDAY	318	361	420	223	301	
TOTAL						G.T.

2. Rule a blank form similar to the first of the tables on this page, copy the data, and find the totals indicated.

WEEKLY SALES REPORT BY DEPARTMENTS

DEPT. No.	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	TOTAL
1	$932.87	$823.75	$239.75	$724.16	$295.16	$840.19	
2	829.47	584.92	395.58	645.97	396.74	492.10	
3	723.85	486.47	523.78	419.52	296.15	492.58	
4	836.47	594.38	723.91	391.16	749.15	581.07	
5	486.74	432.95	312.85	492.43	723.85	239.57	
6	728.74	483.96	472.58	439.17	396.14	211.64	
7	385.75	539.59	542.87	734.67	723.96	447.92	
8	648.57	823.85	514.29	439.76	824.57	238.11	
9	923.37	239.73	629.85	329.54	385.47	294.10	
10	473.48	865.94	518.59	172.39	749.80	663.19	
Total							Grand Total

Problems

Enter the items of the following problems on suitable ruled forms and find the totals as required.

1. In a four day contest to raise funds for charitable purposes the four contesting teams reported the following collections: Compute the

daily totals, the totals for each team for the week and the grand total. Team A: Monday, $1,436; Tuesday, $798; Wednesday, $1,748; Thursday, $1,493. Team B; Monday, $2,473; Tuesday, $1,536; Wednesday, $894; Thursday, $1,592. Team C: Monday, $480; Tuesday, $1,735; Wednesday, $1,849; Thursday, $2,178. Team D: Monday, $485; Tuesday, $1,340; Wednesday, $968; Thursday, $1,745.

2. The daily sales for a week in a department store were as follows:

Shoes, $485.50, $792, $535.50, $815.25, $245, and $380.
Furniture, $1,845, $1,455, $2,580, $960, $3,590, and $1,650.
Groceries, $580.40, $965, $740.30, $380, $875.35, and $760.80.
Hardware, $495.25, $815,30, $748.60, $614.85, $786.30, and $514.85.
Clothing, $985, $1,465.50, $1,730.50, $984.50, $658, and $1,149.50.
Notions, $97.40, $132.80, $218.45, $114.15, $112.50, and $242.30.

Compute the total sales for each day for the store, the total sales of each kind for the week, and the grand total for the week.

It is of the utmost importance that business computations be correct. The computer must check his own work. He must know that his results are accurate. The *habit* of checking all computations should be formed.

There are several methods by which addition may be checked. Some of these checks have been noted. One of the common checks is to add each column twice and in reverse order. *Never try to check by adding in the same order.*

Checking Addition by Casting Out Nines

6. Excess of Nines. The remainder after dividing any number by 9 is called the *Excess of Nines.* If the number 47 is divided by 9, the excess of nines is 2. The excess of nines in 30 is 3, because when 30 is divided by 9, there is a remainder of 3. When we " cast out nines " from a number, we divide the number by 9, and indicate the excess.

What is the excess of nines in 17? 14? 39? 45? 117? 23?

7. Method of Checking. If two numbers, each exactly divisible by 9, are added, their sum also is divisible by 9. If one number with an excess of 3 is added to a number with an excess of 2, the sum will have an excess of 5. *The excess of nines in a sum is equal to the excess in the sum of the excesses of the numbers added.*

Examples. Add 30 and 19; check the result by casting out nines.

SOLUTION: 30 + 19 = 49.
The excess of nines in the sum, 49, is 4.
The excess in 30 is 3;
The excess in 19 is 1.
The sum of the excesses of the numbers added is 4, therefore the addition checks.

For convenience, the work may be arranged as follows:

	Numbers Added	Excesses
a.	30	3
	19	1
Sum	49	4
Excess of sum	4	

	Numbers Added	Excesses
b.	23	5
	13	4
Sum	36	9
Excess of sum	0	0

	Numbers Added	Excesses
c.	49	4
	34	7
Sum	83	11
Excess of sum	2	2

8. Short Method of Finding the Excess. The excess of nines in any number may be found by the following method:

Add the digits composing the number. If this sum is composed of two or more digits add them. Continue until a result of one digit is secured.

Examples. 1. What is the excess of nines in 25?

SOLUTION: 2 + 5 = 7, the excess.

2. What is the excess of nines in 328?

SOLUTION: 3 + 2 + 8 = 13
1 + 3 = 4, the excess.

The addition of digits to find the excess will be further simplified by observing the following suggestions:

a. Ignore 9's, and combinations which add to 9 or multiples of 9.

Example. What is the excess of nines in 9,457?

SOLUTION: Ignore 9 and 5 + 4; the excess is 7.

b. When the addition results in a number of two digits, add the digits and proceed as before.

Example. What is the excess of nines in 7,528?

SOLUTION: $7 + 5 = 12$; $1 + 2 = 3$;
$3 + 2 + 8 = 13$; $1 + 3 = 4$, the excess.

9. Limitations of the Method. Casting out nines is not an absolute check for any process. It will not detect an error or omission of 9 or any multiple of 9, neither will it disclose an interchange of digits, such as 763 for 673. If a result is wrong in such a way as to leave the excesses unchanged, this method of checking will not detect the error.

Written Work

1. Add and check:

NUMBERS TO BE ADDED	EXCESSES
213	?
412	?
617	?
362	?
145	?
215	?
829	?
657	?
923	?
?	?

Add and check by casting out the 9's.

2.	3.	4.	5.
4,796	1,436	7,029	48,987
8,342	9,087	8,516	72,806
7,987	2,635	3,094	39,081
3,246	4,097	7,290	79,420
2,874	386	8,096	63,945

CHECKING BY CASTING OUT ELEVENS

The method of checking addition by casting out elevens will be clear from the following:

The excess of elevens in any number is computed by finding the difference between the sum of the digits in the odd and in the even

places in the number. Thus, the excess of 11's in 47,853 is 15 − 12 or 3 (15 is the sum of the digits in the odd orders, that is, the 1st, 3d, etc., and 12 is the sum of the digits in the even orders). If the sum of the digits in the even orders exceeds the sum of those in the odd orders, add 11, or a multiple of 11, to the sum of the digits in the odd orders. Thus, the excess of 11's in 9,354 is

$$\begin{aligned} 4 + 3 + 11 &= 18 \\ 5 + 9 &= 14 \\ \text{Excess} &\quad 4 \end{aligned}$$

The method is further illustrated by the following:

Number	Excess
478	5
397	1
2,436	5
9,854	9
13,165	20

The excess in the sum (13,165) and in the sum of the excesses (20) is 9.

Add and check by casting out 11's.

1.	2.	3.	4.
7,286	2,908	8,079	19,463
3,974	4,736	3,897	23,879
8,706	7,895	6,543	92,785
4,398	6,789	7,897	43,970
2,764	3,470	3,788	87,698

Add and check by casting out 11's.

5.	6.	7.	8.
1,846	9,086	4,987	894
973	3,908	3,896	1,798
8,427	7,463	7,298	3,984
9,208	9,287	8,407	5,049
764	438	328	7,648
3,792	9,986	1,476	3,507

9. Rule a suitable blank form for the following: At the right of each column of figures rule a narrow column in which to indicate the excesses.

Find the totals as indicated; check additions by casting out nines or elevens.

VALUE OF FARM LANDS, BUILDINGS, AND IMPLEMENTS IN THE UNITED STATES IN A RECENT YEAR

DIVISION	LAND	EX.	BUILDINGS	EX.	IMPLEMENTS	EX.	TOTAL	EX.
New England . . .	382,134,424		336,410,384		50,798,826			
Middle Atlantic. .	1,462,321,005		980,628,098		167,480,384			
E. N. Central. . .	7,231,699,114		1,642,292,480		268,806,550			
W. N. Central . .	10,052,560,913		1,562,104,957		368,635,544			
South Atlantic . .	1,883,349,675		603,086,799		98,230,147			
E. S. Central . . .	1,326,826,864		411,570,975		75,339,333			
W. S. Central. . .	2,716,098,530		412,498,352		119,720,377			
Mountain	1,174,370,096		145,026,777		49,429,975			
Pacific	2,246,313,548		231,832,706		66,408,647			
U. S. Totals . .							Grand Total	

10. The following table shows a method used by department stores to determine the cost of the purchases made for various departments. For example, Invoice No. 1, purchased on January 2, included goods for the six departments of the store. Invoice No. 2 included goods for only three departments.

The total amount of the invoice may be found by horizontal addition.

The total purchases for each department may be found by vertical addition.

Rule a blank suitable for this material, enter the data, and find:

a. The total of each invoice.

b. The total purchases for each department.

c. The total purchases for all departments.

INVOICE NO.	DATE		DEPT. 1	DEPT. 2	DEPT. 3	DEPT. 4	DEPT. 5	DEPT. 6	TOTAL
1	Jan.	2	129 30	340 60	290 39	360 89	240 30	376 80	
2	Jan.	3	276 90		347 80			174 44	
3	Jan.	3		295 67	298 87	350 27	289 67	287 57	
4	Jan.	5	246 32	238 47	350 27	347 25	250 68	276 57	
5	Jan.	6	231 53	350 21		378 89		190 23	
6	Jan.	8	212 23	325 67	345 67	367 78	310 57	185 67	
7	Jan.	8	227 85		346 78	357 89	298 67		
8	Jan.	10						170 67	
9	Jan.	11	160 87	325 67	287 67	367 57	287 00	189 00	
10	Jan.	15	170 97	341 28	321 67	378 98	287 89		
11	Jan.	15	180 89		333 33		278 90	234 86	
12	Jan.	15		345 67	345 67	328 98	300 00	350 90	
Total									

General Review in Addition

Oral Work

1. *a.* $27 + 32 + 14 + 43 + 77$
   b. $73 + 24 + 18 + 39 + 56$
   c. $64 + 29 + 54 + 46 + 18$
   d. $78 + 56 + 47 + 54 + 36$

2. What do you consider the best method for checking addition? Why?

3. Five salesmen for Williams & Company reported daily sales as follows. Compute the total daily sales, the total sales for the week, and the grand total.

Salesman	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Total
M. R. Jones . . .	$248.90	$368.40	$790.30	$240.80	$179.60	$280.90	
J. B. Adams . . .	142.00	263.90	384.20	78.60	324.80	291.40	
S. W. Evans . . .	375.60	264.80	372.95	147.80	265.30	282.90	
A. R. Clark . . .	246.10	312.25	417.19	541.60	324.80	197.60	
L. C. Briggs . . .	472.30	198.70	216.45	325.65	287.30	214.35	

4. The receipts at a retail grocery on Monday were $243.80; Tuesday, $196.50; Wednesday, $342.64; Thursday, $146.90; Friday, $215.80; and Saturday, $280.40. What were the total receipts for the week?

5. The population of the largest ten cities in the United States in 1920 was as indicated below. Compute the total population of these cities.

City	Population	City	Population
New York	5,621,151	St. Louis	772,897
Chicago	2,701,705	Boston	748,060
Philadelphia	1,823,158	Baltimore	733,826
Detroit.	993,739	Pittsburgh.	588,193
Cleveland	796,836	Los Angeles	576,673

CHAPTER II

SUBTRACTION

10. Drill Tables. Daily practice on the following subtraction combinations will increase your speed and accuracy.

Drill on these combinations until you can state all of the results in any order in less than thirty seconds.

9	8	7	9	6	7	4	7	3	6	8	7	8	4	5	8
2	1	5	3	1	2	4	4	3	6	4	3	8	2	1	3
2	7	9	5	9	5	6	5	9	4	4	9	8	2	6	8
1	7	4	5	5	4	3	2	7	4	3	8	5	2	4	6
5	8	4	8	6	7	7	9	8	1	3	3	5	7	9	6
3	8	1	6	2	1	6	1	2	1	2	1	5	7	6	5

Drill on the following until you can state all of the results in any order in less than thirty seconds.

16	14	15	18	13	17	12	11	16	11	12	17	11
8	6	8	9	4	9	5	9	7	6	6	8	4
13	12	15	16	11	12	12	12	11	13	15	11	12
5	3	6	9	3	3	8	7	7	9	7	8	9
11	12	13	14	14	13	14	15	14	11	13	17	18
5	4	7	8	7	6	5	9	9	2	8	6	7

In subtraction "borrowing" is frequently necessary.

Place a pad under these examples, and make the subtractions.

1.	2.	3.	4.	5.	6.
479	536	4,279	14,973	3,084	12,379
284	198	3,682	8,297	1,596	3,084

7.	8.	9.	10.	11.
42,937	946,320	37,649	947,302	794,680
39,048	298,452	20,863	842,943	409,694

11. Checking Subtraction. Subtraction may be conveniently checked by either of the following methods:

(*a*) Difference + Subtrahend = Minuend.
(*b*) Minuend − Difference = Subtrahend.

Example. Subtract 21 from 78.

Solution.		Check. *a.*		*b.*	
78	Minuend	57	Difference	78	Minuend
21	Subtrahend	21	Subtrahend	57	Difference
57	Difference	78	Minuend	21	Subtrahend

Written Work

Perform the subtractions indicated; check the first five by method (*a*), and the last five by method (*b*),

1.	2.	3.	4.	5.
489	7,932	82,578	63,217	49,216
392	494	58,214	41,283	21,953

6.	7.	8.	9.	10.
58,310	40,513	53,821	79,608	184,732
49,168	39,458	41,927	20,973	75,806

Always check results in substraction.

11. The following data show the sales in the various departments of a large store for the month of August, 1921, and the month of August, 1922.

Rule a blank similar to the following:

COMPARATIVE SALES RECORD

DEPARTMENT No.	AUGUST, 1921	AUGUST, 1922	INCREASE OR DECREASE
1	$ 5,483.85	$ 5,834.75	
2	7,239.74	8,238.74	
3	15,493.67	14,835.92	
4	9,834.56	10,385.47	
5	23,842.12	22,147.94	
6	15,858.43	16,614.34	
7	6,395.75	6,114.32	
8	19,432.86	22,324.73	
9	9,445.87	10,302.95	
10	43,221.42	46,932.56	
11	29,496.56	31,416.47	
12	18,853.16	17,542.85	
13	22,542.19	25,436.83	
14	25,746.91	24,427.85	
15	18,422.16	21,541.86	
Total			

Enter the sales in the proper column of the blank.

Find the increase or decrease in the amount of the sales in each department for August, 1922, over the sales in the same department for August, 1921.

Enter increases in black ink; decreases in red ink.

Find the net increase for the entire store.

Do the results check?

12. The following shows the total population of the United States for the years indicated.

1790 — 3,929,214; 1840 — 17,069,453; 1890 — 62,947,714;
1800 — 5,308,483; 1850 — 23,191,876; 1900 — 75,994,575;
1810 — 7,239,881; 1860 — 31,443,321; 1910 — 91,972,266;
1820 — 9,638,453; 1870 — 38,558,371; 1920 — 105,708,771.
1830 — 12,866,020; 1880 — 50,155,783;

Rule a blank form to show the year the census was taken, the population each census year, and the increase in population for each interval of 10 years.

See how simple you can make this blank. Check the results.

13. Rule a form, enter the following statistics, find the gross profit of each department, and the gross profit of the entire store.

GROSS PROFITS OF A DEPARTMENT STORE

DEPT. NO.	SALES	COST OF GOODS SOLD	GROSS PROFIT
1	$ 5,629.80	$4,984.37	
2	7,358.92	6,295.16	
3	4,916.09	4,192.86	
4	7,329.16	6,593.54	
5	10,609.15	9,835.17	
6	6,123.18	5,902.52	
7	7,212.47	6,275.43	
8	9,475.37	8,594.48	
9	4,238.16	3,725.91	
10	5,824.78	5,014.78	
Total			

14. Rule a suitable form, enter the following statistics, and find:

a. The gross profit by departments.

b. The gross profit of the entire store (two ways).

c. The net profit or net loss by departments; enter net profit in black, net loss in red.

d. Net profit or net loss of the entire store (two ways).

DEPT. NO.	SALES	COST OF GOODS SOLD	GROSS PROFIT	EXPENSES	NET PROFIT OR LOSS
1	$7,816.40	$6,715.32		$423.85	
2	9,317.42	8,492.17		530.08	
3	6,842.56	5,914.72		731.96	
4	7,319.62	6,593.57		671.42	
5	8,295.17	7,942.17		456.72	
6	5,732.88	5,101.59		322.75	
7	9,514.82	8,899.55		693.15	
8	2,289.74	1,856.80		216.77	
9	5,793.66	5,135.60		788.51	
10	9,559.38	8,625.50		523.80	

12. Subtracting by Adding Complements. A series of additions and subtractions may be performed by the method of adding complements.

The *complement* of a number is the number which must be added to get the next higher power of ten. Thus, the complement of 6 is 4; the complement of 83 is 17.

If, instead of subtracting a number less than ten from a given number, its complement be added, the result will be 10 too large. Thus, $13 - 6 = 7$ or $13 + 4 = 17$ (a result 10 too large).

If, instead of subtracting *two* numbers less than ten from a given number, their complements are added, the result will be 20 too large; if the complements of three such numbers are added, the result will be 30 too large, etc.

Examples. 1. $29 - 6 - 7 = ?$

SOLUTION. $29 + 4 + 3 = 36$. Since two complements were added, the result is 20 too large. Therefore, subtract 20, leaving 16.

2. $38 - 4 - 7 - 8 = ?$

SOLUTION. $38 + 6 + 3 + 2 = 49$ Since three complements were added, subtract 30, leaving 19.

3. $46 - 8 + 4 - 7 = ?$

SOLUTION. $46 + 2 + 4 + 3 = 55$. Since two complements were added, 20 must be subtracted, leaving 35.

The practical value of this method will be shown by the following examples.

1. $36 - 24 + 19 - 12 + 21 - 13 - 6 = ?$

SOLUTION. By combining as indicated in the units' column, beginning at the top (complements are marked with an *) we have

$$6 + 6^* + 9 + 8^* + 1 + 7^* + 4^* = 41$$

Since four complements were added, the result is 40 too large. Therefore write 1 and drop the 4.

By combining as indicated in the tens' column we have

$$3 + 8^* + 1 + 9^* + 2 + 9^* = 32$$

Since three complements were added, the result is 30 too large. Write 2 and drop the 3.

Result, 21.

$$\begin{array}{r} 36 \\ -\ 24 \\ 19 \\ -\ 12 \\ 21 \\ -\ 13 \\ -\ 6 \\ \hline 21 \end{array}$$

2. $985 + 234 - 126 - 34 - 125 - 174 + 386 = ?$

985
234
− 126
− 34
− 125
− 174
386
1,146

SOLUTION. Combining in the units' column, beginning at the top we have

$$5 + 4 + 4^* + 6^* + 5^* + 6^* + 6 = 36$$

Write the 6. Since four complements were added, the result is 40 too large. Therefore we must drop the 3 in 36, and also subtract 1 from the 8 at the top of the tens' column.

Combining in the tens' column we have

$$7 + 3 + 8^* + 7^* + 8^* + 3^* + 8 = 44$$

Since four complements were added, we must deduct 40, leaving 4 which is written in the tens' column of the result.

Combining in the hundreds' column we have

$$9 + 2 + 9^* + 9^* + 9^* + 3 = 41$$

Three complements were added, the result is therefore 30 too large. Subtract 30 and write 11.

Result, 1146.

The complement method is valuable when an occasional subtraction is to be made in work which consists largely of addition. Subtractions are made on the adding machine by adding the complement of the number to be subtracted, and deducting one from the next higher order.

Find the results by the addition of complements:

1. $46 - 4 - 7 =$
2. $72 - 8 - 6 - 3 =$
3. $39 - 7 + 3 - 9 =$
4. $48 - 17 - 6 =$
5. $81 - 23 - 42 =$
6. $49 - 22 + 15 - 11 =$
7. $85 - 26 - 35 + 62 - 16 =$
8. $643 - 289 + 364 =$
9. $781 - 247 + 64 =$
10. $1046 - 987 + 649 =$

11.	12.	13.
2,346	1,932	923,578
+ 1,267	+ 9,845	− 5,284
− 321	− 932	+ 28,956
+ 6,964	+ 2,122	− 123,749
− 1,235	− 2,375	+ 83,219
− 2,960	+ 692	+ 125,982
− 985	− 1,243	− 429,764

Written Work

1. The following model shows the ruling of a ledger in which a bank keeps its accounts with depositors. Deposits are added to the balance of the previous day, and checks are subtracted, to find the new balance to the credit of the depositor's account.

Find the daily balances:

WILLIAM HATFIELD

DATE	DEPOSITS	CHECKS	BALANCE
June 1			$328.57
	$ 85.68	$ 14.25	?
2	125.80	65.90	
		12.73	?
3	245.85	127.89	
		114.56	
		35.87	?
4	319.45	38.51	
	26.82	102.50	?
5		95.90	
		141.66	
		95.68	?
6	450.00	132.88	
		6.20	?
8	139.75	216.45	
		39.46	
		19.99	
		213.55	
		66.82	?

2. The following blank shows a convenient method of keeping a record of cash received and paid by a small business.

Rule a blank similar to the model and enter the data.

DATE	CASH RECEIVED		CASH PAID			DAILY CASH
	Cash Sales	On Account	Purchases	On Account	Expenses	Balance
Feb. 1						$216.27
1	$215.78	$124.35	$ 80.75	$ 68.55	$ 8.75	?
2	294.80	95.88	157.47	113.25	3.73	?
3	188.47	148.23	95.48	316.57	27.49	?
4	218.43	49.47	17.49	42.39	9.12	?
5	388.92	112.67	221.81	76.29	75.84	?
6	478.07	290.04	88.12	448.14	6.90	?
	?	?	?	?	?	?

a. Find the daily cash balances.

To the balance of the preceding day add the receipts from cash sales and receipts on account, and subtract the various amounts listed under "Cash Paid."

Thus, \$216.27 + 215.78 + 124.35 − 80.75 − 68.55 − 8.75 = ?

b. Find the totals for each column.

3 Add upward; subtract across. Do the results check?

a.			*b.*		
7,463	− 2,847	=	174,638	− 94,273	=
5,928	− 3,804	=	38,270	− 27,409	=
9,604	− 2,870	=	70,563	− 2,879	=
3,962	− 1,436	=	924,360	− 14,287	=
9,287	− 5,426	=	7,503	− 2,769	=
	−	=		−	=

In weighing articles contained in bags, barrels, or any container whose weight is not to be included, the total weight is called the *gross weight* and the amount to be deducted is called *tare.* The difference between gross weight and tare is called *net weight.*

Thus, if the gross weight of a wagon loaded with wheat is 3,840 pounds and the weight of the wagon is 1,360 pounds, the net weight is 2,480 pounds.

4. From a certain farm the following loads of wheat were hauled to an elevator. The gross weight includes the weight of the wagon. Compute the number of bushels of wheat hauled. Gross weight 3,840, tare 1,380; gross weight 3,960, tare 1,420; gross weight 4,285, tare 1,490; gross weight 4,180, tare 1,410; gross weight 4,585, tare 1,450.

Rule blank forms, enter the data, and make the required computations in the first two problems below.

1. On September 22, Adams and Johnson opened business with \$242.18 cash. The receipts and payments during the day were as follows: Receipts, \$42.50, \$18.30, \$112.90, \$5.80, \$11.48, \$79.43, \$106.35, \$25.40, \$10.80. Payments, \$38.50, \$109.45, \$6.40, \$2.50, \$71.95, \$14.60, \$165.80. How much cash did they have at the end of the day?

2. On October 3d George Scott had a bank balance of \$432.85. The following checks were drawn and deposits made. Checks, October 4, \$82.40, \$41.50; October 5, \$114.35; October 11, \$145.90; October 12, \$5.60, \$8.45, \$7.50; Deposits, October 5, \$94.80; October 6

$124.10, October 8, $34.67, $118.11; October 10, $14.92, $103.87. What was his balance on October 12th?

3. A contractor is to remove 17,840 cubic yards of earth. During 9 weeks he removes the following number of cubic yards: 482; 697; 365; 495; 672; 543; 748; 836; 765. How many cubic yards are still to be removed?

4. A wheat elevator contains 242,800 bushels of wheat. The following number of bushels are taken from the elevator and loaded into cars: 1,520, 1,430, 1,465, 1,560, 1,610, 1,490, 1,572, 1,463, 1,524, 1,585, 1,605, 1,425, 1,495, 1,580, 1,535. How many bushels of wheat are left in the elevator?

5. The following table gives the population of certain states for 1910 and 1920. Compute the increase in each by horizontal subtraction and add the increases. Devise a method for checking the totals.

	1910	1920	Increase
California	2,377,549	3,426,861	
Georgia	2,609,121	2,895,832	
Kansas	1,690,949	1,769,257	
Missouri	3,293,335	3,404,055	
New York	9,113,614	10,384,829	
Ohio	4,767,121	5,759,394	
Pennsylvania	7,665,111	8,720,017	
Total			

General Review of Addition and Subtraction

Subtract and check results.

1. 47,806 30,597	2. 70,409 38,763	3. 18,947 3,879	4. 90,260 83,976
5. $879.40 438.90	6. $1,878.30 1,499.80	7. $2,840.36 1,092.09	8. $4,086.50 2,989.70

9. The following table gives the population of the twelve largest cities of the United States in 1920 and the population of these cities in 1910. Compute the increase for each city, the total increase, and the total population of the twelve cities for 1920 and 1910.

City	1910	1920	Increase
New York	4,766,883	5,621,151	
Chicago	2,185,283	2,701,705	
Philadelphia	1,549,008	1,823,158	
Detroit	465,766	993,739	
Cleveland	560,663	796,836	
St. Louis	687,029	772,897	
Boston	670,585	748,060	
Baltimore	558,485	733,826	
Pittsburgh	533,905	588,193	
Los Angeles	319,198	576,673	
San Francisco	416,912	508,410	
Buffalo	423,715	506,775	
Total			

10. State the difference between each pair of numbers, as 14 and 87.

a	14	87	46	92	86	92	39	82	80	47	28	96
b	45	64	97	83	27	69	63	78	92	86	47	39
c	47	82	26	31	43	72	74	91	37	49	87	46
d	39	91	84	37	81	46	87	98	87	78	39	14

11. Add each pair of numbers, in the above list, as 14 and 87.

12. What change should be given for a five-dollar bill if the following purchases were made?

a.	$1.54	*e.*	$ .87	*i.*	$ .49	*m.*	$1.40
b.	1.30	*f.*	.45	*j.*	1.16	*n.*	.82
c.	1.18	*g.*	1.72	*k.*	1.58	*o.*	.15
d.	.95	*h.*	1.18	*l.*	.83	*p.*	1.28

13. Find the results by the complement method:

a. 142 + 187 + 274 − 85 − 28
b. 948 + 738 − 460 + 895
c. 348 + 749 − 685 + 480
d. 1,546 + 1,197 − 1,430 + 840

14. A real estate agent bought a tract of land containing 100,000 square feet. He sold lots containing the following number of square

feet: 9,000, 9,750, 10,500, 7,500, 15,000, 12,000, 12,300. How many square feet of the tract were not sold?

15. Add orally:

a. $34 + 62 + 18 + 37 + 43 + 29$
b. $83 + 26 + 42 + 39 + 72 + 43$
c. $25 + 34 + 62 + 37 + 46 + 82$
d. $63 + 38 + 41 + 64 + 55 + 27$

16 Make a problem involving addition of at least five numbers of four digits each and check the result by casting out nines.

17. Make another problem as required above and check the result by casting out elevens.

18. State several methods for checking addition. Which method do you *habitually* use for checking column addition? Why?

CHAPTER III

MULTIPLICATION

ONE engaged in business uses multiplication frequently, and the efficient clerk and accountant must have acquired speed and accuracy in the process.

The student has learned the multiplication tables to 10×10 or to 12×12. The following is the multiplication tables to 15×15. This should be thoroughly mastered.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225

NOTE. Drill on the more difficult combinations until they are thoroughly mastered.

The result obtained by multiplication is called the *product*. The multiplicand and the multiplier are called the *factors* of the product. The product divided by either factor gives the other factor.

Written Multiplication

Multiply 387 by 426:

$$\begin{array}{r} 387 \\ 426 \\ \hline 2322 \\ 774 \\ 1548 \\ \hline 164862 \end{array}$$

Multiply 426 by 387:

$$\begin{array}{r} 426 \\ 387 \\ \hline 2982 \\ 3408 \\ 1278 \\ \hline 164862 \end{array}$$

It is apparent from the illustration above that the product is not altered if the order of the factors is reversed. This method is sometimes used in checking multiplication.

Make the following multiplications and add the products. Add the multiplicands and multiply this total by 473.

1.	2.	3.	4.
3,842	3,097	9,270	13,768
473	473	473	473

Make the following multiplications and add the products. Add the multiplicands and multiply this total by 709.

5.	6.	7.	8.
3,846	1,908	4,362	2,940
709	709	709	709

NOTE. The teacher is urged to assign several examples similar to those suggested above, as drill in multiplication.

13. Drill Tables.

Multiply across; add upward:

1.	2.	3.	4.
$74 \times 436 =$	$83 \times 423 =$	$25 \times 624 =$	$35 \times 624 =$
$74 \times 523 =$	$83 \times 157 =$	$25 \times 726 =$	$35 \times 706 =$
$74 \times 287 =$	$83 \times 284 =$	$25 \times 37 =$	$35 \times 753 =$
$74 \times 492 =$	$83 \times 307 =$	$25 \times 869 =$	$35 \times 496 =$
$74 \times ? =$	$83 \times 596 =$	$25 \times 493 =$	$35 \times 548 =$
	$83 \times ? =$	$25 \times 468 =$	$35 \times 784 =$
		$25 \times ? =$	$35 \times ? =$

Oral Work

Use 2, 3, 4, 5, 6, 7, 8, and 9 as multipliers. State the results for each column in less than 20 seconds.

1.	2.	3.	4.	5.
3	18	5	4	9
5	10	8	9	11
7	4	12	15	16
14	7	15	20	30
11	9	7	17	25
6	15	30	8	14
8	16	22	13	17
9	20	16	18	11
15	12	25	22	16

Written Work

1. A factory made an investigation of the number of articles of a certain kind manufactured by each of its employees. It found that:

9 men produced 46 articles each.
9 men produced 48 articles each.
11 men produced 53 articles each.
15 men produced 55 articles each.
18 men produced 59 articles each.
23 men produced 61 articles each.
25 men produced 62 articles each.
25 men produced 63 articles each.
21 men produced 64 articles each.
17 men produced 65 articles each.
17 men produced 66 articles each.
14 men produced 68 articles each.
12 men produced 69 articles each.
8 men produced 70 articles each.

Rule a form with a heading similar to the following:

PRODUCTION RECORD

NUMBER OF MEN	NUMBER OF ARTICLES MADE BY EACH	TOTAL

Enter the statistics, find the number of articles produced by each group of employees, and the total number of articles produced by all of the employees.

2. Nine workmen were employed in the manufacture of different articles, and were paid a certain number of cents for each piece completed. Complete the following table, finding the wages earned by each workman.

DAILY PIECEWORK LABOR COST

WORKMAN No.	NUMBER OF ARTICLES MADE	WAGE RATE PER PIECE	WAGES EARNED
1	17	$0.27	
2	19	.22	
3	38	.16	
4	79	.06	
5	32	.15	
6	28	.18	
7	64	.07	
8	81	.05	
9	29	.21	

14. Checking Multiplication. Multiplication may be checked by several methods. The following methods are commonly used.

a. Repeating the multiplication and assuming that if the same product is obtained the work is correct. This is not a reliable check because an error is likely to be repeated.

b. Interchanging multiplicand and multiplier.

c. Dividing the product by the multiplier to obtain the multiplicand, or by the multiplicand to obtain the multiplier.

d. Casting out nines.

e. Casting out elevens.

15. Casting Out Nines. The method is as follows: *Find the excess of nines in the multiplicand and in the multiplier. Find the product of these excesses. Find the excess of nines in this product. It should equal the excess of nines in the result.*

Example. Multiply 23 by 16; check by casting out nines.

SOLUTION. 23 5 = the excess of nines in 23.
16 7 = the excess of nines in 16.
368 35 = the product of these excesses.
CHECK. 8 = the excess in this product.
The excess of nines in 368 is also 8. The multiplication, therefore, checks.

16. Casting Out Elevens. The check is illustrated below.

1874 4
976 8
11244 32 10
13118
16866
1829024 10

Written Work

Without performing the multiplications determine the probable correctness of the following products, by means of casting out nines:

1.	2.	3.	4.	5.	6.	7.
25	82	36	286	4,172	344	733
38	35	87	37	39	281	492
950	2,870	3,132	10,382	162,698	96,664	362,636

8.	9.	10.
398	43,962	34,276
241	47,835	21,578
95,918	2,102,922,270	738,627,528

Multiply and check by casting out elevens:

11.	12.	13.	14.	15.	16.
347	279	627	132,879	63,154	893,567
861	439	123	2,378	9,837	17,839

17. One square mile contains 640 acres. How many acres in each of the following states?

State	Area in Square Miles
Rhode Island	1,248
New York	49,204
Pennsylvania	45,126
Maryland	12,327
Georgia	59,265
Ohio	41,040
Illinois	56,665
Minnesota	84,682
Colorado	103,948
California	158,297

If your state is not included in the above list, compute the number of acres in it.

18. A book of 294 pages contains an average of 348 words to a page. How many words are in the book?

19. Each page of the book contains an average of 34 lines. How many lines are in the book?

20. A certain field yielded 52 bushels of corn to the acre. How many bushels did the field of 19 acres yield?

21. Determine the present retail prices of each of the following and compute the total cost.

7 lb. coffee	17 lb. chocolate	25 lb. flour
100 lb. sugar	11 lb. cocoa	12 lb. navy beans
2 lb. raisins	21 lb. rice	14 bu. potatoes
4 lb. tea	13 lb. starch	

22. Determine the present retail price of each of the following and compute the total cost.

17 yd. percale	18 yd. flannel
13 yd. denim	17 yd. muslin (unbleached)
12 yd. madras	18 yd. cheese cloth

23. Multiply each of the following numbers by 347, then add the multiplicands and multiply this sum by 347. Does the result check?

847 1,926 427 893

24. How many of these examples in multiplication can you copy and perform in six minutes?

a. 7,480 392	*b.* 1,706 927	*c.* 4,957 307	*d.* 2,759 629
e. 7,986 640	*f.* 9,043 792	*g.* 8,739 708	*h.* 7,382 547
i. 9,086 429	*j.* 6,980 609	*k.* 3,708 473	*l.* 9,287 654

25. Check each product in example 24.

26. What method do you *habitually* use in checking multiplication? Why?

Short Methods of Multiplication

These short methods will be found to be very practical. Master two or three of them thoroughly before taking up others. Use those that you have mastered whenever you have opportunity to do so. It is not necessary that all of these short methods be mastered before the succeeding chapters are studied.

17. To multiply by 10, 100, 1,000, 10,000, etc.

a. When the multiplicand is an integer. *Annex to the multiplicand as many zeros as there are zeros in the multiplier.*

Thus, to multiply an integer by 10, annex one zero; to multiply by 100, annex two zeros.

Examples. 1. $37 \times 10 = 370$. 2. $29 \times 100 = 2{,}900$.

b. When the multiplicand is a decimal fraction. *Move the decimal point as many places to the right as there are zeros in the multiplier.*

It may be necessary to annex zeros in order to move the decimal point the desired number of places.

Examples. 1. Multiply .1357 by 100.

SOLUTION. Move the decimal point two places to the right, 13.57.

2. Multiply 32.46 by 1000.

SOLUTION. In order to move the decimal point three places to the right it is necessary to annex one zero. The result is 32,460.

Oral Work

Multiply as indicated:

1. $37{,}946 \times 100$.
2. $5{,}293 \times 10{,}000$.
3. $639 \times 100{,}000$.
4. $120 \times 1{,}000$.
5. $.376 \times 1{,}000$.
6. $1.349 \times 1{,}000$.
7. 27.9637×100.
8. $.00932 \times 10$.
9. $.0873 \times 1{,}000$.
10. $.7032 \times 1{,}000$.
11. $3.69 \times 1{,}000$.
12. $.0027 \times 100{,}000$.
13. $1{,}427.834 \times 10{,}000$.
14. $625.086 \times 1{,}000$.

18. To multiply numbers ending with zeros.

a. When both numbers are integers. *Multiply the numbers represented by the significant figures. To the product thus obtained, annex as many zeros as there are final zeros in both the multiplicand and multiplier*

Example. Multiply 3,400 by 1,200.

SOLUTION. $34 \times 12 = 408$. Annexing four zeros, we obtain the product 4080000.

Perform the following multiplications. Whenever possible, do the work orally.

1. 169×300.
2. 210×300.
3. $4{,}567 \times 700$.
4. $1{,}390 \times 1{,}200$.
5. $2{,}300 \times 1{,}500$.
6. $19{,}000 \times 16$.
7. $3{,}194 \times 23{,}000$.
8. $420 \times 3{,}400$.

b. When one of the numbers is a decimal fraction. *Multiply the numbers represented by the significant figures. Move the decimal point as many places to the right in the product as there are final zeros in the integer.* (This may necessitate annexing zeros.)

Examples.

1. Multiply .486 by 300.

SOLUTION. $3 \times .486 = 1.458$.
Move the decimal point two places to the right, the result is 145.8.

2. Multiply 3.2 by 400.

SOLUTION. $4 \times 3.2 = 12.8$.
Move the decimal point two places to the right, the result is 1280.

Written Work

Perform the following multiplications:

1. $.47 \times 200$.
2. $3.786 \times 4{,}000$.
3. 17.682×500.
4. $.0746 \times 3{,}000$.
5. $.072 \times 6{,}000$.
6. $.382 \times 1{,}200$.
7. $.0837 \times 14{,}000$.
8. $.042 \times 170$.
9. $.0036 \times 1{,}500$.
10. $4.26 \times 7{,}000$.

19. To multiply by 9, 99, 999, etc.

a. To multiply by 9. *Annex one zero to the number to be multiplied, thus multiplying it by* 10; *from this result subtract the number to be multiplied.*

Example. Multiply 346 by 9.

SOLUTION.

$$\begin{array}{r} 3460 \\ 346 \\ \hline 3114 \end{array}$$

b. To multiply by 99. *Annex two zeros to the number to be multiplied, thus multiplying it by* 100; *from this result subtract the number to be multiplied.*

Example. Multiply 293 by 99.

SOLUTION.

$$\begin{array}{r} 29300 \\ 293 \\ \hline 29007 \end{array}$$

Writ en Work

Multiply each of the following numbers by 9, 99, and 999:

1. 632. **2.** 748. **3.** 135. **4.** 737. **5.** 427. **6.** 166.

20. To multiply by numbers slightly smaller than 10, 100, 1,000, 10,000, etc.

A modification of the short method explained in the preceding section may be used to multiply by numbers slightly smaller than 10, 100, 1,000, 10,000, etc.

To multiply by 98. *Annex two zeros to the number to be multiplied and from this result subtract twice the number to be multiplied.*

How can the short method be used if you are to multiply by 97, 96, 95, or 998?

Written Work

Apply short methods to the following examples:

1. 675×96.	**2.** 350×95.	**3.** 535×91.
4. 687×97.	**5.** 94×3.4.	**6.** 995×82.
7. 634×994.	**8.** $.48 \times 997$.	**9.** 23×99.
10. 48×997.	**11.** 12×988.	**12.** 47×998.

21. To multiply by 11.

a. When the multiplicand contains two digits.

Place between these two digits, their sum.

Example. $34 \times 11 = 374$.

When the sum of the two digits is 10 or more, 1 must be carried to the digit at the left.

Example. $68 \times 11 = 748$.

Written Work

Multiply each of the following numbers by 11:

1. 27.	**2.** 63.	**3.** 93.	**4.** 74.	**5.** 26.	**6.** 35.
7. 22.	**8.** 87.	**9.** 28.	**10.** 46.	**11.** 75.	**12.** 96.
13. 37.	**14.** 36.	**15.** 57.	**16.** 85.	**17.** 98.	**18.** 72.

b. When the multiplicand contains three or more digits:

The units' digit of the multiplicand is the units' digit of the product; the sum of the units' and tens' digits is the tens' digit of the product; the sum of the tens' and hundreds' digits is the hundreds' digit of the product, etc.

Whenever the sum of two digits is ten or more, 1 must be carried.

Example. 1. Multiply 793 by 11.

SOLUTION.

3 (the units' digit of the multiplicand) becomes the units' digit of the product.

$9 + 3 = 12$ (carry the 1)

$1 + 7 + 9 = 17$ (carry the 1)

$1 + 7 = 8$

8723

2. Multiply 52,635 by 11.

SOLUTION. $52635 \times 11 = 578985$.

Written Work

Multiply each of the following by 11:

1. 363.	**2.** 271.
3. 823.	**4.** 456.
5. 3,742.	**6.** 876,394.
7. 3,578,962.	**8.** 34,579.
9. 263,789.	**10.** 123,496,287.

22. To multiply by 111.

$362{,}941 \times 111 = ?$

When the multiplication is performed in the customary manner, the multiplicand is repeated as follows:

362,941
362,941
362,941

When the short method is applied, *the units' digit of the multiplicand is the units' digit of the product; the sum of the units' and tens' digits of the multiplicand is the tens' digit of the product; the sum of the units', tens', and hundreds' digits of the multiplicand is the hundreds' digit of the product; the sum of the tens', hundreds', and thousands' digits of the multiplicand is the thousands' digit of the product, etc.*

The excess above 10 is always to be carried to the next sum.

Without recopying, write the products obtained by multiplying each of the following by 111:

1. 729,361.	**2.** 124,396.	**3.** 1,793,862.
4. 5,374.	**5.** 235,692.	**6.** 8,354,927.

23. To multiply two numbers ending in 5.

a. When the sum of the digits at the left of the 5's is an even number.

Multiply the digits at the left of the 5's; to this product add one half the sum of these digits; to this result annex 25.

Examples. 1. Multiply 65 by 25.

SOLUTION. $2 \times 6 = 12$, the product of the digits at the left of the 5's.
$\frac{1}{2}$ of $(2 + 6) = 4$
16
1625, result obtained by annexing 25.

2. Multiply 625 by 445.

SOLUTION. $62 \times 44 = 2728$
$\frac{1}{2}$ of $(62 + 44) = 53$
2781
278125, result obtained by annexing 25.

Written Work

Multiply as indicated:

1. 35×75.	**2.** 95×75.
3. 35×55.	**4.** 25×45.
5. 325×45.	**6.** 725×65.
7. 835×175.	**8.** 145×165.
9. 195×115.	**10.** 225×185.

b. When the sum of the digits at the left of the 5's is an odd number.

Multiply the digits at the left of the 5's; to this product add one half the sum of these digits, dropping the ½; to this result annex 75.

Examples. 1. Multiply 75 by 45.

SOLUTION. $4 \times 7 = 28$, the product of the digits at the left of the 5's.

$\frac{1}{2}$ of $(4 + 7) = 5\frac{1}{2}$

$33\frac{1}{2}$

3375, the result obtained by dropping the fraction ½, and annexing 75.

2. Multiply 325 by 475.

SOLUTION. $32 \times 47 = 1504$

$\frac{1}{2}$ of $(32 + 47) = 39\frac{1}{2}$

1543 adding and dropping the ½.

154375, result secured by annexing 75.

Written Work

Multiply as indicated:

1. 75×65.
2. 125×135.
3. 95×85.
4. 145×175.
5. 225×75.
6. 165×135.
7. 145×215.
8. 435×125.

24. To multiply two numbers, when certain digits of the multiplier are contained an integral number of times in other digits of the multiplier.

Multiply 224 by 279.

Since 9 is contained 3 times in 27, first multiply 224 by 9, then multiply this product by 3.

Thus,

$$\begin{array}{rl} 224 & \\ 279 & \\ \hline 2016 & = 9 \times 224 \\ 6048 & = 3 \times 2016 \\ \hline 62496 & \end{array}$$

When using this method, be careful to place the product of the second multiplication in the proper position.

Example. 1. Multiply 341 by 618.

341	
618	
2046	$= 6 \times 341$
6138	$= 3 \times 2046$
210738	

SOLUTION. Since 18 is a multiple of 6, multiply first by 6. Place the right-hand figure of the product, 2046, directly under the 6 of the multiplier. The product of 18×341 can now be obtained by multiplying 2046 by 3. The right-hand figure of this product, 6138, is placed directly under the 8 of the multiplier.

Written Work

Multiply the following, stating by what numbers you multiplied in order to take advantage of the short method:

1. 468×243.	**2.** $1{,}235 \times 981$.	**3.** 719×427.
4. 687×654.	**5.** 739×848.	**6.** $7{,}362 \times 1{,}248$.
7. $1{,}247 \times 1{,}864$.	**8.** $146{,}387 \times 315$.	**9.** $1{,}235 \times 819$.

25. The **supplement** of a number is the difference between the number and the next lower power of 10. The supplement of 15 is 5; the supplement of 134 is 34; the supplement of 1,042 is 42.

26. To multiply two numbers each of which is a little larger than 100.

To either of the numbers add the supplement of the other; to this sum annex the product of the supplements.

Example. Multiply 131 by 103.

SOLUTION. $131 + 3 = 134$
(or $103 + 31 = 134$).

Annex 93 (31×3).
The result is 13493.

The same rule may be applied to numbers a little larger than 1,000.

Example. Multiply 1,062 by 1,006.

SOLUTION

$1062 + 6 = 1068$
$62 \times 6 = \quad 372$
1068372

NOTE. When the supplements are based on 100 and the product of the supplements is a number of only one digit, a zero must be put in the tens' place. For example: $102 \times 103 = 10{,}506$. Similarly, when the supplements are based on 1,000 and the product of the supplements is a number of less than three digits, zeros must be put in the proper places.

Written Work

1. 104×120.	2. 127×102.	3. 113×106.
4. 114×105.	5. 114×106.	6. 109×106.
7. 126×107.	8. 109×111.	9. $1{,}007 \times 1{,}003$.
10. $1{,}009 \times 1{,}012$.	11. $1{,}214 \times 1{,}006$.	12. $1{,}206 \times 1{,}012$.
13. $1{,}112 \times 1{,}006$.	14. $1{,}416 \times 1{,}009$.	15. $1{,}374 \times 1{,}005$.
16. $1{,}674 \times 1{,}012$.		

27. The **complement** of a number is the difference between the next higher power of 10 and the number. The complement of 92 is 8; the complement of 89 is 11; the complement of 996 is 4.

28. To multiply two numbers both slightly less than 100.

From either number subtract the complement of the other number. To this result annex the product of the complements.

Example. Multiply 96 by 93.

SOLUTION. $96 - 7 = 89$ (or $93 - 4 = 89$).
$4 \times 7 = 28$ (product of the complements).
Annex 28 to 89 and the result is 8,928.

NOTE. When the complements are based on 100 and the product of the complements is less than 10, a zero must be put in tens' place. For illustration: $98 \times 97 = 9{,}506$. Similarly, the tens' and hundreds' places must be filled when the complements are based on 1,000.

Written Work

Multiply as indicated. Perform orally as much of this work as possible.

1. 98×95.	2. 97×84.	3. 87×91.
4. 95×80.	5. 93×85.	6. 87×97.
7. 88×87.	8. 91×92.	9. 86×94.

The same method may be applied to numbers slightly less than 1,000.

Example. Multiply 996 by 987.

SOLUTION. $996 - 13$ (the complement of 987) $= 983$.
$4 \times 13 = 52$ (the product of the complements).
Since the complements are based on 1000 and the product of the complements is a number of only two digits, a zero must be put in hundreds' place. The result is 983052.

Written Work

Multiply:

1. 994 992	**2.** 996 984	**3.** 983 991	**4.** 987 992
5. 991 981	**6.** 983 982	**7.** 978 983	**8.** 981 988

29. To multiply any two numbers in the teens.

To either of the numbers add the units' digit of the other number and annex a zero.

To this result add the product of the units' digits.

Example. Multiply 15 by 17.

SOLUTION. $15 + 7 = 22$; annex a zero, 220
$5 \times 7 =$ 35
255

Oral Work

Multiply:

1. 17 12	**2.** 13 12	**3.** 16 18	**4.** 17 13	**5.** 14 13	**6.** 19 15
7. 13 16	**8.** 17 19	**9.** 18 12	**10.** 13 19	**11.** 17 16	**12.** 15 17

30. To multiply by 25, 50, or 75.

Since 25 is $\frac{1}{4}$ of 100, the product obtained by multiplying any number by 25 is $\frac{1}{4}$ of the product obtained by multiplying the number by 100.

The student should be asked to discover a short method to be used when 50 or 75 are multipliers.

Written Work

Multiply each number in the first row by 25, each in the second row by 50, and in the third by 75.

a.	482	398	1,436	297	849
b.	154	697	2,843	792	786
c.	4,092	378	2,864	923	871

CHAPTER IV

DIVISION

Oral Work

31. Short Division.

1. Divide by 2: 16, 28, 76, 248, 368, 926, 1,054.
2. Divide by 3: 27, 36, 57, 75, 417, 732, 873.
3. Divide by 4: 32, 48, 64, 72, 96, 196, 384, 748.
4. Divide by 5: 75, 95, 145, 545, 725, 965, 1,025, 1,370.
5. Divide by 6: 72, 96, 126, 366, 528, 732, 1,044.
6. Divide by 7: 56, 91, 147, 203, 476, 924, 1,575.
7. Divide by 8: 72, 96, 144, 360, 424, 792, 1,240.
8. Divide by 9: 54, 98, 171, 243, 378, 567, 981.

State the remainder when each of the following numbers is divided by 3, 4, 5, 7, 8, and 9.

9.	10.	11.	12.	13.
473	9,846	2,638	4,020	19,206
692	3,723	7,284	7,302	47,308
876	9,264	3,047	7,960	23,074
479	3,749	2,636	8,403	95,306

14. How many pounds of meat can be bought for \$1.26 at 18¢ per pound?

15. How many weeks in 91 days? 175 days? 266 days?

16. The dividend is 176 and the quotient is 8. What is the divisor?

17. The divisor is 9, the quotient is 7, and the remainder is 3. What is the dividend?

18. Six dozen oranges were bought for \$2.40. What was the price per dozen?

19. How many yards in 186 feet? 924 feet? 5,280 feet?

20. A grocer buys 9 dozen eggs for \$2.88. What is the cost per dozen?

21. If you are to discharge a debt of \$8.40 in 6 equal weekly payments, how much must you pay each week?

32. Long Division.

Examples. 1. Divide 4,625 by 37. 2. Divide 47,285 by 327.

SOLUTION.

```
      125
37 | 4625
     37
     ---
      92
      74
     ---
     185
     185
     ---
```

SOLUTION.

```
      144 11/327
327 | 47285
      327
      ----
      1458
      1308
      ----
      1505
      1308
      ----
       197
```

(The quotient should be written above the dividend in long division.)

33. Checks. Division may be checked by several methods. The method stated below is easily understood and generally used.

Multiply the quotient by the divisor and add the remainder, if any, to this product to obtain the dividend.

Division may also be checked by the following methods:

1. Divide the dividend by the quotient to get the divisor.
2. Check by casting out the 9's or the 11's.
3. Divide the dividend in turn by the various factors of the divisor.

Written Work

Perform the following divisions and check each result:

1. 1,758 ÷ 144.
2. 15,762 ÷ 37.
3. 73,627 ÷ 425.
4. 78,264 ÷ 738.
5. 508,573 ÷ 97.
6. 66,816 ÷ 928.
7. 41,228 ÷ 44.
8. 2,235,812 ÷ 284.
9. Divide the sum of 478, 392, 648, 971, and 1,483 by 27.
10. Divide the sum of 3,024, 4,763, 8,297, 9,468, and 293 by 38.
11. Complete:

$37 \times 29 =$
$42 \times 29 =$
$36 \times 29 =$
$94 \times 29 =$
$402 \times 29 =$
Total $=$ _____

12. The total circulation of a certain daily newspaper for 23 consecutive days was 524,423 copies. What was the average daily circulation?

13. At $2.18 a yard, how many yards of cloth can be bought for $37.06?

14. The dividend is 14,286, the quotient is 18, and the remainder is 12. What is the divisor?

15. The product of two numbers is 135,468. One of the numbers is 426. What is the other?

16. In a factory 437 men produce 5,681 articles in a week. What is the average weekly production for each employee?

17. At the rate of 43 miles an hour, how long will it take a train to run 473 miles?

18. Into how many states as large as Rhode Island (1,248 sq. mi.) could Texas (265,896 sq. mi.) be divided?

19. The expenses for 5 months for a family of 5 amounted to $258 for food, $170 for rent, $216 for clothes, $147 for amusements and other expenses. What was the average monthly expense for the family and the average for each person?

34. Pointing Off the Quotient. When it is necessary to continue a division to tenths or hundredths, place a decimal point to the right of the units' figure of the dividend and the quotient, annex zeros to the dividend, and continue the process.

Example. Divide 2,916 by 26.

SOLUTION.

$$26 \overline{)2916.0} \quad 112.1$$

Written Work

The following table gives the area of each of the states in square miles, and also the population of each state.

Review the instructions given on page 10 for ruling a blank form and for entering statistics.

Prepare a form similar to the following:

State and Division	Land Area in Square Miles	Population 1920	Population per Square Mile
New England			
Maine	29,895	768,014	
New Hampshire	9,031	443,083	
Vermont	9,124	352,428	
Massachusetts	8,039	3,852.356	
Rhode Island	1,067	604,397	
Connecticut	4,820	1,380,631	
Middle Atlantic			
New York	47,654	10,384,829	
New Jersey	7,514	3,155,900	
Pennsylvania	44,832	8,720,017	
East North Central			
Ohio	40,740	5,759,394	
Indiana	36,045	2,930,390	
Illinois	56,043	6,485,280	
Michigan	57,480	3,668,412	
Wisconsin	55,256	2,632,067	
West North Central			
Minnesota	80,858	2,387,125	
Iowa	55,586	2,404,021	
Missouri	68,727	3,404,055	
North Dakota	70,183	645,680	
South Dakota	76,868	636,547	
Nebraska	76,808	1,296,372	
Kansas	81,774	1,769,257	
South Atlantic			
Delaware	1,965	223,003	
Maryland	9,941	1,449,661	
District of Columbia	60	437,571	
Virginia	40,262	2,309,187	
West Virginia	24,022	1,463,701	
North Carolina	48,740	2,559,123	
South Carolina	30,495	1,683,724	
Georgia	58,725	2,895,832	
Florida	54,861	968,470	
East South Central			
Kentucky	40,181	2,416,630	
Tennessee	41,687	2,337,885	
Alabama	51,279	2,348,174	
Mississippi	46,362	1,790,618	

State and Division	Land Area in Square Miles	Population 1920	Population per Square Mile
West South Central			
Arkansas	52,525	1,752,204	
Louisiana	45,409	1,798,509	
Oklahoma	69,414	2,028,283	
Texas	262,398	4,663,228	
Mountain			
Montana	146,201	548,889	
Idaho	83,354	431,866	
Wyoming	97,594	194,402	
Colorado	103,658	939,629	
New Mexico	122,503	360,350	
Arizona	113,810	333,903	
Utah	82,184	449,396	
Nevada	109,821	77,407	
Pacific			
Washington	66,836	1,356,621	
Oregon	95,607	783,389	
California	155,652	3,426,861	

Note. The class should be divided into groups and each group held responsible for a geographical division.

Complete as much of the table as the teacher assigns, showing the population per square mile of each state, of each section, and of the United States.

Carry results to one decimal place.

General Exercises for Drill

I. Estimated Time 7 Minutes

1. Add:

a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*	*h.*
27	14	29	73	29	83	72	94
36	33	43	99	48	92	86	55
92	27	38	26	73	17	93	76
23	19	64	10	27	64	14	87
49	86	27	18	65	20	27	31
85	35	85	35	42	78	38	49

2. Multiply:

a.	*b.*	*c.*
4,736	9,463	7,427
3	5	3

3. Subtract:

a.	*b.*	*c.*	*d.*
72,049	4,706	32,964	92,807
32,608	2,985	7,193	1,968

4. Add across and down:

a.	*b.*
7 + 3 + 9 + 2 =	24 + 7 + 6 =
5 + 4 + 8 + 3 =	13 + 3 + 3 =
4 + 5 + 1 + 9 =	17 + 4 + 7 =
7 + 6 + 5 + 1 =	9 + 15 + 3 =
1 + 9 + 3 + 4 =	18 + 9 + 5 =
8 + 1 + 2 + 5 =	13 + 2 + 1 =
3 + 2 + 7 + 8 =	7 + 19 + 8 =
9 + 7 + 4 + 6 = ___	15 + 3 + 9 = ___
+ + + =	+ + =

II. Estimated Time 9 Minutes

1. Add across and down:

741 + 437 + 563 + 234
526 + 318 + 725 + 238
382 + 746 + 128 + 407
387 + 209 + 504 + 916
328 + 460 + 937 + 659
473 + 295 + 243 + 175
___ + + + +

2. Multiply:

a.	*b.*	*c*	*d.*
7,460	9,047	739	7,036
38	29	86	43

3. $143 \times 6 =$
$427 \times 5 =$
$793 \times 8 =$
$429 \times 7 =$
$384 \times 3 =$
$827 \times 6 =$
$538 \times 9 =$

4. Divide:

a. $326 \overline{)6{,}846}$ *b.* $22 \overline{)6{,}996}$

III. Estimated Time 8 Minutes

Add across and down:

7	9	7	3	5	8	1	8
4	2	5	9	2	4	7	3
3	8	3	6	4	5	3	6
6	4	7	7	8	9	8	9
2	3	9	4	7	7	6	7
9	5	8	8	9	6	4	2
8	7	6	5	3	3	5	4
4	6	4	1	1	4	9	8
7	1	9	2	2	1	2	5
6	9	2	8	6	2	7	1

Subtract:

a.	*b.*	*c.*
74,036	92,407	769,082
21,809	38,795	432,767

Multiply:

a.	*b.*	*c.*
3,742	9,427	3,897
37	45	28

Divide:

a. $6 \overline{)748{,}632}$ *b.* $8 \overline{)938{,}424}$

IV. Estimated Time 8 Minutes

1. Add:

a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*	*h.*
37	24	33	51	26	72	43	18
24	13	15	32	39	46	75	37
36	43	31	44	72	92	76	11
15	25	17	65	43	54	28	26
13	17	65	23	86	28	35	34
11	38	22	25	54	63	54	83

2. Subtract:

a.	*b.*	*c.*	*d.*
4,372	7,246	9,264	3,164
2,439	3,817	5,728	1,476

3. Add from left to right:

a.	*b.*
7 + 3 + 2 + 4 =	6 + 9 + 4 + 8 =
3 + 7 + 5 + 6 =	13 + 17 + 19 + 12 =
4 + 8 + 9 + 1 =	15 + 13 + 12 + 16 =
3 + 7 + 2 + 4 =	5 + 7 + 8 + 6 =
8 + 3 + 7 + 5 =	13 + 27 + 32 + 17 =
9 + 3 + 2 + 8 =	48 + 27 + 3 + 2 =
5 + 7 + 3 + 8 =	7 + 9 + 2 + 3 =

4. Multiply:

a.	*b.*	*c.*	*d.*
3,724	4,923	6,728	3,107
2	3	2	3

V. Estimated Time 12 Minutes

1. Divide:

a.	*b.*	*c.*
5 ⟌ 742,385	8 ⟌ 943,824	9 ⟌ 736,848

2. Multiply:

a.	*b.*	*c.*
1,437	1,943	1,048
246	632	453

3. Subtract:

a.	*b.*	*c.*	*d.*
472,387	940,360	472,863	796,048
29,389	397,685	30,940	397,059

4. Add across and down:

$$\begin{array}{l} 943 + 790 + 408 + 941 = \\ 276 + 463 + 312 + 806 = \\ 841 + 287 + 637 + 420 = \\ 732 + 934 + 924 + 387 = \\ \underline{286} + \underline{742} + \underline{836} + \underline{209} = \underline{\quad} \\ \quad + \quad + \quad + \quad = \end{array}$$

5. Divide:

a. $423 \overline{)13{,}536}$ *b.* $91 \overline{)46{,}228}$

General Exercises in the Fundamental Operations

1. $439 \times 57 =$
$826 \times 57 =$
$409 \times 57 =$
$392 \times 57 =$
$870 \times 57 =$
$\underline{439 \times 57 = \quad}$
$\underline{? \times 57 = ?}$

2. $409 \times 36 =$
$872 \times 36 =$
$791 \times 36 =$
$879 \times 36 =$
$472 \times 36 =$
$\underline{748 \times 36 = \quad}$
$\underline{? \times 36 = ?}$

3. $807 \times 83 =$
$904 \times 83 =$
$296 \times 83 =$
$475 \times 83 =$
$917 \times 83 =$
$\underline{796 \times 83 = \quad}$
$\underline{? \times 83 = ?}$

4. Add without rewriting:

a. $47 + 36 + 82 + 94$
b. $83 + 21 + 72 + 76$
c. $42 + 63 + 17 + 84$
d. $56 + 43 + 27 + 18$
e. $26 + 35 + 42 + 23$
f. $81 + 42 + 63 + 35$

5. The number of automobiles in use in certain states in 1919 and 1920 is indicated in the following table. Compute the increase in each

state for 1920, and compute the total number in use in the states listed.

Do the results check?

	1919	1920	Gain
Connecticut	84,902	100,550	
Delaware	12,066	16,600	
Idaho	31,925	46,360	
Illinois	389,135	497,318	
Iowa	327,500	405,182	
Kansas	186,109	253,896	
Texas	250,083	298,234	
Wisconsin	196,844	280,452	

6. The highest and the lowest points of each continent are stated in the table below. Compute the greatest elevation above the lowest point for each continent.

	Highest Point (Ft.)	Lowest Point (Ft.) (Below sea level)
North America	20,300	276
South America	23,080	Sea level
Europe	15,782	86
Asia	29,002	1,290
Africa	19,320	150

7. How many of the following examples in division can you finish in six minutes?

a. 89,175 ÷ 39
b. 2,137,512 ÷ 372
c. 187,931 ÷ 68
d. 784,267 ÷ 269
e. 199,988 ÷ 578
f. 1,728 ÷ 144
g. 148,785 ÷ 455
h. 3,157 ÷ 287
i. 275,258 ÷ 458
j. 198,001 ÷ 67
k. 210,897 ÷ 321
l. 64,170 ÷ 345

8. If you are told the divisor, quotient, and remainder how can you find the dividend?

9. If you know your balance in the bank at the end of October, the amount withdrawn during the month and the amount deposited

during the month, how can you compute the balance which you had in the bank at the beginning of the month?

10. If you are told the remainder, the quotient, and the dividend, how can you find the divisor?

11. What do you consider the best check for addition? subtraction? multiplication? division?

12. A plumber has the following stock record of pipe at the time of taking inventory. Complete the table by filling the blanks.

Kind of Pipe	Feet On Hand Jan. 1	Feet Bought during Year	Used during Year	Inventory Dec. 31
½ in.	745	2,230		540
¾ in.	1,472	1,750		1,200
1 in.	2,485	1,480		1,260
1¼ in.	890	2,690		480
1½ in.	2,150	1,890		1,050
2 in.	1,340	2,400		1,280
3 in.	520	1,200		215
4 in.	260	200		120

13. A cashier in a lunch room takes in checks as indicated below. Compute the total amount taken in.

Check	Number	Check	Number	Check	Number
15¢	18	25¢	47	60¢	9
18¢	7	34¢	18	72¢	3
20¢	14	45¢	17	75¢	5
23¢	34	55¢	13	82¢	4

14. A steamer has a storage capacity of 192,000 cubic feet for coal. How many long tons of coal can be carried in the steamer, if a long ton occupies 43 cubic feet?

CHAPTER V

AVERAGE

35. Simple Average. A boy walked 9 miles on Monday, 5 miles on Tuesday, and 10 miles on Wednesday. If he had divided the trip into three equal distances, how many miles would he have walked each day? In other words, what was the *average* distance walked each day?

A man owned a five-acre field. One acre yielded 71 bushels of potatoes, another 77 bushels, the third 85 bushels, the fourth 93 bushels, and the fifth 64 bushels. What was the total crop? What was the *average* crop per acre?

What two processes are usually involved in computing an *average?*

The average of several quantities is determined by dividing the sum of the quantities by the number of quantities added.

Written Work

1. The noon temperatures in a certain city for a week were as follows: Monday, 62°; Tuesday, 70°; Wednesday, 74°; Thursday, 68°; Friday, 62°; Saturday, 70°; Sunday, 76°. What was the average noon temperature for the week?

2. A newsdealer sold papers as indicated in the table. Find the average number of each sold.

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Avg.
Sentinel . . .	39	47	63	82	74	67	
Argus	13	19	27	31	29	26	
Tribune . . .	39	42	48	51	39	41	
Recorder. . .	28	43	56	29	37	29	

3. The receipts of a certain department of a store for one week were as follows: Monday, $396.40; Tuesday, $283.70; Wednesday, $468.14; Thursday, $184.68; Friday, $369.58; Saturday, $296.40. Compute the average daily receipts.

4. A certain city reported the rainfall by months, beginning with January, as follows: 1.2, 1.4, 1.4, 1.8, 1.0, .4, .5, .7, 1.0, 1.2, 1.3, 1.5 inches. Compute the average monthly rainfall.

5. A polar expedition recorded the noon temperatures for six consecutive days as follows: 45°, 52°, 48°, 47°, 51°, and 55°. All temperatures were below zero. Compute the average daily temperature.

6. A subscription was taken to secure funds to purchase a gift.

3 men gave $1.00 each
1 man gave .75
2 men gave 1.50 each
1 man gave .60
4 men gave .90 each

Find the average amount given.

7. The following table shows the number of men employed and the total weekly wages in each of the four departments of a factory. Find the average wage in each department and the average wage for the four departments.

Department	Number of Men Employed	Total Wage	Average Wage
A	37	$ 960	
B	12	290	
C	42	1,054	
D	16	318	

Oral Review

1. What is the difference in the meaning of the following statements?

a. Each of the 300 employees in our factory earns $2.50 per day.

b. The average daily wages of employees in our factory is $2.50 per day.

2. What would you have to know and how would you proceed to find:

a. The average weight of twenty boxes?

b. The average value of a herd of cattle?

c. The average number of miles a train traveled per hour, going from Chicago to St. Louis?

d. The average age of the students in your class?

e. The average daily sales of a clerk?

3. What could you find if you were told:

a. The average value of farm land per acre in your state, and the number of acres of farm land?

b. The average daily sales of a clerk during the week days of June?

c. The average monthly grocery bill of your family?

4. If you were told:

a. The daily circulation of the *Evening Herald* for the twenty-six week days of July, and the average daily circulation of the *Evening Transcript* for the same month, how could you compare their circulation?

b. The average daily wages of A, who worked 265 days last year and the average daily wages of B, who worked 303 days, how could you find the yearly wages of each, and which one earned the larger amount?

c. The difference between the average daily outputs of two shoe factories, how could you find the difference in their production for a year of 300 working days?

5. What would you have to know and what would you do to find the average daily speed of an ocean liner on a given voyage?

6. Several boys worked for a farmer picking strawberries. One earned $5.00 more than the average of the other boys' earnings. What else must you know and what would you do to find the total earnings of all the boys? What else would you have to know and how would you find the average wage cost of picking a quart of strawberries?

7. What facts would you need to know and how would you proceed to find the average height and the average weight of the boys and of the girls in your class?

Written Work

1. A merchant kept a record of the deliveries of goods made to his customers for a week. The record follows:

Day	Deliveries
Monday	213
Tuesday	187
Wednesday	208
Thursday	221
Friday	168
Saturday	251

a. What was the average number of deliveries per day?

b. The expense of running the delivery cars for a week, including care of cars, repairs and wages, and interest on the money invested in cars, was $84.75. What was the average cost per delivery?

2. Five clerks in a store sold the following amounts of goods during a month:

A.	$1,246.50
B.	1,076.85
C.	944.90
D.	1,388.20
E.	1,109.75

a. What was the average amount of sales per clerk?

b. Which clerks sold more than the average? Which clerks sold less than the average? Did any clerk sell just the average?

CHAPTER VI

FACTORS AND MULTIPLES

36. Terms. An **integer** is a number of whole units, such as 1, 7, 12.

The **factors** of a number are the integers which, multiplied together, produce the given number. Thus, 3 and 5 are factors of 15; 3 and 6 are factors of 18.

A **factor** of a number is a divisor of that number.

A number which is not exactly divisible by any other number (except 1) is called a **prime number.** Thus, 1, 3, 5, 7, 11, and 13 are prime numbers.

Numbers are said to be **prime to each other** when they have no common factor except 1. Thus, 10 and 27 are **prime to each other,** although neither is a prime number.

37. Test of Divisibility of Numbers. A number is divisible by:

a. Two, if it ends with 0, 2, 4, 6, or 8.

b. Three, if the sum of its digits is divisible by 3.

c. Four, if the number expressed by its last two digits is divisible by 4.

d. Five, if it ends in 0 or 5.

e. Six, if it is divisible by both 2 and 3.

f. Eight, if the number expressed by its last three digits is divisible by 8.

g. Nine, if the sum of its digits is divisible by 9.

h. Ten, if its right-hand digit is zero.

i. Eleven, if the difference between the sums of the numbers represented by the odd and even orders of digits is divisible by 11. Thus, 16,280 is divisible by 11, since $(8 + 6) - (0 + 2 + 1)$ is divisible by 11.

There is no simple method for testing divisibility by 7.

38. Factoring is the process of separating a number into its factors.

Oral Work

1. Learn the prime numbers from 1 to 100 so that you can recognize them at sight.

2. Apply the tests of divisibility in finding the prime factors of the following:

28	160	728	478	76	720
42	320	640	96	84	37
72	48	386	84	90	145
36	360	31	92	360	390
98	280	100	81	760	625

3. What numbers between 161 and 200 are divisible by 9?

4. What numbers between 746 and 800 are divisible by 6?

5. Name the factors of 36 which are not prime to each other.

39. Cancellation is the process of shortening certain computations involving division by removing or canceling equal factors from both dividend and divisor.

Example. Divide the product of 4, 9, 8, 36, 24, and 7 by the product of 18, 2, 8, 3, 14, and 4.

SOLUTION.
$$\frac{\overset{2}{\cancel{4}} \times 9 \times \cancel{8} \times \overset{18}{\cancel{36}} \times \overset{12}{\cancel{24}} \times 7}{\underset{2}{\cancel{18}} \times 2 \times \cancel{8} \times 3 \times \underset{2}{\cancel{14}} \times \cancel{4}} = 36.$$

In all computations involving only multiplication and division, cancellation should be used when possible. Indicate the multiplication and division as in the illustration above, then cancel the common factors.

Written Work

Use cancellation when possible

1. $\dfrac{24 \times 36 \times 15}{16 \times 5 \times 9}$.

2. $\dfrac{4 \times 37 \times 16 \times 5 \times 60}{48 \times 32 \times 74}$.

3. $\dfrac{48 \times 32 \times 100 \times 360}{16 \times 50 \times 72}$.

4. $\dfrac{27 \times 64 \times 96 \times 38}{19 \times 16 \times 9 \times 2}$.

5. $\dfrac{130 \times 14 \times 18 \times 121 \times 15}{7 \times 27 \times 13 \times 11}$.

6. $\dfrac{144 \times 32 \times 63 \times 7}{16 \times 9 \times 28}$.

7. $\dfrac{1{,}728 \times 360 \times 100 \times 32 \times 3}{18 \times 144 \times 64 \times 75}$.

8. $\dfrac{21 \times 72 \times 160 \times 340 \times 27}{180 \times 36 \times 35}$.

40. Greatest Common Divisor. An integer that is a factor of two or more numbers is called a **common divisor,** or a **common factor** of those numbers.

The **greatest common divisor** of two or more numbers is the greatest factor common to the numbers. "Greatest common divisor" is usually expressed as g. c. d.

Example. Find the greatest common divisor of 12, 20, and 36.

SOLUTION.
$$12 = 2 \times 2 \times 3$$
$$20 = 2 \times 2 \times 5$$
$$36 = 2 \times 2 \times 3 \times 3.$$

The factor 2 occurs twice in all the numbers and none of the other factors occurs in all the numbers, hence, 4 is the g. c. d. of 12, 20, and 36.

To find the g. c. d. of two or more numbers, *separate the numbers into prime factors and find the product of the prime factors common to the numbers.*

Written Work

Find the g. c. d. of:

1. 12, 18, 24.
2. 24, 60, 72.
3. 15, 20, 30.
4. 60, 90, 100.
5. 84, 32, 60.
6. 60, 96, 120.
7. 18, 32, 48.
8. 27, 36, 45.
9. 360, 120, 40.
10. 121, 88, 242.
11. Find the g. c. d. of 8 ft. and 12 ft.
12. Find the g. c. d. of \$48 and \$60.
13. What is the length of the longest logs, all of the same length, that can be cut from pieces of timber whose lengths are 36, 54, and 72 feet respectively?

41. Least Common Multiple. A **multiple** of a number is an integral number of times that number. Thus, 28 is a multiple of 7. 60 is a multiple of 12.

A **common multiple** of two or more numbers is a number that is a multiple of each of them. It is therefore divisible by each of them.

The **least common multiple** of two or more numbers is the least number that is a multiple of each of them.

Thus, 60 is the least common multiple of 12, 15, and 30.

Example. Find the least common multiple (l. c. m.) of 18, 20, and 24.

SOLUTION.

$$18 = 2 \times 3 \times 3$$
$$20 = 2 \times 2 \times 5$$
$$24 = 2 \times 2 \times 2 \times 3.$$
$$\text{l.c.m.} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360.$$

To find the least common multiple of two or more numbers, *separate each number into its prime factors. Find the product of these factors, using each factor the greatest number of times it occurs in any one of the given numbers.*

Written Work

Find the l. c. m. of the following:

1. 12, 15.
2. 8, 12.
3. 6, 15.
4. 7, 8, 12.
5. 8, 9, 12, 15.
6. 24, 36, 60.
7. 8, 12, 16.
8. 36, 24, 75.
9. 360, 345.
10. 75, 130, 190.
11. 425, 345, 336.
12. 360, 240, 420.

Find the g. c. d. and the l. c. m. of the following:

13. 60, 80, 95.
14. 36, 75, 48.
15. 480, 360, 120.
16. 24, 30, 36.

CHAPTER VII

COMMON FRACTIONS

42. Terms. A **fraction** is one or more of the equal parts of a unit.

A **common fraction** is usually expressed by writing one figure above and one below a short line; thus, $\frac{2}{3}$ or $^2/_3$.

The **numerator** of a fraction is the number which shows how many of the equal parts of the unit are taken. It is written above the line.

The **denominator** of a fraction is the number which shows into how many equal parts the unit is divided. It is written below the line.

The numerator and the denominator are called the **terms** of the fraction. Thus, 3 and 4 are the terms of the fraction $\frac{3}{4}$.

A common fraction may be either *proper* or *improper*.

A **proper fraction** is one whose numerator is less than its denominator, as $\frac{3}{4}$.

An **improper fraction** is one whose numerator is equal to or greater than its denominator as $\frac{4}{4}$ or $\frac{5}{4}$.

A **mixed number** consists of a whole number and a fraction, as $5\frac{2}{3}$.

Oral Work

Which is greater, $\frac{1}{4}$ or $\frac{1}{7}$? $\frac{1}{9}$ or $\frac{1}{10}$? $\frac{1}{20}$ or $\frac{1}{12}$?

How is the value of a fraction affected by increasing the numerator, the denominator remaining the same?

Reduction of Fractions

43. Reducing Fractions to Lower Terms. When the numerator and the denominator contain one or more common factors, the fraction may be reduced to a fraction of equivalent value expressed in lower terms.

Thus, $\frac{10}{12}$ may be reduced to the equivalent fraction $\frac{5}{6}$, by dividing both terms by 2. Similarly, $\frac{18}{24} = \frac{3}{4}$; $\frac{7}{21} = \frac{1}{3}$.

State a rule for reducing fractions to lower terms.

44. Reducing Fractions to Higher Terms. Fractions may be raised to equivalent fractions in higher terms, *by multiplying both numerator and denominator by the same number.*

Example. Express $\frac{3}{5}$ as an equivalent fraction whose denominator is 60.

SOLUTION. The given denominator, 5, must be multiplied by 12 to obtain the desired denominator, 60. Therefore, multiply both terms of the fraction by 12, and obtain the equivalent fraction $\frac{36}{60}$.

Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.

45. Reducing Improper Fractions to Mixed Numbers. An improper fraction may be reduced to a mixed number. The following example shows the method:

Example. Reduce $\frac{19}{4}$ to a mixed number.

SOLUTION. $19 \div 4 = 4\frac{3}{4}$.

State a rule for changing an improper fraction to a mixed number.

46. Reducing Mixed Numbers to Improper Fractions. A mixed number may be reduced to an improper fraction. The following example shows the method.

Example. Reduce $13\frac{3}{7}$ to an improper fraction.

SOLUTION. $7 \times 13 = 91$. $91 + 3 = 94$, the numerator of the improper fraction. The denominator of the fraction in the mixed number is retained as the denominator of the improper fraction.

$$13\tfrac{3}{7} = \tfrac{94}{7}.$$

Oral Work

1. Reduce to lowest terms: $\frac{4}{12}$, $\frac{3}{18}$, $\frac{5}{10}$, $\frac{6}{24}$, $\frac{18}{30}$, $\frac{24}{28}$, $\frac{15}{30}$, $\frac{72}{84}$.
2. Express each of the following fractions with the denominator 16: $\frac{5}{8}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{4}{8}$, $\frac{2}{4}$, $\frac{7}{8}$, $\frac{5}{2}$, $\frac{5}{1}$, 3.
3. Express with the denominator 36: $\frac{4}{9}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{7}{12}$, $\frac{4}{12}$, $\frac{1}{3}$, $\frac{8}{9}$, $\frac{7}{3}$, $\frac{8}{4}$, $\frac{1}{9}$, $\frac{7}{2}$.
4. Reduce to twenty-fourths: $\frac{5}{6}$, $\frac{7}{12}$, $\frac{2}{3}$, $\frac{3}{2}$, $\frac{7}{6}$, $\frac{1}{12}$, 2.
5. Reduce to lowest terms: $\frac{8}{12}$, $\frac{9}{15}$, $\frac{4}{16}$, $\frac{8}{10}$, $\frac{9}{12}$, $\frac{14}{16}$.
6. Reduce to fiftieths: $\frac{2}{5}$, $\frac{9}{10}$, $\frac{7}{25}$, $\frac{19}{10}$, $\frac{5}{2}$, $\frac{7}{10}$.
7. Reduce to improper fractions: $3\frac{1}{5}$, $5\frac{1}{3}$, $7\frac{1}{4}$, $8\frac{2}{3}$, $4\frac{3}{5}$, $7\frac{2}{7}$, $9\frac{4}{5}$, $8\frac{1}{4}$.
8. Reduce the following fractions to equivalent fractions in lowest terms: $\frac{9}{24}$, $\frac{16}{88}$, $\frac{20}{85}$, $\frac{18}{144}$, $\frac{28}{210}$, $\frac{27}{63}$, $\frac{30}{150}$, $\frac{21}{15}$, $\frac{72}{180}$, $\frac{24}{36}$, $\frac{8}{12}$, $\frac{18}{21}$.
9. Change the following fractions as indicated:

$\frac{3}{4}$ to an equivalent fraction whose denominator is 20.
$\frac{7}{8}$ to an equivalent fraction whose denominator is 64.
$\frac{4}{9}$ to an equivalent fraction whose denominator is 63.
$\frac{2}{18}$ to an equivalent fraction whose denominator is 72.
$\frac{6}{11}$ to an equivalent fraction whose denominator is 121.
$\frac{5}{12}$ to an equivalent fraction whose denominator is 84.

10. Change the following improper fractions to mixed numbers: $\frac{8}{3}$, $\frac{17}{6}$, $\frac{39}{7}$, $\frac{25}{9}$, $\frac{31}{2}$, $\frac{64}{11}$, $\frac{488}{15}$, $\frac{300}{29}$, $\frac{341}{17}$, $\frac{83}{13}$, $\frac{216}{13}$, $\frac{235}{12}$.

11. Change the following mixed numbers to equivalent improper fractions: $3\frac{2}{3}$, $6\frac{4}{7}$, $8\frac{1}{8}$, $4\frac{3}{5}$, $8\frac{16}{25}$, $10\frac{17}{35}$, $14\frac{8}{13}$, $12\frac{7}{12}$, $18\frac{3}{20}$, $9\frac{5}{24}$.

12. When we reduce $\frac{8}{12}$ to $\frac{2}{3}$, do we increase or decrease the size of the fractional unit? Do we increase or decrease the number of fractional units?

13. Which fraction is the greatest, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{13}{15}$?

Addition and Subtraction of Fractions

47. Finding the Least Common Denominator. Only like quantities can be added or subtracted. It is not possible to add or subtract 4 lb. and 2 oz. or $\frac{1}{2}$ and $\frac{1}{4}$ until they are reduced to the same denomination.

The common denominator should be the smallest number which will exactly contain all of the denominators. It may usually be found by inspection.

Any number which contains all of the factors of another number will also contain that number. Thus, 24 contains all of the factors of 12, and it therefore contains 12.

The common denominator must, therefore, contain the prime factors of the denominators.

Hence, to find the least common denominator of two or more fractions, *find the prime factors of the denominators; multiply the different prime factors, using each factor the greatest number of times it is contained in any denominator.*

Problem. What is the least common denominator of fractions with denominators 8, 9, and 12?

Solution.

$8 = 2 \times 2 \times 2.$
$9 = 3 \times 3.$
$12 = 2 \times 2 \times 3.$

l.c.m. of denominators $= 2 \times 2 \times 2 \times 3 \times 3.$

48. Reducing Fractions to a Common Denominator. After the least common denominator has been found, the fractions may be reduced to this denominator.

Example. Express $\frac{3}{5}$, $\frac{2}{3}$, and $\frac{5}{6}$ with a common denominator.

SOLUTION. The least common denominator is 30.

$$\frac{3}{5} = \frac{18}{30}.$$
$$\frac{2}{3} = \frac{20}{30}.$$
$$\frac{5}{6} = \frac{25}{30}.$$

State the process of reducing fractions to a common denominator.

Written Work

Express the fractions in each example as fractions having the least common denominator:

1. $\frac{1}{2}$ and $\frac{1}{3}$.
2. $\frac{1}{3}$ and $\frac{1}{4}$.
3. $\frac{1}{4}$ and $\frac{1}{8}$.
4. $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{1}{12}$.
5. $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{20}$.
6. $\frac{2}{3}$, $\frac{4}{9}$, $\frac{5}{12}$.
7. $\frac{3}{7}$, $\frac{2}{11}$, $\frac{5}{42}$.
8. $\frac{7}{12}$, $\frac{9}{32}$, $\frac{13}{56}$, $\frac{4}{9}$, $\frac{5}{14}$.
9. $\frac{3}{20}$, $\frac{17}{36}$, $\frac{8}{45}$, $\frac{12}{35}$.
10. $\frac{11}{18}$, $\frac{9}{28}$, $\frac{1}{42}$, $\frac{11}{15}$.
11. $\frac{17}{96}$, $\frac{23}{42}$, $\frac{25}{63}$, $\frac{19}{27}$.

49. Rule for Addition of Fractions.

Reduce the fractions to a common denominator. Add the numerators to form the numerator of the sum. The common denominator is the denominator of the sum. Reduce the result to the simplest form.

Proper fractions should be reduced to their lowest terms.
Improper fractions should be changed to mixed numbers.

Examples. 1. Add $\frac{2}{3}$ and $\frac{3}{4}$.

SOLUTION. The least common multiple of the denominators is 12.

$$\frac{2}{3} = \frac{8}{12}.$$
$$\frac{3}{4} = \frac{9}{12}.$$
$$\frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\frac{5}{12}.$$

2. Add $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{10}$.

SOLUTION. The least common multiple of the denominators is 60.

$$\frac{3}{4} = \frac{45}{60}.$$
$$\frac{5}{6} = \frac{50}{60}.$$
$$\frac{7}{10} = \frac{42}{60}.$$
$$\frac{45}{60} + \frac{50}{60} + \frac{42}{60} = \frac{137}{60} = 2\frac{17}{60}.$$

50. Rule for Subtraction of Fractions.

The process is the same as for Addition of Fractions except that the numerators are subtracted instead of added after the fractions have been reduced to a common denominator.

Oral Work

State the sum, then the difference of the following:

1. $\frac{1}{2}$, $\frac{1}{3}$.	**2.** $\frac{1}{3}$, $\frac{1}{4}$.	**3.** $\frac{1}{5}$, $\frac{1}{7}$.	**4.** $\frac{1}{4}$, $\frac{1}{6}$.	**5.** $\frac{1}{3}$, $\frac{1}{5}$.
6. $\frac{1}{2}$, $\frac{1}{8}$.	**7.** $\frac{1}{7}$, $\frac{1}{9}$.	**8.** $\frac{1}{9}$, $\frac{1}{10}$.	**9.** $\frac{2}{3}$, $\frac{2}{7}$.	**10.** $\frac{5}{6}$, $\frac{3}{4}$.
11. $\frac{7}{9}$, $\frac{2}{3}$.	**12.** $\frac{5}{6}$, $\frac{4}{5}$.	**13.** $\frac{4}{7}$, $\frac{1}{3}$.	**14.** $\frac{3}{4}$, $\frac{3}{5}$.	**15.** $\frac{3}{5}$, $\frac{2}{7}$.

NOTE. The Short Methods for the Addition and Subtraction of Fractions, explained on pages 72–75, may be studied at this time.

51. Rule for Addition of Mixed Numbers.

Add the fractions.
If the sum is an improper fraction, reduce it to a mixed number.
Add the sum of the fractions to the sum of the integers.

Example. Add $3\frac{3}{4}$ and $4\frac{7}{8}$.

SOLUTION.
$$\tfrac{3}{4} + \tfrac{7}{8} = \tfrac{13}{8} = 1\tfrac{5}{8}.$$
$$3 + 4 = 7.$$
$$7 + 1\tfrac{5}{8} = 8\tfrac{5}{8}.$$

Written Work

Find the sums:

1. $4\frac{1}{2} + 3\frac{1}{3}$.
2. $2\frac{1}{5} + 5\frac{1}{7}$.
3. $7\frac{1}{8} + 5\frac{2}{3}$.
4. $17\frac{1}{5} + 18\frac{2}{7}$.
5. $275\frac{2}{3} + 78\frac{1}{5}$.
6. $3\frac{7}{8} + 5\frac{1}{4}$.
7. $9\frac{1}{10} + 17\frac{2}{5}$.
8. $3\frac{1}{3} + 4\frac{7}{8} + 3\frac{5}{12}$.
9. $23\frac{1}{3} + 16\frac{5}{12} + 7\frac{3}{4}$.
10. $18\frac{5}{6} + 42\frac{1}{10} + 37\frac{3}{5}$.

52. Rule for Subtraction of Mixed Numbers.

If the fraction of the minuend is larger than that of the subtrahend, the above rule for addition can be easily changed to apply to subtraction.

Example. Subtract $4\frac{2}{5}$ from $7\frac{5}{6}$.

SOLUTION.
$$\tfrac{5}{6} - \tfrac{2}{5} = \tfrac{13}{30}.$$
$$7 - 4 = 3.$$
Result $3\frac{13}{30}$.

If the fraction of the minuend is smaller than that of the subtrahend, we may proceed as follows:

Example. Subtract $4\frac{2}{3}$ from $9\frac{1}{5}$.

SOLUTION. $\frac{2}{3}$ cannot be subtracted from $\frac{1}{5}$, but $9\frac{1}{5}$ may be regarded as $8\frac{6}{5}$. $\frac{6}{5} - \frac{2}{3} = \frac{8}{15}$. Therefore, $9\frac{1}{5} - 4\frac{2}{3} = 4\frac{8}{15}$.

Written Work

Add as indicated:

1. $\frac{3}{7} + \frac{2}{5}$.
2. $\frac{6}{13} + \frac{5}{9}$.
3. $\frac{7}{9} + \frac{4}{15}$.
4. $\frac{2}{15} + \frac{7}{24}$.
5. $\frac{3}{5} + \frac{9}{25} + \frac{9}{16}$
6. $\frac{15}{32} + \frac{7}{24} + \frac{11}{36}$.
7. $3\frac{5}{8} + \frac{1}{7}$.
8. $12\frac{9}{16} + \frac{7}{18}$.
9. $13\frac{5}{7} + 22\frac{9}{15}$.

Subtract as indicated:

10. $\frac{4}{7} - \frac{3}{11}$.
11. $\frac{15}{32} - \frac{7}{20}$.
12. $2\frac{9}{14} - \frac{3}{16}$.
13. $4\frac{7}{8} - \frac{6}{7}$.
14. $3\frac{2}{3} - 2\frac{1}{2}$.
15. $4\frac{7}{9} - 2\frac{1}{6}$.
16. $5\frac{9}{25} - 2\frac{8}{9}$.

Add upward and across:

17.
$$\begin{array}{l} \frac{1}{4} + \frac{1}{3} + \frac{1}{2} \\ \frac{2}{3} + \frac{3}{4} + \frac{5}{12} \\ \frac{5}{6} + \frac{7}{8} + \frac{7}{6} \\ \hline ? + ? + ? \end{array}$$

18.
$$\begin{array}{l} \frac{5}{8} + \frac{3}{16} + \frac{2}{3} \\ \frac{7}{24} + \frac{5}{6} + \frac{1}{24} \\ \frac{1}{8} + \frac{5}{6} + \frac{7}{4} \\ \hline ? + ? + ? \end{array}$$

19. Add upward; subtract across:

$$\begin{array}{l} 14\frac{2}{3} - 4\frac{1}{5} \\ 18\frac{5}{6} - 7\frac{2}{3} \\ 5\frac{1}{12} - 3\frac{1}{4} \\ 15\frac{3}{4} - 7\frac{5}{6} \\ \hline ? \quad - \quad ? \end{array}$$

MULTIPLICATION AND DIVISION OF FRACTIONS

53. Rule for Multiplication of Fractions.

Multiply the numerators to form the numerator of the product. Multiply the denominators to form the denominator of the product. Reduce the product to its simplest form.

Thus, $\frac{3}{5} \times \frac{4}{7} = \frac{12}{35}$.

NOTE. — The word "of" placed between fractions indicates multiplication.

Mixed numbers should be changed to improper fractions before multiplying.

Example. $8 \times 6\frac{3}{4} \times \frac{2}{3} \times 3\frac{1}{5} = ?$

SOLUTION. Reducing mixed numbers to improper fractions:

$$\frac{8}{1} \times \frac{27}{4} \times \frac{2}{3} \times \frac{19}{5} = ?$$

$$\frac{\overset{2}{\cancel{8}}}{1} \times \frac{27}{\underset{1}{\cancel{4}}} \times \frac{2}{\underset{1}{\cancel{3}}} \times \frac{19}{5}.$$

Multiplying the resulting numerators: $2 \times 9 \times 2 \times 19 = 684$, the numerator of the product.

Multiplying the resulting denominators: $1 \times 5 = 5$, the denominator of the product.

Changing to a mixed number: $\frac{684}{5} = 136\frac{4}{5}$, the product.

Written Work

Perform the following multiplications:

1. $\frac{2}{3} \times \frac{4}{5}$.
2. $\frac{3}{8} \times \frac{5}{7}$.
3. $\frac{6}{5} \times \frac{8}{7}$.
4. $\frac{1}{2} \times 1\frac{5}{12}$.
5. $\frac{3}{4} \times \frac{7}{8}$.
6. $3\frac{1}{3} \times 4\frac{2}{3}$.
7. $\frac{8}{9} \times \frac{3}{4}$.
8. $\frac{2}{3}$ of $\frac{3}{4} \times \frac{1}{2}$.
9. $\frac{3}{7} \times \frac{3}{5}$ of $\frac{4}{3}$.
10. $\frac{8}{12} \times \frac{5}{7} \times \frac{4}{9} \times \frac{3}{10}$.
11. $5\frac{2}{3} \times 3\frac{1}{4} \times 4\frac{2}{5}$.
12. $\frac{3}{8} \times 4 \times \frac{7}{2} \times \frac{3}{15}$.

54. Rule for Division of Fractions.

Invert the divisor, and multiply the fractions, using cancellation when possible.

Examples. 1. $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$.

2. $\frac{8}{9} \div \frac{4}{5} = \frac{8}{9} \times \frac{5}{4} = \frac{10}{9} = 1\frac{1}{9}$.

3. $8\frac{2}{3} \div \frac{4}{9} = ?$

SOLUTION.

$$8\frac{2}{3} \div \frac{4}{9} = \frac{\overset{13}{\cancel{26}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{4}}} = \frac{39}{2} = 19\frac{1}{2}.$$

Written Work

1. $\frac{3}{5} \div \frac{2}{3}$.
2. $\frac{4}{7} \times \frac{9}{8}$.
3. $\frac{2}{3}$ of $\frac{3}{4} \times \frac{1}{6}$.
4. $5\frac{1}{2} \div 4\frac{2}{3}$.
5. $7\frac{1}{8} \div 3\frac{1}{4}$.
6. $\frac{3}{5} + \frac{2}{3}$.
7. $\frac{4}{7} - \frac{2}{5}$.
8. $3\frac{1}{5} + 5\frac{2}{3}$.
9. $\frac{3}{4}$ of $\frac{3}{7}$.
10. $9\frac{1}{2} \times 12\frac{2}{3} \div 3\frac{1}{4}$.
11. $\frac{7}{8}$ of $\frac{2}{11} \div \frac{14}{33}$.
12. $\frac{3}{5} + \frac{2}{9}$.
13. $4\frac{2}{5} - 3\frac{1}{3} + 5\frac{3}{4}$.
14. $17\frac{2}{3} \times 5\frac{1}{2} \div 4\frac{5}{12}$.
15. $\frac{3}{8}$ of $\frac{2}{3} \div \frac{2}{5}$ of $\frac{4}{9}$.

Review Exercises

1. Multiply each of the fractions in the column at the left by each of the fractions at the top. Enter the products in the spaces formed by the horizontal and vertical lines. (For example, the space marked "x" is to contain the product of $\frac{5}{8}$ and $\frac{7}{8}$.)

MULTIPLY	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{7}{8}$
$\frac{5}{6}$				
$3\frac{1}{3}$				
$\frac{5}{8}$				×
$\frac{7}{8}$				

2. In the following form divide each of the fractions in the column at the left by the fractions in the upper row. Rule a blank similar to the form given and record the quotients in the proper spaces.

	$\frac{3}{5}$	$\frac{7}{8}$
$\frac{5}{6}$		
$\frac{8}{3}$		
$\frac{3}{4}$		
$\frac{2}{5}$		
$\frac{4}{9}$		
$\frac{5}{4}$		
$\frac{3}{8}$		
$\frac{5}{12}$		
$\frac{8}{9}$		

Miscellaneous Problems

1. If $\frac{1}{3}$ of a number is 5, what is the number?

2. If $\frac{2}{3}$ of a number is 12, what is the number?

3. If $\frac{4}{7}$ of a number is 20, what is the number?

4. If $\frac{3}{5}$ of $\frac{2}{3}$ of a number is 15, what is the number?

5. After spending $\frac{1}{4}$ of his monthly salary for board and room, and $\frac{1}{10}$ of it for clothes, a man has \$78 left. What is his salary?

6. If 5 yards of cloth cost $47\frac{1}{2}$¢, what will 1 yard cost at the same rate?

7. A train runs $\frac{2}{3}$ of a mile in $\frac{5}{6}$ of a minute. What is the rate per hour?

8. Which will give the larger result, multiplying or dividing an integer by a proper fraction? Multiplying or dividing by an improper fraction?

9. If you are told the amount of a man's wages and the fractional part of his wages which he spent, how can you find the fractional part which he saved?

10. State two ways of finding the amount saved, and state which you think is the easier.

11. If you were told what $\frac{3}{4}$ of $\frac{2}{3}$ of a number is, how could you find the number?

12. A man owned $\frac{3}{4}$ of a farm and sold $\frac{1}{3}$ of what he owned. If he received \$3,000 for the land sold, what was the value of the entire farm at the same rate?

13. A mechanic works 8 hours a day. Two and one half hours are spent at a bench and the remainder at a machine. What fractional part of the day is spent at a machine?

14. In a certain family $\frac{1}{4}$ of the yearly income is spent for rent, $\frac{1}{5}$ for clothes, $\frac{1}{4}$ for food, and $\frac{1}{2}$ of the remainder for travel. If the yearly income is \$3,000, how much is spent for travel?

15. A man sold $\frac{2}{3}$ of an acre of land for \$76. At that rate what is his farm of 120 acres worth?

16. $\frac{4}{7}$ of the number of students in a high school are girls. The number of boys is 120. How many students are there in the school?

17. Mr. Jones bought some land and sold it so as to realize $\frac{1}{5}$ more than the cost. If the selling price was \$360, what did he pay for the land?

18. Mr. Williams sold some goods for $\frac{1}{6}$ less than the cost. If he received \$70 for the goods, what was the cost?

19. In making up a certain cake recipe for 6 people, $1\frac{1}{2}$ cups of sugar and $\frac{2}{3}$ of a cup of butter are used. How much sugar and butter should be used in making up the recipe for 4 people?

20. A boy works $3\frac{1}{2}$ days at the rate of \$6.50 a week of 6 days. How much does he earn?

21. Find the total cost of $8\frac{2}{3}$ yards of ribbon at 28 cents per yard, $3\frac{1}{2}$ yards of insertion at $12\frac{1}{2}$ cents per yard, and $5\frac{1}{3}$ yards of silk at \$1.35 per yard.

22. A dealer bought oranges at the rate of 4 for 10 cents and sold them at the rate of 3 for 10 cents. How much did he gain on each orange?

23. Mr. Rankin owned $\frac{2}{3}$ of a store and sold $\frac{3}{5}$ of his share to Mr. Johnson for \$3,600. At this rate what was the store worth?

24. If a man drives his automobile 18 miles in $\frac{2}{3}$ of an hour, how long would he require, at the same rate, to travel 60 miles?

25. When $\frac{3}{4}$ of a yard of cloth costs \$1.80, what is the price of $\frac{1}{3}$ of a yard?

26. A girl grew $1\frac{1}{4}$ inches during one year and $\frac{5}{8}$ of an inch during the next year. How much more did she grow during the first year than during the second?

27. The entire length of a skirt is to be $24\frac{3}{4}$ inches, and the ruffle at the bottom is to be $2\frac{7}{8}$ inches wide. What will be the length of the skirt above the ruffle?

28. The record for the hundred yard dash is $9\frac{3}{5}$ seconds. A boy can run the distance in $11\frac{1}{5}$ seconds. What is the difference between his time and the record time?

29. A farmer raised $296\frac{1}{3}$ bushels of wheat on $15\frac{1}{4}$ acres. What was the average yield per acre?

30. Three and one half bushels of seed were sown and the yield was $21\frac{1}{2}$ bushels. What was the average yield per bushel of seed?

Short Methods Involving Fractions

55. To add fractions when the numerators are the same.

Add the denominators and multiply this sum by the common numerator to obtain the numerator of the result. Multiply the denominators to obtain the denominator of the result. Reduce to simplest form.

Examples. 1. Add $\frac{1}{3}$ and $\frac{1}{7}$.

Solution. $3 + 7 = 10$, the numerator of the result.
$3 \times 7 = 21$, the denominator of the result.
Hence, $\frac{1}{3} + \frac{1}{7} = \frac{10}{21}$.

2. Add $\frac{2}{5}$ and $\frac{3}{9}$.

SOLUTION. $5 + 9 = 14$. $14 \times 2 = 28$, the numerator of the result.
$5 \times 9 = 45$, the denominator of the result.
Hence, $\frac{2}{5} + \frac{3}{9} = \frac{28}{45}$.

Oral Work

Add as indicated:

1. $\frac{1}{5} + \frac{1}{3}$.	**2.** $\frac{1}{7} + \frac{1}{9}$.	**3.** $\frac{1}{5} + \frac{1}{8}$.	**4.** $\frac{1}{9} + \frac{1}{16}$.
5. $\frac{1}{7} + \frac{1}{8}$.	**6.** $\frac{1}{12} + \frac{1}{7}$.	**7.** $\frac{1}{9} + \frac{1}{14}$.	**8.** $\frac{2}{7} + \frac{2}{13}$.
9. $\frac{1}{3} + \frac{1}{6}$.	**10.** $\frac{2}{3} + \frac{2}{7}$.	**11.** $\frac{2}{11} + \frac{2}{7}$.	**12.** $\frac{3}{5} + \frac{3}{7}$.
13. $\frac{3}{8} + \frac{3}{16}$.	**14.** $\frac{3}{8} + \frac{3}{11}$.	**15.** $\frac{3}{8} + \frac{3}{5}$.	**16.** $\frac{2}{3} + \frac{2}{5}$.

56. To subtract fractions when the numerators are the same.

Subtract the denominators and multiply this difference by the common numerator to form the numerator of the result. Multiply the denominators to form the denominator of the result. Reduce to simplest form.

Example. Subtract $\frac{2}{5}$ from $\frac{2}{3}$.

SOLUTION. $5 - 3 = 2$. $2 \times 2 = 4$, the numerator of the result.
$5 \times 3 = 15$, the denominator of the result.
Hence, $\frac{2}{3} - \frac{2}{5} = \frac{4}{15}$.

Oral Work

Perform the following subtractions:

1. $\frac{1}{6} - \frac{1}{8}$.	**2.** $\frac{1}{7} - \frac{1}{13}$.	**3.** $\frac{1}{8} - \frac{1}{12}$.
4. $\frac{2}{7} - \frac{2}{15}$.	**5.** $\frac{3}{7} - \frac{3}{16}$.	**6.** $\frac{5}{9} - \frac{5}{14}$.

57. To add two fractions by "cross multiplication."

Multiply the numerator of each fraction by the denominator of the other fraction. Add these products to form the numerator. Multiply the denominators to form the denominator. Reduce to lowest terms.

Example. Add $\frac{2}{3}$ and $\frac{3}{4}$.

SOLUTION. $2 \times 4 = 8$.
$3 \times 3 = 9$.
$9 + 8 = 17$, the numerator.
$3 \times 4 = 12$, the denominator.
Hence, $\frac{2}{3} + \frac{3}{4} = \frac{17}{12}$, $\frac{17}{12} = 1\frac{5}{12}$.

Oral Work

Add as indicated:

1. $\frac{3}{5} + \frac{2}{7}$.
2. $\frac{2}{3} + \frac{3}{7}$.
3. $\frac{1}{2} + \frac{3}{8}$.
4. $\frac{3}{10} + \frac{7}{16}$.
5. $\frac{3}{9} + \frac{7}{8}$.
6. $\frac{3}{5} + \frac{7}{8}$.
7. $\frac{5}{12} + \frac{3}{8}$.
8. $\frac{6}{7} + \frac{6}{11}$
9. $\frac{5}{7} + \frac{9}{16}$.

58. To subtract fractions by "cross multiplication."

Multiply the numerator of each fraction by the denominator of the other fraction. Subtract these products to form the numerator. Multiply the denominators to form the denominator. Reduce to lowest terms.

Example. Subtract $\frac{2}{3}$ from $\frac{5}{6}$.

SOLUTION.
$5 \times 3 = 15$.
$6 \times 2 = 12$.
$15 - 12 = 3$, the numerator.
$6 \times 3 = 18$, the denominator.

HENCE, $\frac{5}{6} - \frac{2}{3} = \frac{3}{18}$, $\frac{3}{18} = \frac{1}{6}$.

Written Work

Subtract as indicated:

1. $\frac{4}{7} - \frac{1}{3}$.
2. $\frac{8}{11} - \frac{3}{5}$.
3. $\frac{5}{8} - \frac{3}{10}$.
4. $\frac{7}{8} - \frac{7}{11}$.
5. $\frac{8}{15} - \frac{4}{11}$.
6. $\frac{13}{25} - \frac{4}{11}$.
7. $\frac{5}{9} - \frac{6}{7}$.
8. $\frac{4}{5} - \frac{2}{7}$.

59. To find the approximate product of mixed numbers.

Multiply the integers. Multiply each integer by the other fraction to the nearest unit. Add these three products.

Example. Multiply $13\frac{1}{2}$ by $6\frac{2}{3}$.

SOLUTION.

$$\begin{array}{r} 13\frac{1}{2} \\ 6\frac{2}{3} \\ \hline 13 \times 6 = 78 \\ \frac{1}{2} \text{ of } 6 = 3 \\ \frac{2}{3} \text{ of } 13 = 9 \text{ (to the nearest unit).} \\ \hline 90 \end{array}$$

Written Work

Find approximate products:

1. $43\frac{1}{3}$	2. $89\frac{1}{4}$	3. $76\frac{1}{4}$	4. $15\frac{2}{5}$	5. $98\frac{1}{6}$
$62\frac{1}{5}$	$7\frac{1}{2}$	$82\frac{1}{7}$	$40\frac{2}{7}$	$47\frac{1}{3}$
6. $17\frac{3}{5}$	7. $13\frac{3}{7}$	8. $79\frac{2}{5}$	9. $38\frac{2}{5}$	10. $73\frac{5}{8}$
$46\frac{4}{5}$	$16\frac{1}{4}$	$63\frac{1}{5}$	$71\frac{1}{6}$	$96\frac{7}{8}$

60. To find the product of any two numbers ending in $\frac{1}{2}$.

(*a*) When the sum of the integers is an even number.

To the product of the integers, add one half of their sum and annex $\frac{1}{4}$ to the result.

Example. Multiply $39\frac{1}{2}$ by $3\frac{1}{2}$.

SOLUTION.

$39\frac{1}{2}$
$3\frac{1}{2}$
$39 \times 3 = 117$
$\frac{1}{2}$ the sum of 39 and 3 = 21
$138\frac{1}{4}$ *Result.*

Written Work

Multiply:

1. $44\frac{1}{2}$	**2.** $18\frac{1}{2}$	**3.** $38\frac{1}{2}$	**4.** $75\frac{1}{2}$	**5.** $29\frac{1}{2}$	**6.** $81\frac{1}{2}$
$64\frac{1}{2}$	$12\frac{1}{2}$	$14\frac{1}{2}$	$35\frac{1}{2}$	$33\frac{1}{2}$	$75\frac{1}{2}$

(*b*) When the sum of the integers is an odd number.

To the product of the integers, add the result obtained by taking half of one less than their sum. To this result annex $\frac{3}{4}$.

Example. Multiply $39\frac{1}{2}$ by $6\frac{1}{2}$.

SOLUTION.

$39\frac{1}{2}$
$6\frac{1}{2}$
234
22
$256\frac{3}{4}$

Written Work

Multiply:

1. $73\frac{1}{2}$	**2.** $57\frac{1}{2}$	**3.** $92\frac{1}{2}$	**4.** $35\frac{1}{2}$	**5.** $98\frac{1}{2}$
$42\frac{1}{2}$	$24\frac{1}{2}$	$47\frac{1}{2}$	$58\frac{1}{2}$	$17\frac{1}{2}$
6. $29\frac{1}{2}$	**7.** $83\frac{1}{2}$	**8.** $76\frac{1}{2}$	**9.** $43\frac{1}{2}$	**10.** $98\frac{1}{2}$
$36\frac{1}{2}$	$96\frac{1}{2}$	$19\frac{1}{2}$	$46\frac{1}{2}$	$23\frac{1}{2}$
11. $48\frac{1}{2}$	**12.** $95\frac{1}{2}$	**13.** $73\frac{1}{2}$	**14.** $47\frac{1}{2}$	**15.** $27\frac{1}{2}$
$72\frac{1}{2}$	$81\frac{1}{2}$	$71\frac{1}{2}$	$33\frac{1}{2}$	$71\frac{1}{2}$

General Review of Common Fractions and the Fundamental Operations

Add the following:

1. $\frac{1}{5}+\frac{1}{6}$	$\frac{1}{7}+\frac{1}{8}$	$32\frac{1}{2}+17\frac{1}{3}$	$43\frac{3}{4}+9\frac{2}{5}$
2. $\frac{1}{2}+\frac{1}{3}$	$\frac{1}{6}+\frac{3}{4}$	$37\frac{2}{3}+24\frac{1}{2}$	$82\frac{2}{5}+37\frac{7}{10}$
3. $\frac{4}{5}+\frac{2}{3}$	$\frac{1}{2}+\frac{1}{5}$	$42\frac{4}{5}+31\frac{3}{4}$	$46\frac{5}{6}+17\frac{2}{3}$

Subtract the following:

4. $\frac{3}{5}-\frac{1}{2}$	$47\frac{4}{5}-31\frac{1}{2}$	$198\frac{4}{5}-74\frac{2}{3}$	$92\frac{3}{4}-74\frac{1}{8}$
5. $\frac{2}{5}-\frac{1}{8}$	$26\frac{3}{4}-18\frac{2}{5}$	$149\frac{3}{4}-34\frac{7}{8}$	$35\frac{2}{3}-18\frac{1}{5}$
6. $\frac{1}{3}-\frac{1}{5}$	$83\frac{2}{3}-27\frac{1}{2}$	$187\frac{2}{3}-83\frac{5}{6}$	$43\frac{5}{6}-29\frac{2}{3}$

Multiply the following:

7. $\frac{7}{8}\times\frac{3}{16}$	$3\frac{1}{3}\times 62\frac{1}{2}$	$72\frac{4}{5}\times 45$	$142\frac{1}{2}\times 408\frac{2}{3}$
8. $\frac{2}{3}\times\frac{4}{15}$	$5\frac{1}{2}\times 48$	$38\times 17\frac{5}{6}$	$36\frac{1}{3}\times 60\frac{1}{2}$
9. $\frac{5}{6}\times 12\frac{1}{2}$	$6\frac{1}{3}\times 7\frac{1}{4}$	$24\frac{1}{5}\times 4\frac{1}{4}$	$16\frac{2}{3}\times 37\frac{1}{2}$

Divide the following:

10. $\frac{3}{4}\div\frac{7}{8}$	$2\frac{1}{2}\div\frac{7}{8}$	$16\frac{2}{3}\div\frac{2}{3}$	$18\frac{2}{3}\div 4$
11. $\frac{5}{8}\div\frac{2}{3}$	$3\frac{3}{4}\div 5\frac{1}{2}$	$87\frac{1}{2}\div\frac{5}{4}$	$\frac{2}{3}\div 4\frac{2}{3}$
12. $\frac{7}{8}\div\frac{4}{5}$	$\frac{1}{3}\div 2\frac{2}{5}$	$160\div\frac{2}{5}$	$160\div 3\frac{1}{3}$

13. A woman ordered the following amounts of meat during a week: $1\frac{3}{4}$ lb., $1\frac{1}{2}$ lb., $2\frac{3}{4}$ lb., 1 lb., $1\frac{7}{8}$ lb., $1\frac{2}{3}$ lb., $2\frac{1}{3}$ lb. How many pounds of meat did she buy?

14. How many pieces of cloth each $3\frac{1}{3}$ yards long can be cut from a piece of goods containing 40 yards?

15. In drawing plans for a house an architect wishes to draw a line that shall be $\frac{2}{3}$ as long as one that is $\frac{7}{8}$ in. long. How long shall he draw the line?

16. A fourpenny nail is $1\frac{3}{8}$ in. long and a sixpenny nail is $\frac{5}{8}$ in. longer. How much shorter is the sixpenny nail than a tenpenny nail the length of which is $2\frac{3}{4}$ in.?

17. A turkey weighing $9\frac{3}{4}$ lb. was bought at $31\frac{1}{2}$ cents per pound. What did the turkey cost?

Add and check:

18.	**19.**	**20.**	**21.**
$473\frac{1}{2}$	$7{,}098\frac{2}{5}$	$478\frac{5}{6}$	$8{,}372\frac{1}{4}$
$289\frac{1}{3}$	$4{,}860\frac{1}{3}$	$362\frac{2}{3}$	$4{,}326\frac{1}{5}$
$706\frac{2}{5}$	$9{,}287\frac{5}{8}$	$748\frac{5}{12}$	$7{,}029\frac{3}{4}$
$483\frac{5}{8}$	$746\frac{5}{16}$	$392\frac{4}{5}$	$8{,}270\frac{5}{6}$
$729\frac{3}{4}$	$982\frac{1}{6}$	$807\frac{1}{6}$	$4{,}207\frac{1}{3}$

22. What is the effect on the value of a proper fraction when each term is multiplied or divided by the same number? Illustrate.

What is the effect when the numerator only is multiplied by a number? Illustrate.

23. Make a blank form, record the following purchases, and compute the total amount:

$17\frac{1}{2}$ yd.	muslin	at	17	cents	per yard
$15\frac{3}{4}$ yd.	lace	at	48	"	" "
$5\frac{2}{3}$ yd.	silk	at	$1.68	"	" "
$6\frac{1}{2}$ yd.	serge	at	1.56	"	" "
$3\frac{2}{3}$ yd.	ribbon	at	65	"	" "

24. For a period of four years beginning with his thirteenth birthday a boy grew $1\frac{3}{4}$ inches the first year, $1\frac{1}{2}$ inches the second year, $1\frac{3}{4}$ inches the third year, and $1\frac{7}{8}$ in. the fourth year. How many inches did he grow during the four years?

25. A gallon of water weighs $8\frac{1}{3}$ pounds and a gallon of mercury is $13\frac{3}{5}$ times as heavy. What is the weight of a gallon of mercury?

26. An auto truck travelled $102\frac{5}{8}$ miles on Monday, $104\frac{2}{3}$ miles on Tuesday, $83\frac{1}{2}$ miles on Wednesday, $94\frac{5}{6}$ miles on Thursday, $71\frac{5}{6}$ miles on Friday, and $83\frac{1}{4}$ miles on Saturday. Compute the total distance travelled during the six days.

27. The rear wheel of a certain bicycle is $7\frac{1}{3}$ feet in circumference, and revolves $2\frac{1}{2}$ times as often as the pedals. How many miles are travelled in 2,000 revolutions of the pedals? (1 mile is 5,280 feet.)

28. How many acres are there in five tracts of land, the first of which contains $135\frac{2}{3}$ acres, the second, $82\frac{5}{8}$ acres, the third, $253\frac{1}{2}$ acres, the fourth, $48\frac{7}{8}$ acres, and the fifth, $76\frac{1}{5}$ acres?

29. Add across and down. Do the results check?

a.
$$47\tfrac{1}{2} + 23\tfrac{1}{3} + 15\tfrac{1}{6}$$
$$23\tfrac{2}{3} + 39\tfrac{5}{8} + 18\tfrac{1}{3}$$
$$37\tfrac{5}{8} + 82\tfrac{2}{3} + 54\tfrac{2}{3}$$
$$46\tfrac{1}{2} + 87\tfrac{2}{3} + 42\tfrac{5}{6}$$

b.
$$37\tfrac{3}{4} + 18\tfrac{2}{3} + 19\tfrac{5}{12}$$
$$42\tfrac{2}{3} + 37\tfrac{1}{6} + 72\tfrac{3}{4}$$
$$17\tfrac{5}{6} + 29\tfrac{3}{4} + 47\tfrac{1}{6}$$
$$31\tfrac{1}{4} + 92\tfrac{5}{6} + 35\tfrac{2}{3}$$

Copy, perform the multiplication or division and check the following:

30. 4,786 × 392

31. 8,409 × 756

32. 3,892 × 708

33. 42,873 × 5,496

34. $287\overline{)18{,}476}$

35. $429\overline{)30{,}871}$

36. $403\overline{)72{,}876}$

37. $921\overline{)341{,}278}$

38. A plate of glass $22\frac{3}{4}$ inches by $28\frac{1}{8}$ inches is set in a picture frame that covers it $\frac{3}{8}$ inches from each edge. What are the dimensions of the glass not covered by the frame?

39. A certain recipe calls for 1 cup molasses, $\frac{1}{2}$ cup shortening, $2\frac{1}{2}$ cups flour, 1 tablespoon ginger, 1 tablespoon soda, 2 tablespoons milk, 1 teaspoon salt. What quantities of each should be used for one and one-half portions of this recipe?

40. The following is a recipe for spiced currants:

7 lbs. currants	4 tablespoons cinnamon
5 lbs. brown sugar	3 tablespoons cloves
	1 pint vinegar

Determine the quantities to be used in making a one-third portion of this recipe; also for one and one-half portions.

41. A cubic foot of pure ice weighs 56 pounds and a cubic foot of water weighs $62\frac{1}{2}$ pounds. Each weight is what fractional part of the other?

42. By how much does the thickness of a $1\frac{1}{2}$ inch board exceed the length of a $\frac{7}{8}$ inch screw?

43. If you know $\frac{3}{4}$ of the cost of a lot, what multiplication would you perform to find the entire cost?

44. Which is the larger of each of the following pairs of fractions? What is the difference between the two fractions?

a. $\frac{1}{2}$ or $\frac{1}{3}$	*e.* $\frac{5}{8}$ or $\frac{3}{4}$	*i.* $\frac{7}{8}$ or $1\frac{3}{16}$
b. $\frac{1}{8}$ or $\frac{1}{7}$	*f.* $\frac{9}{16}$ or $\frac{2}{3}$	*j.* $\frac{5}{8}$ or $\frac{7}{12}$
c. $\frac{1}{5}$ or $\frac{1}{10}$	*g.* $\frac{7}{10}$ or $\frac{2}{3}$	*k.* $\frac{3}{8}$ or $\frac{4}{9}$
d. $\frac{3}{5}$ or $\frac{3}{4}$	*h.* $\frac{3}{5}$ or $\frac{5}{8}$	*l.* $\frac{5}{12}$ or $\frac{3}{8}$

45. Under what conditions will the quotient be larger than the dividend? Give several illustrations.

46. Under what conditions will the quotient be smaller than the dividend? Give several illustrations.

Problems without Numbers

1. If you know the quotient of two fractions and one of the fractions, how can you find the other fraction?

2. If you know the product of two fractions and one of the fractions, how can you find the other fraction?

3. What must you know and what must you do to find the average size of several tracts of land?

4. If you know the total cost of some land and the cost per acre, how do you find the number of acres?

5. If you know the quotient, remainder, and dividend, how would you find the divisor?

6. If you add the same number to both terms of a proper fraction, what is the effect on the value of the fraction? Illustrate.

7. If you know the amount that each of three persons invests in a certain partnership, how could you find the part of the profits to which each is entitled on the basis of investment?

8. If you know the circumference of an automobile wheel, how could you find how many times the wheel will turn in going half a mile?

9. If you know what part of an hour it takes a train to go a mile, how could you find how far the train would go in an hour at the same rate?

10. What are the advantages of cancellation? On what principle does it depend? Illustrate.

11. If you know the number of cubic feet of water in a tank and the weight of a cubic foot of water, how could you find the weight of the water in the tank?

CHAPTER VIII

DECIMAL FRACTIONS

61. Importance of Decimal Fractions. Decimal Fractions, because of their frequent use in business, are really the common fractions of today. It is of the greatest importance that the student of business arithmetic acquire speed and accuracy in computations involving decimal fractions.

62. Comparison of Common and Decimal Fractions.

The denominator of a common fraction may be any number, and it is always expressed.

The denominator of a decimal fraction is always some power of 10, and the power is indicated by the number of figures to the right of the decimal point.

Thus, $.7 = \frac{7}{10}$. $.0007 = \frac{7}{10,000}$.

$.07 = \frac{7}{100}$. $.423 = \frac{423}{1,000}$.

$.007 = \frac{7}{1,000}$. $.027 = \frac{27}{1,000}$.

63. Reading and Writing Decimals.

The names of the various decimal orders are stated in the following table:

.	1	2	3	4	5	6	7
	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths	Ten-millionths

The names of the various orders should be learned. You should know that the *name* of the third decimal order is thousandths; that of the fifth is hundred-thousandths; that of the second is hundredths, etc.

To read a decimal fraction, read the number as if it were an integer and then state the name of the last decimal order which a digit of the number occupies.

Pronounce the word "and" at the decimal point and omit it elsewhere, except in reading complex decimals, as noted below:

.045 is read forty-five thousandths.
.505 is read five hundred five thousandths.
500.005 is read five hundred *and* five thousandths.
.4206 is read four thousand two hundred six ten-thousandths.
.0$\frac{2}{3}$ is read two thirds of a tenth.
2.0$\frac{2}{3}$ is read two *and* two thirds of a tenth.
.2$\frac{2}{3}$ is read two and two thirds tenths (a complex decimal).

Read:

1. .043.	**2.** .1764.	**3.** 2.04.
4. 500.17.	**5.** 400.004.	**6.** .00$\frac{2}{3}$.
7. 214.0014.	**8.** .000562.	**9.** 64.3.
10. 14.003.	**11.** .003726.	**12.** 542.54.
13. 3,005.016.	**14.** .00024$\frac{1}{3}$.	**15.** 7.00707.

Write decimally:

1. Seven tenths; forty-two thousandths; five hundred four thousandths.

2. Four hundred and three thousandths; eight hundred two ten-thousandths.

3. Three fourths of a tenth; two and one seventh tenths.

4. Twenty-four ten-thousandths; five millionths; seventeen hundred-thousandths; two hundred and three hundredths.

5. Five hundred seventeen millionths; sixteen ten-thousandths; eighteen hundred and fifteen ten-thousandths.

6. Five thousand four hundred-millionths; eighteen and four thousandths.

64. Addition of Decimal Fractions.

Place the decimals so that the decimal points fall vertically. Add as with integers. Place the decimal point of the sum directly below those in the numbers added.

Example. Find the sum of 4.023, .507, 2.003, and 1.125.

SOLUTION.

$$\begin{array}{r} 4.023 \\ .507 \\ 2.003 \\ 1.125 \\ \hline 7.658 \end{array}$$

Written Work

1. Add 2,569.327, 1,462.978, 47.9634, 2,693.072, and .019.
2. Add 27.9548, 91.0005, 37.427, and 27,563.974.
3. Add 2,752.9374, .0003, 23.247, and 259.6347.

Write the following common fractions as decimal fractions, and add:

4. $\frac{24}{100}, \frac{372}{1{,}000}, \frac{19}{10}, \frac{254}{100}$.
5. $\frac{3}{1{,}000}, \frac{24}{10{,}000}, \frac{675}{10}, \frac{170}{10{,}000}$.
6. $\frac{36.5479}{100}, \frac{762}{1{,}000}, \frac{7642}{10{,}000}, \frac{271}{1{,}000}$.
7. Copy, find the totals required, and check:

$ 324.80	$3,764.20	$7,436.08	?
17.04	762.83	427.36	?
9,320.87	9,402.03	92.18	?
473.26	1.18	4,724.63	?
791.14	13,420.03	924.56	?
?	?	?	?

8. The receipts in a department store are shown in the table below. What were the total receipts for each day, for each department, and for the week?

	DEP'T. I	DEP'T. II	DEP'T. III	DEP'T. IV	DEP'T. V	TOTAL
Monday	$418.30	$ 67.08	$492.70	$702.30	$280.40	
Tuesday	214.06	43.90	386.40	347.20	165.90	
Wednesday	185.30	120.85	297.80	436.10	380.45	
Thursday	542.65	106.92	380.70	290.18	260.92	
Friday	320.80	231.40	920.40	482.70	192.73	
Saturday	435.90	192.60	870.60	395.17	243.65	
Total						

65. Subtraction of Decimal Fractions.

Place the decimals so that the points fall vertically. Subtract as with integers, then place the decimal point in the remainder immediately below the decimal points above.

Example. Subtract 23.213 from 34.047.

SOLUTION.

$$\begin{array}{r} 34.047 \\ 23.213 \\ \hline 10.834 \end{array}$$

Subtract as indicated. Check the results.

1. 394.237 − 1.027.
2. .47 − .003.
3. .394237 − .15.
4. 7,763.421 − 28.796.
5. 3.027 − .489
6. 87.5932 − 2.3579.

Write as decimal fractions and subtract:

7. $\frac{465}{1,000} - \frac{23}{100}$.
8. $\frac{7,6943}{100} - \frac{235}{1,000}$.
9. $\frac{8,732}{10} - \frac{236,547}{100,000}$.

10. The sum of two numbers is 342.086. One of the numbers is 206.78. What is the other number?

11. A man deposited \$5,764.80 in a bank. He later drew out \$3,780.92 and then deposited \$4,814.60. How much had he then?

12. The daily bank deposits and withdrawals of a merchant during a week were as follows:

DAYS	DEPOSITS	WITHDRAWALS
Monday	\$486.30	\$211.85
Tuesday	718.90	392.18
Wednesday	143.65	417.21
Thursday	638.95	243.62
Friday	531.80	428.30
Saturday	782.45	1,243.90

Compute the difference between the deposits and the withdrawals for the week.

66. Multiplication of Decimal Fractions.

Multiply as with integers. From the right of the product, point off as many places as the sum of the number of decimal places in the multiplicand and the multiplier.

Examples. 1. Multiply 4.625 by .05.

SOLUTION.

$$\begin{array}{r} 4.625 \\ .05 \\ \hline .23125 \end{array}$$

2. Multiply .00362 by .06.

SOLUTION.

$$\begin{array}{r} .00362 \\ .06 \\ \hline .0002172 \end{array}$$

Oral Work

1. $.7 \times .03$.	**2.** $.08 \times .4$.	**3.** $.06 \times .09$.
4. $.25 \times .06$.	**5.** $.026 \times .5$.	**6.** 4.5×4.
7. $.009 \times .8$.	**8.** $.025 \times .10$.	**9.** $.25 \times .008$.
10. $.030 \times .09$.	**11.** $.08 \times .018$.	**12.** $.0125 \times .4$.

Written Work

Multiply as indicated. Check the results.

1. $25.763 \times .1463$.	**2.** $75.46 \times .03$.
3. $.2462 \times .347$.	**4.** $.083 \times .0462$.
5. $78.23 \times .000007$.	**6.** $17.13 \times .042$.
7. $570.04 \times .00326$.	**8.** $\$374.26 \times .055$.
9. $\$9{,}037.28 \times .035$.	**10.** $\$473.54 \times .335$.

11. Find the cost of 46 lots at an average cost of $847.60.

12. Find the cost of 143 barrels of apples at an average price of $5.64 per barrel.

67. Division of Decimal Fractions.

a. When the divisor is an integer.

Examples. 1. $.12 \div 4 = .03$; just as $\$12 \div 4 = \$3$.

2. $.426 \div 3 = .142$; just as $\$426 \div 3 = \$142$.

In making the division the decimal point should be written in the quotient when it is reached in the dividend.

3. Divide 157.25 by 37.

SOLUTION.

$$\begin{array}{r} 4.25 \\ 37\overline{)157.25} \\ 148 \\ \hline 92 \\ 74 \\ \hline 185 \\ 185 \\ \hline \end{array}$$

b. When the divisor is not an integer.

Multiply both dividend and divisor by the least power of ten that will make the divisor an integer. Divide as in case *a*.

Example. Divide .028 by .4.

SOLUTION. First multiply both dividend and divisor by 10, to make the divisor an integer. Then divide the resulting dividend, .28, by 4. The result is .07.

Similarly, $.0456 \div .03 = 4.56 \div 3 = 1.52$.
Similarly, $5.64 \div .0004 = 56{,}400 \div 4 = 14{,}100$.

If there is a remainder after all of the decimal places in the dividend have been used, zeros may be annexed to the dividend and the division may be carried as far as is desired.

Oral Work

1. .08 by .4.	**2.** .014 by .2.	**3.** .364 by .04.
4. 3,300 by .11.	**5.** 8.48 by .008.	**6.** .220 by .0011.
7. 1 by .01.	**8.** .10 by .1.	**9.** .018 by 18.
10. .0001 by .00001.	**11.** .042 by 2.1.	**12.** 1.1 by .011.

Written Work

1. 268,632 ÷ 36.9.	**2.** 26,863.2 ÷ 36.9.
3. 26,863.2 ÷ 369.	**4.** 2,686.32 ÷ 36.9.
5. 26.8632 ÷ 36.9.	**6.** 2,686.32 ÷ .369.
7. 26.8632 ÷ .369.	**8.** 26.8632 ÷ .0369.
9. 2.68632 ÷ .369.	**10.** 2.68632 ÷ 3.69.
11. 2.68632 ÷ 36.9.	**12.** 2.68632 ÷ 369.
13. .268632 ÷ 369.	**14.** 2.68632 ÷ 3690.
15. 293.45 ÷ 14.24.	**16.** 2.734 ÷ 1.32.
17. .73469 ÷ 127.9638.	**18.** 762.397 ÷ 36,947.28.
19. .1479376 ÷ 293.4798.	**20.** .0097 ÷ 12.34692.

21. The multiplier is .045, the product is .01665. What is the multiplicand?

22. At \$.44 per dozen, how many eggs can be bought for \$3.52?

23. If a man's annual income is \$5,420 and his annual expenses are \$4,262, what are his average weekly savings?

24. What number is $\frac{3}{7}$ as large as .0427?

25. What number divided by 4.28 gives a quotient of .07 and a remainder of .04?

26. In a certain factory 30 men are employed at \$2.15 per day, 10 men at \$3.60 per day, 28 men at \$2.90 a day, and 12 men at \$1.80 a day. Find the average daily wage.

27. Using the current market price find the cost of the following: 1 pk. of apples, $\frac{1}{2}$ bu. of sweet potatoes, $\frac{1}{2}$ lb. of tea, 6 bars of laundry soap, $1\frac{1}{2}$ lb. porterhouse steak.

28. A train ran at the rate of .87 mile a minute. At this rate how many miles would it travel in $3\frac{1}{2}$ minutes? In 10 minutes? In 1 hour?

29. An automobile traveled 72.25 miles in $2\frac{1}{3}$ hours. What was the average rate of speed per hour?

68. Changing Decimal Fractions to Equivalent Common Fractions.

Omit the decimal point; write for the denominator 1 *with as many zeros annexed as there are places in the decimal. Reduce to lowest terms.*

Examples. 1. Change .75 to a common fraction.

SOLUTION. $.75 = \frac{75}{100} = \frac{3}{4}$.

2. Change .00864 to a common fraction.

SOLUTION. $.00864 = \frac{864}{100000} = \frac{27}{3125}$.

Written Work

Change the following decimals to equivalent common fractions and reduce to lowest terms:

1. .48.	**2.** .095.	**3.** .3705.	**4.** .0012.	**5.** 3.6425.
6. .125.	**7.** .9825.	**8.** .625.	**9.** .1875.	**10.** .0015.
11. $.06\frac{1}{4}$.	**12.** $.002\frac{1}{2}$.	**13.** $.37\frac{1}{2}$.	**14.** .333.	**15.** $.012\frac{1}{2}$.
16. $.06\frac{2}{3}$.	**17.** .00625.	**18.** .0036.	**19.** $.008\frac{1}{3}$.	**20.** $.009\frac{1}{11}$.
21. $.01\frac{1}{4}$.	**22.** $.08\frac{1}{3}$.	**23.** $.83\frac{1}{3}$.	**24.** $.87\frac{1}{2}$.	**25.** $.11\frac{1}{9}$.

69. Changing Common Fractions to Equivalent Decimal Fractions.

It is frequently desirable to reduce common to decimal fractions in order that the required operations may be performed more easily and in order that the values may be compared more easily.

Since a common fraction may be regarded as an indicated division,

the reduction may be made by the methods of division previously explained. Although most common fractions do not have an exact decimal equivalent, it is possible by continuing the division to find the equivalent as approximately as necessity requires.

Example. Change $\frac{4}{5}$ to a decimal fraction.

SOLUTION. $\frac{4}{5} = \frac{1}{5}$ of 4. Place a decimal point to the right of the 4, annex zeros, and divide.

$$5\overline{)4.0} \quad .8$$

Similarly, $\frac{7}{8} = \frac{1}{8}$ of 7.

$$8\overline{)7.000} \quad .875$$

Similarly, $\frac{2}{3}\frac{3}{7}\frac{7}{5} = \frac{1}{3}\frac{1}{7}\frac{1}{5}$ of 2.

$$375\overline{)2.0000} \quad .0053+$$

$$\begin{array}{r} 1875 \\ \hline 1250 \\ 1125 \\ \hline 125 \end{array}$$

It is usually not necessary to carry the division more than three or four places.

For example, $\frac{1}{7}$ may be expressed decimally as $.142\frac{8}{7}$ or as .1429–.

There are some common fractions whose decimal equivalents should be memorized. These are listed below and are further discussed in Chapter IX.

IMPORTANT EQUIVALENTS

$.50 = \frac{1}{2}$.	$.16\frac{2}{3} = \frac{1}{6}$.	$.06\frac{2}{3} = \frac{1}{15}$.	$.37\frac{1}{2} = \frac{3}{8}$.
$.33 = \frac{1}{3}$.	$.12\frac{1}{2} = \frac{1}{8}$.	$.06\frac{1}{4} = \frac{1}{16}$.	$.62\frac{1}{2} = \frac{5}{8}$.
$.25 = \frac{1}{4}$.	$.10 = \frac{1}{10}$.	$.05 = \frac{1}{20}$.	$.87\frac{1}{2} = \frac{7}{8}$.
$.20 = \frac{1}{5}$.	$.08\frac{1}{3} = \frac{1}{12}$.	$.25 = \frac{1}{25}$.	$.83\frac{1}{3} = \frac{5}{6}$.

Written Work

Considering the figures in the top row of the following form as numerators, and those at the left as denominators, change the common fractions thus formed to decimals, entering the results in the proper places. Carry the results to three decimal places.

Prepare the work on a carefully ruled form.

Denominators	Numerators								
	3	7	6	26	47	126	79	814	1216
18									
27									
94									
38									
172									
9									
66									
213									
793									
2361									
43									
109									

70. Approximate Results. For most practical purposes it is unnecessary to carry a result to more than three decimal places. Thus, \$25.86347 is stated with sufficient accuracy as \$25.86.

In finding approximate decimal fractions, follow these directions.

Carry the division at least one more place than the number of digits desired in the result. If the digit at the right of the last place desired in the result is less than 5, *drop it; if it is* 5 *or larger than* 5, *increase the preceding digit by* 1.

Review of Decimal Fractions

Written Work

1. If a boy's average gain in weight is 8.25 lb. per year, how much will he gain in weight in $2\frac{1}{2}$ yr.?

2. If one turn of a screw advances the point .14 in., how far will 7 turns advance it?

3. A gallon of milk weighs 8.622 lb. and a gallon of water weighs 8.355 lb. How much more does $\frac{1}{2}$ gal. of milk weigh than $\frac{1}{2}$ gal. of water?

4. A boy can run 100 yd. in 11.2 sec. At this rate, how long would it take him to run 80 yd.?

5. In a certain city the rainfall during March was 2.46 in., during April 3.15 in., and during May 1.09 in. Find the average for these months.

6. A man sold 24 dozen eggs, which was .4 of all that he had. How many did he have?

7. If .375 of an acre of land sells for \$48, what should $2\frac{1}{2}$ acres sell for, at the same rate?

8. An English pound contains 113.00001 grains of pure gold, and an American gold dollar contains 23.22 grains of pure gold. Find the par value of an English pound in American money.

9. A nautical mile is 6,080 feet. Express this as a decimal of the statute mile (5,280 feet).

10. The record for the 100 yard dash is $9\frac{3}{5}$ seconds. How many yards per second is this?

11. An American bushel contains 2,150.42 cubic inches. What decimal part of a bushel is a cubic foot (1,728 cubic inches)?

12. If your school team played 11 games during the season and won 8 of them, what part of the games played did the team win? (State the result to the nearest hundredth.)

13. The heating surface of a certain boiler is 4,195.25 sq. feet and the grate surface is 86.5 sq. feet. How many square feet of heating surface are there to each square foot of grate surface?

14. The distance which a freely falling object will fall from any height in any number of seconds may be found by multiplying 16.08 feet by the square of the number of seconds. (The square of a number is the number multiplied by itself.) Compute the distance that an object will fall in 2 seconds; in 5 seconds; in 12 seconds?

Compute the distance that an object will fall during the third second.

15. A man paid \$34.50 for an automobile tire. At the end of 5,480 miles the tire is worthless. Compute the cost per mile for this tire.

16. The total area of the United States is 3,026,788 square miles and the population in 1920 was 105,708,771. What was the population per square mile?

17. The land area of Massachusetts is 8,039 square miles and the population in 1920 was 3,852,356. What was the population per square mile? (Carry results to decimals.)

The land area of Texas is 262,398 square miles. If Texas were as densely populated as Massachusetts what would its population be?

What would the population of your state be, if it were as densely populated as Massachusetts?

18. A gallon contains 231 cubic inches; a cubic foot, 1,728 cubic inches. How many gallons of water can be placed in a vessel which has a capacity of 1 cubic foot?

19. Compute the weight of a gallon of water, if a cubic foot of water weighs 62.5 pounds.

20. The following table shows the number of games won and the number lost by each of the teams of a certain school during a season. Which made the best record? (Express the fractions as decimals.)

TEAMS	GAMES WON	GAMES LOST
Reds	7	5
Whites	6	4
Blues	2	7
Blacks	5	7
Greens	7	4

CHAPTER IX

SHORT METHODS INVOLVING ALIQUOT PARTS

An *aliquot part* of a number is contained in the given number an integral number of times. Thus, 2, $2\frac{1}{2}$, $3\frac{1}{3}$, and 5 are aliquot parts of 10. Similarly, 10, $12\frac{1}{2}$, $16\frac{2}{3}$, 20, and 50, are aliquot parts of 100.

71. Equivalent Common and Decimal Fractions in Frequent Use.

$.50 = \frac{1}{2}$.	$.12\frac{1}{2} = \frac{1}{8}$.	$.06\frac{1}{4} = \frac{1}{16}$.	$.60 = \frac{3}{5}$.
$.33\frac{1}{3} = \frac{1}{3}$.	$.11\frac{1}{9} = \frac{1}{9}$.	$.05 = \frac{1}{20}$.	$.62\frac{1}{2} = \frac{5}{8}$.
$.25 = \frac{1}{4}$.	$.10 = \frac{1}{10}$.	$.04 = \frac{1}{25}$.	$.75 = \frac{3}{4}$.
$.20 = \frac{1}{5}$.	$.09\frac{1}{11} = \frac{1}{11}$.	$.03\frac{1}{3} = \frac{1}{30}$.	$.80 = \frac{4}{5}$.
$.16\frac{2}{3} = \frac{1}{6}$.	$.08\frac{1}{3} = \frac{1}{12}$.	$.40 = \frac{2}{5}$.	$.83\frac{1}{3} = \frac{5}{6}$.
$.14\frac{2}{7} = \frac{1}{7}$.	$.06\frac{2}{3} = \frac{1}{15}$.	$.37\frac{1}{2} = \frac{3}{8}$.	$.87\frac{1}{2} = \frac{7}{8}$.

These equivalents should be memorized, as they can be used to shorten many computations.

72. A Short Method of Multiplying by These Fractions.

Which is easier, $1{,}248 \times .16\frac{2}{3} = 208$; or $\frac{1}{6}$ of $1{,}248 = 208$?

State a short method of multiplying by each of the decimal fractions in the table above.

Exercise

Multiply:

1. $642 \times .50$.
2. $936 \times .33\frac{1}{3}$.
3. $488 \times .25$.
4. $650 \times .20$.
5. $366 \times .16\frac{2}{3}$.
6. $840 \times .14\frac{2}{7}$.
7. $720 \times .12\frac{1}{2}$.
8. $456 \times .08\frac{1}{3}$.
9. $930 \times .06\frac{2}{3}$.
10. $320 \times .06\frac{1}{4}$.

The results in the preceding exercise are integers. The same method may be used when the result is a fraction.

Example. Multiply 4,634 by .25.

SOLUTION. $.25 = \frac{1}{4}$.
$4634 \div 4 = 1158.5$.

Exercise

Without copying, state the results of the following:

1. $8{,}735 \times .50$.
2. $3{,}940 \times .33\frac{1}{3}$.
3. $267 \times .25$.
4. $379.3 \times .16\frac{2}{3}$.
5. $842 \times .20$.
6. $3{,}824 \times .14\frac{2}{7}$.
7. $5{,}316 \times .12\frac{1}{2}$.
8. $4{,}026 \times .08\frac{1}{3}$.
9. $1{,}848 \times .06\frac{2}{3}$.
10. $1{,}940 \times .06\frac{1}{4}$.
11. $35.69 \times .33\frac{1}{3}$.
12. $649.3 \times .25$.
13. $.8347 \times .50$.
14. $723.68 \times .20$.
15. $36.92 \times .14\frac{2}{7}$.
16. $56.32 \times .12\frac{1}{2}$.
17. $7{,}836 \times .06\frac{1}{4}$.
18. $1{,}223 \times .16\frac{2}{3}$.
19. $3{,}380 \times .06\frac{2}{3}$.
20. $290 \times .06\frac{2}{3}$.

21. Find the value of each of the following:

48 lb. at $37\frac{1}{2}¢$ per pound.
32 lb. at $12\frac{1}{2}¢$ per pound.
21 lb. at $87\frac{1}{2}¢$ per pound.
24 yd. at $12\frac{1}{2}¢$ per yard.
30 yd. at $62\frac{1}{2}¢$ per yard.
21 lb. at $14\frac{2}{7}¢$ per pound.
48 yd. at $6\frac{2}{3}¢$ per yard.
96 lb. at $83\frac{1}{3}¢$ per pound.

73. A Short Method of Dividing by These Fractions.

Which is easier, $6 \div .25 = 24$; or $6 \times 4 = 24$?

State a short method of dividing by each of the decimal fractions in the table on page 91.

Exercise

Without copying, state the results of the following:

1. $7 \div .12\frac{1}{2}$.
2. $49 \div .50$.
3. $126 \div .33\frac{1}{3}$.
4. $63 \div .14\frac{2}{7}$.
5. $8 \div .06\frac{1}{4}$.
6. $10 \div .06\frac{2}{3}$.
7. $15 \div .25$.
8. $1{,}627 \div .20$.
9. $13.63 \div .16\frac{2}{3}$.
10. $1.43 \div .25$.
11. $3.249 \div .50$.
12. $.023 \div .08\frac{1}{3}$.
13. $63.98 \div .2$.
14. $.043 \div .16\frac{2}{3}$.

74. Aliquot Parts which are not Unit Fractions.

State a short method of multiplying by $.37\frac{1}{2}$; by $.66\frac{2}{3}$; by .75; by $.87\frac{1}{2}$.

State a short method of dividing by .37½; by .66⅔; by .75; by .87½.

When dividing by one number and multiplying by another, observe the following directions:

Divide first if the division results in an integer. If the division would result in a mixed number, multiply first.

Examples. 1. Multiply 32 by .37½.

SOLUTION. .37½ = ⅜.
32 ÷ 8 = 4.
4 × 3 = 12. Since 8 is a factor of 32, the division is performed first.

2. Divide 16 by .37½.

SOLUTION. Substituting the equivalent common fraction, we have
16 ÷ ⅜ = ?
or 16 × ⅜ = ? Inverting the divisor.
16 × 8 = 128.
128 ÷ 3 = 42⅔. Since 3 is not a factor of 16, the multiplication is performed first.

What advantage is there in observing these directions?

Exercise

Without recopying, find the results of the following:

1. 64 × .375.	2. 96 × .66⅔.	3. 480 × .75.
4. 168 × .875.	5. 248 × .87½.	6. 3,260 × .75.
7. 4,864 × .37½.	8. 639 × .66⅔.	9. 42 ÷ .37½.
10. 56 ÷ .87½.	11. 40 ÷ .66⅔.	12. 36 ÷ .75.
13. 1,462 ÷ .66⅔.	14. 465 ÷ .375.	15. 11,322 ÷ .75.
16. 994 ÷ .875.	17. 426 × .875.	18. 143 × .37½.
19. 1,347 × .75.	20. 539 × .66⅔.	21. 3.29 × .75.
22. 1.267 × .875.	23. 139 ÷ .87½.	24. 1,426 ÷ .37½.
25. 139 ÷ .66⅔.	26. 46 ÷ .75.	27. 46.9 ÷ .37½.
28. .429 ÷ .66⅔.		

75. General Table of Aliquot Parts. The following table shows the most frequently used decimal parts of 1, 10, 100, and 1,000.

Fractions	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{6}$	$\frac{5}{6}$
of 1	.50	$.33\frac{1}{3}$	$.66\frac{2}{3}$	.25	.75	$.16\frac{2}{3}$	$.83\frac{1}{3}$
of 10	5.	$3.33\frac{1}{3}$	$6.66\frac{2}{3}$	2.5	7.5	$1.6\frac{2}{3}$	$8.33\frac{1}{3}$
of 100	50.	$33.33\frac{1}{3}$	$66.66\frac{2}{3}$	25.	75.	$16\frac{2}{3}$	$83.3\frac{1}{3}$
of 1,000	500.	$333\frac{1}{3}$	$666\frac{2}{3}$	250.	750.	$166\frac{2}{3}$	$833.3\frac{1}{3}$

Fractions	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{16}$
of 1	$.14\frac{2}{7}$	.125	.375	.625	.875	$.08\frac{1}{3}$	$.06\frac{2}{3}$	$.06\frac{1}{4}$
of 10	1.42	1.25	3.75	6.25	8.75	$.83\frac{1}{3}$	$.66\frac{2}{3}$	.625
of 100	14.28	12.5	37.5	62.5	87.5	$8.3\frac{1}{3}$	$6.6\frac{2}{3}$	6.25
of 1,000	142.8	125.	375.	625.	875.	$83\frac{1}{3}$	$66\frac{2}{3}$	62.5

Study this table until you can recognize quickly these fractional parts of 1, 10, 100, and 1,000.

76. Rules for Multiplication by Aliquot Parts of 1, 10, 100, and 1,000.

a. To multiply by fractional parts of 10.

Multiply by the equivalent fraction; multiply this product by 10, *by moving the decimal point one place to the right, annexing a zero, if necessary.*

Examples. 1. Multiply 16 by $2\frac{1}{2}$.

Solution. $2\frac{1}{2}$ is $\frac{1}{4}$ of 10. $\frac{1}{4}$ of 16 = 4. Multiply by 10, by annexing a zero; the result is 40.

2. Multiply 26 by 1.25.

Solution. 1.25 is $\frac{1}{8}$ of 10. $\frac{1}{8}$ of 26 = 3.25. Move the decimal point one place to the right; the result is 32.5.

b. To multiply by fractional parts of 100.

Multiply by the equivalent common fraction; multiply this product by 100.

Examples. 1. Multiply 24 by $87\frac{1}{2}$.

Solution.
$87\frac{1}{2}$ is $\frac{7}{8}$ of 100.
$\frac{7}{8}$ of 24 = 21.
$21 \times 100 = 2100$.

2. Multiply 4,252 by 12.5.

Solution.
12.5 is $\frac{1}{8}$ of 100.
$\frac{1}{8}$ of 4,252 = 531.5.
$531.5 \times 100 = 53150$.

Exercise

State a short method of multiplying by each of the following.

$.06\frac{1}{4}$;	2.50;	$6\frac{2}{3}$;	$33\frac{1}{3}$;	250;	$87\frac{1}{2}$;	6.25;
$37\frac{1}{2}$;	7.50;	5;	$333\frac{1}{3}$;	75;	125;	$8\frac{1}{3}$;
$2\frac{1}{2}$;	12.50;	$6\frac{1}{4}$;	$62\frac{1}{2}$;	$1\frac{1}{4}$;	$8\frac{3}{4}$;	25;
$66\frac{2}{3}$;	375;	750;	875;	37.50;	7500;	12,500.

Find the products of the following. Do the work orally when possible. Be prepared to state how the short method was applied in each example.

1. 32×25.	**2.** $111 \times 3\frac{1}{3}$.	**3.** $390 \times 66\frac{2}{3}$.
4. 648×250.	**5.** 724×75.	**6.** $832 \times 1\frac{1}{8}$.
7. $96 \times 87\frac{1}{2}$.	**8.** $108 \times 83\frac{1}{3}$.	**9.** $3{,}264 \times .625$.
10. $144 \times 8\frac{1}{3}$.	**11.** 80×875.	**12.** 48×375.
13. 464×12.50.	**14.** $24 \times 7\frac{1}{2}$.	**15.** 3.20×25.
16. $.27 \times 6\frac{2}{3}$.	**17.** 8.4×3.75.	**18.** 640×6.25.
19. $3.612 \times .83\frac{1}{3}$.	**20.** $208 \times 3\frac{3}{4}$.	**21.** 143×250.
22. $1{,}936 \times 62.50$.	**23.** $1{,}721 \times 7.50$.	**24.** $1{,}239 \times 125$.
25. 426×750.	**26.** $420 \times .87\frac{1}{2}$.	**27.** $369 \times 33\frac{1}{3}$.

77. Interchanging Multiplicand and Multiplier. When the multiplicand and multiplier are abstract numbers, they may be interchanged.

Example. Multiply 25 by 428.

SOLUTION. Interchanging, 428×25
$\frac{1}{4}$ of 428 = 107.
$107 \times 100 = 10{,}700$.

Exercise

Perform the following multiplications as indicated. State how the short method was applied in each example.

1. 924×12.50.	**2.** 864×750.	**3.** 125×488.
4. $264 \times .12\frac{1}{2}$.	**5.** $45 \times .06\frac{2}{3}$.	**6.** $250 \times 3{,}288$.
7. 62.50×648.	**8.** $8\frac{1}{3} \times 3.60$.	**9.** 342×87.50.
10. 375×112.	**11.** $6\frac{2}{3} \times 456$.	**12.** $1{,}468 \times 250$.
13. 875×88.	**14.** $1{,}250 \times 96$.	**15.** $1{,}776 \times .625$.
16. $7.2 \times 8\frac{3}{4}$.	**17.** $1\frac{1}{4} \times 104$.	**18.** 125×1.248.
19. 66×875.	**20.** $83\frac{1}{3} \times 9.6$.	**21.** $62\frac{1}{2} \times 20$.

Find the cost of the following:

22. 12 articles at \$.50 each
16 articles at .25 each
18 articles at .33⅓ each
30 articles at .40 each
16 articles at .75 each
Total _____

23. 64 doz. articles at \$ .62½ per dozen
76 doz. articles at 8.75 per dozen
58 doz. articles at .75 per dozen
125 doz. articles at .16 per dozen
6¼ doz. articles at .32 per dozen
Total _____

24. 72 lb. at \$ .83⅓ per pound
52 lb. at 1.25 per pound
44 lb. at .37½ per pound
90 lb. at .12½ per pound
112 lb. at .06¼ per pound
Total _____

25. 80 yd. at \$ .62½ per yard
66⅔ yd. at 1.20 per yard
86 yd. at 1.25 per yard
25 yd. at .78 per yard
75 yd. at .66⅔ per yard
Total _____

26. 83⅓ yd. at \$1.80 per yard
12½ doz. at 1.60 per dozen
8¾ lb. at 1.60 per pound
6⅔ yd. at 2.10 per yard
3⅓ yd. at 1.80 per yard
Total _____

27. 135 yd. at \$.06¼ per yard
56 yd. at .08¾ per yard
96 yd. at .16⅔ per yard
152 yd. at .12½ per yard
24 yd. at .07½ per yard
Total _____

78. Rules for Division by Aliquot Parts of 1, 10, 100, 1,000.

a. To divide by fractional parts of 1.

Divide by the equivalent common fraction.

Example. Divide 13 by $.12\frac{1}{2}$. SOLUTION. $.12\frac{1}{2} = \frac{1}{8}$.
$13 \div \frac{1}{8} = 13 \times \frac{8}{1} = 104.$

b. To divide by fractional parts of 10.
Divide by 10; *divide this quotient by the equivalent common fraction.*

Example. Divide 80 by 2.50. SOLUTION. 2.50 is $\frac{1}{4}$ of 10.
$80 \div 10 = 8.$
$8 \div \frac{1}{4} = 8 \times \frac{4}{1} = 32.$

c. To divide by fractional parts of 100.

Divide by 100; *divide this quotient by the equivalent common fraction.*

Exercise

1. State a rule for dividing by fractional parts of 1,000.
2. \$100 will pay for how many articles at $\$.33\frac{1}{3}$ each?
3. \$12 will pay for how many yards of cloth at \$.25 per yard?
4. Which is easier —

$14 \div .75 = ?$ or $14 \div \frac{3}{4} = ?$

5. State how to find the total cost of some articles, when the number of articles bought is given and the cost of each is $8\frac{1}{3}¢$; $14\frac{2}{7}¢$; $87\frac{1}{2}¢$; $16\frac{2}{3}¢$; $33\frac{1}{3}¢$; $62\frac{1}{2}¢$; $37\frac{1}{2}¢$.

Without copying, state results for the following examples. State the method of operation in each.

6. $15 \div .12\frac{1}{2}$.
7. $14 \div .06\frac{1}{4}$.
8. $9 \div .25$.
9. \$6 ÷ \$.75.
10. \$14 ÷ \$.87½.
11. \$8 ÷ \$1.33⅓.

State the quotients of each of the following:

	DIVIDEND	DIVISOR	QUOTIENT		DIVIDEND	DIVISOR	QUOTIENT
12.	42	$.33\frac{1}{3}$		13.	13	.25	
14.	22	$.66\frac{2}{3}$		15.	36	.75	
16.	5	$.06\frac{1}{4}$		17.	45	$.37\frac{1}{2}$	
18.	17	.125		19.	28	87.5	
20.	15	1.25		21.	900	6.25	
22.	400	$8.33\frac{1}{3}$		23.	150	$.06\frac{2}{3}$	

Exercise

Complete the following:

	Total Cost	Price Each	Number Purchased
1.	$ 5.25	$ .37½	
2.	20.25	.75	
3.	56.00	.66⅔	
4.	40.00	1.25	
5.	77.00	.87½	
6.	212.50	2.50	
7.	97.50	3.75	

8. Divide each of the following by .12½:

.37½; .33⅓; .625; .87½; .25; 1.14²⁄₇.

9. Find the sum of ¼, ⅕, ⅜, ⅝, ⅞, and ½ and express the sum decimally.

10. Divide 36 by each of the following:

.37½; .12½; .14²⁄₇; .50; .06¼.

11. How many yards can be bought in each case?

Total Cost	Cost per Yard	Number of Yards
$16	$ .25	
24	.33⅓	
36	.37½	
20	.62½	
32	.50	
26	.66⅔	
56	1.25	
44	1.33⅓	
24	2.66⅔	

Review of Short Methods

Perform the following operations, stating what short method was used in each case:

1. $360 \div .37\frac{1}{2}$.	2. $\frac{2}{7} + \frac{3}{8}$.	3. 436×11.
4. 65×25.	5. 463×287.	6. $1{,}016 \times 1{,}005$.
7. $3{,}654 \times 2{,}700$.	8. 105×126.	9. 95×87.
10. $\frac{1}{3} - \frac{1}{5}$.	11. $48\frac{1}{4} \times 63\frac{7}{8}$.	12. $61\frac{1}{2} \times 12\frac{1}{2}$.
13. 109×113.	14. $\frac{9}{16} + \frac{8}{11}$.	15. 463×416.

16. 96×94.
17. $1{,}015 \times 1{,}014$.
18. 13×18.
19. $6\frac{1}{2} \times 4\frac{1}{2}$.
20. 825×927.
21. $.000825 \times 100$.
22. 17×12.
23. 724×999.
24. 116×104.
25. $\frac{3}{5} \div \frac{7}{8}$.
26. 113×102.
27. $1{,}026 \times 1{,}003$.
28. $\frac{1}{5} + \frac{1}{3}$.
29. $5{,}362 \times 111$.
30. 433×99.
31. 109×112.
32. 976×990.
33. $333 \times 12\frac{1}{2}$.
34. 17×19.
35. $13\frac{1}{2} \times 14\frac{2}{3}$.
36. $\frac{3}{7} + \frac{3}{8}$.
37. $\frac{3}{7} - \frac{3}{8}$.
38. $15\frac{1}{2} \times 18\frac{1}{2}$.
39. 96×88.
40. $905.387 \times 3{,}000$.
41. 986×993.
42. 16×18.

General Review Exercises

1. $7{,}205 \div 87\frac{1}{2}$. $360 \times 16\frac{2}{3}$. $648 \times 12\frac{1}{2}$.
   $1{,}240 \div 83\frac{1}{3}$. $48 \times .66\frac{2}{3}$. $246 \div 37\frac{1}{2}$.
   $1{,}940 \times 62\frac{1}{2}$. $189 \times .33\frac{1}{3}$. $147 \div .87\frac{1}{2}$.

2. Find the cost of:

156 articles at	\$12.50 each	420 articles at	\$.37½ each
116 articles at	2.50 each	96 articles at	.66⅔ each
48 articles at	3.33 each	360 articles at	.62½ each

3. At $12\frac{1}{2}$ cents per yard how many yards of cloth can be bought for each of the following amounts: \$4? \$8.50? \$.50? \$10? \$6.50?

4. Copy from dictation, multiply, and check:

a.	*b.*	*c.*	*d.*
140.78	70.926	3.6409	428.5
32.5	1.248	20.8	7.48
e.	*f.*	*g.*	*h.*
87.34	9.284	409.63	246.3
.005	4.56	5.09	.258

5. The monthly rainfall by inches in a certain place was as follows: 2.1, 1.9, 1.8, 2.3, 2, 2.1, 1.4, 1.5, 1.3, 1.5, 1.4, 1.1. What was the average monthly rainfall?

6. At $66\frac{2}{3}$ cents each how many articles will \$8 buy? \$14? \$126? \$180?

7. A girl used seven balls of worsted yarn in making a sweater. Each ball weighed $1\frac{1}{2}$ ounces. What was the weight of the sweater and what did the material cost, the price of yarn being 65 cents a ball?

8. A certain piece of beef lost $\frac{1}{5}$ of its weight in roasting. The roasted piece weighs $3\frac{3}{4}$ lb. What was the weight before roasting?

9. Add and check:

a.	*b.*	*c.*	*d.*
4,876	2,089	9,076	4,807
3,294	7,648	5,492	3,928
7,386	3,975	398	7,096
9,275	8,437	7,863	4,279
8,143	2,978	47	847

UNITS OF MEASURE AND THEIR APPLICATIONS

CHAPTER X

DENOMINATE NUMBERS

MANY of the purchases and sales in modern business life involve measurement. We find it necessary to use frequently the various units to measure weight, distance, area, volume, value, and time. A mastery of the most commonly used tables is necessary.

In the tables the units which are most frequently used are printed in heavy type.

79. Linear Measure.

12 **inches** (in.)	= 1 **foot** (ft.)
3 feet	= 1 **yard** (yd.)
5½ yards, or 16½ feet	= 1 **rod** (rd.)
320 rods, or 5280 feet	= 1 **mile** (mi.)
1,760 yards	= 1 mile

Architects, carpenters, and mechanics frequently write ′ for foot, and ″ for inch. Thus 8′ 7″ means 8 ft. 7 in.

In engineering it is customary to divide the foot and the inch into tenths and hundredths, instead of into halves and fourths.

Other measures of length are:

1 hand = 4 in. Used in measuring the height of horses.
1 fathom = 6 ft. Used in measuring depths at sea.
1 knot, nautical or geographical mile = 1.1526⅞ mi. or 6,089 ft.

The knot is used in measuring distances at sea. It is equivalent to 1 min. of longitude at the equator.

80. Square Measure.

144 **square inches** (sq. in.)	= 1 **square foot** (sq. ft.)
9 square feet	= 1 **square yard** (sq. yd.)
30¼ square yards	= 1 **square rod** (sq. rd.)
160 square rods	= 1 **acre** (A.)
640 acres	= 1 **square mile** (sq. mi.)

Sq. ′ and sq. ″ are frequently used for square foot and square inch. Thus, 15 sq. ′ 6 sq. ″ means 15 sq. ft. 6 sq. in.

An acre contains 43.560 sq. ft.

A *square* is 100 sq. ft. It is used in measuring roofing.

81. Cubic Measure.

1,728 **cubic inches** (cu. in.)	= 1	**cubic foot** (cu. ft.)
27 cubic feet	= 1	**cubic yard** (cu. yd.)
128 cubic feet	= 1	cord

A cubic yard (of earth) is considered a load.

$24\frac{3}{4}$ cubic feet	= 1	perch (P.)

A perch of stone or masonry is $16\frac{1}{2}$ ft. (1 rd.) long, $1\frac{1}{2}$ ft. wide, and 1 ft. high.

82. Avoirdupois Weight.

16 **ounces** (oz.)	= 1	**pound** (lb.)
100 pounds	= 1	hundredweight (cwt.)
2,000 pounds	= 1	**short ton** (T.)
2,240 pounds	= 1	long ton (T.)

Avoirdupois weight is used in weighing all ordinary substances except precious metals, jewels, and drugs at retail.

The long ton is used in measuring coal and iron and in the United States customhouse.

83. Liquid Measure.

4 gills (gi.)	= 1	**pint** (pt.)
2 pints	= 1	**quart** (qt.)
4 quarts	= 1	**gallon** (gal.)
$31\frac{1}{2}$ gallons	= 1	**barrel** (bbl.)

A gallon contains 231 cu. in.

63 gal. = 1 hogshead (hhd.).

In practice hogsheads and barrels differ in size. The standard gasoline barrel has a capacity of 56 gallons.

One cubic foot is $7\frac{1}{2}$ gal. approximately.

84. Dry Measure.

2 **pints** (pt.)	= 1	**quart** (qt.)
8 quarts	= 1	**peck** (pk.)
4 pecks	= 1	**bushel** (bu.)

A bushel contains 2,150.42 cu. in.

A heaped bushel contains 2,747.71 cu. in.

A dry quart contains 67.2 cu. in.

A liquid quart contains 57.75 cu. in.

Many of the states have fixed by statute the weight of a bushel of certain grains and seeds and these must be used in buying and selling unless specific agreements to the contrary are made. Determine these specifications for your state. See Appendix.

85. Measures of Time.

60	**seconds** (sec.)	= 1 **minute** (min.)
60	minutes	= 1 **hour** (hr.)
24	hours	= 1 **day** (da.)
7	days	= 1 **week** (wk.)
30	days	= 1 commercial **month** (mo.)
52	weeks	= 1 **year** (yr.)
12	months	= 1 year
360	days	= 1 commercial year
365	days	= 1 common year
366	days	= 1 leap year
100	years	= 1 century

86. Counting.

12	units	= 1 **dozen** (doz.)
12	dozen	= 1 **gross** (gro.)
12	gross	= 1 great gross (gr. gro.)
20	units	= 1 score

87. Wood Measure.

A straight pile of wood, 4 ft. × 4 ft. × 1 ft., contains 1 cord foot. 8 cord feet, or 128 cubic feet = 1 cord.

88. United States Money.

The principal units of United States money are the cent and the dollar.

10	mills	= 1 **cent**
5	cents	= 1 **nickel**
10	cents	= 1 **dime**
25	cents	= 1 **quarter**
50	cents	= 1 **half dollar**
100	cents	= 1 **dollar**
10	dollars	= 1 eagle

The mill is not a coin, but the term is frequently used in computations such as the levying of taxes. The term "eagle" is not common in business.

89. English Money.

4 farthings (far.) = 1 **penny** (*d.*), (plural, pence)
12 pence = 1 **shilling** (*s.*)
20 shillings = 1 **pound sterling** (£)

1 pound sterling at par is $4.8665. The actual exchange value fluctuates.

90. French Money.

10 millimes (m.) = 1 **centime** (c.)
10 centimes = 1 decime (d.)
10 decimes = 1 **franc** (F.)

The par value of a franc is $.193.

91. German Money.

100 **pfennigs** (pf.) = 1 **mark** (M.)

The par value of a mark is $.238.

UNITS OF MONEY IN OTHER COUNTRIES

The following table gives the principal units of value in several other countries with their approximate par value in United States money.

COUNTRY	PRINCIPAL UNIT	PAR VALUE
Argentine Republic	Peso	$ .96
Brazil	Milrei	.55
Denmark	Krone	.27
Ecuador	Libra	4.86
Japan	Yen	.50
Mexico	Peso (Dollar)	.49
Netherlands	Guilder	.40
Peru	Libra	4.86
Russia	Ruble	.52
Turkey	Lira	4.40

Belgium, Greece, Italy, Roumania, Servia, and Spain use the same monetary system as France. Norway and Sweden use the same system as Denmark.

Written Work

1. Draw a line on the blackboard 1 foot long. Try to form a definite mental picture of this length so that you may be able to estimate lengths with a fair degree of accuracy.

2. Draw lines of the following lengths without aid, then check each estimate.

$\frac{1}{2}$ ft., 2 ft., 3 ft., $\frac{2}{3}$ ft., $1\frac{1}{2}$ ft., 7 ft., 3 in., 9 in., 15 in.

3. Estimate the length and width of your desk; the length and width and height of the classroom; the dimensions of the school building and grounds. Check your estimates.

4. Practice estimating and stepping off various distances until you can do so with a fair degree of accuracy.

5. Lay off on the school grounds a square rod and, if possible, an acre. Estimate the areas of various lots, parks, or fields, and check your results, when possible.

By lifting various units of weights and then checking your own estimates of the weights of numerous objects, you can acquire the ability to estimate certain weights with but a small error.

92. Reduction of Denominate Numbers. A denominate number is reduced by changing the units in which it is expressed, without changing the value. When the units are reduced to those of lower denomination the process is called descending reduction. When they are reduced to units of a higher denomination the process is called ascending reduction.

93. Changing to Lower Denominations.

Example. How many quarts in 3 bu. 3 pk. 5 qt.?

SOLUTION.

3 bu. = 12 pk.
12 pk. + 3 pk. = 15 pk.
15 pk. = 120 qt.
120 qt. + 5 qt. = 125 qt.
Therefore, 3 bu. 3 pk. 5 qt. = 125 qt.

After studying this solution, state a method for reducing measured quantities to lower denominations.

Oral Work

Reduce as indicated; do as much of the work orally as possible.

1. $15\frac{1}{3}$ yd. to feet.
2. $4\frac{2}{3}$ ft. to inches.
3. 5 gal. 3 qt. 1 pt. to pints.
4. 4 bu. 3 pk. 5 qt. to quarts.
5. 5 lb. 9 oz. to ounces (avoir.).

Example. Reduce .345 mi. to lower denominations.

SOLUTION.

1 mi. = 320 rd.
.345 mi. = 110.4 rd
1 rd. = 5.5 yd.
.4 rd. = 2.2 yd.
1 yd. = 3 ft.
.2 yd. = .6 ft.
1 ft. = 12 in.
.6 ft. = 7.2 in.

Therefore, .345 mi. = 110 rd. 2 yd. 7.2 in.

After studying this solution, state a method for reducing fractions of a large denomination to units of lower denominations.

Written Work

Reduce:

1. 8 mi. to rods.
2. .236 mi. to feet.
3. .89 bu. to quarts.
4. $\frac{1}{24}$ sq. mi. to square rods.
5. 62 rds. to feet.
6. .85 gal. to pints.
7. 4.3 mi. to feet.
8. 43 A. to square feet.

94. Changing to Higher Denominations. When a quantity is expressed in a low denomination, it may be desired to express it in higher denominations.

Examples. 1. Express 1,316 pints in higher denominations.

SOLUTION. Since 2 pints = 1 quart.
1316 pints = 658 quarts.
Since 4 quarts = 1 gallon,
658 quarts = 164 gals., and 2 qt. remaining.
Therefore, 1316 pints = 164 gal., 2 qt.

After this studying solution, state a method for reducing quantities to units of higher denominations.

2. Reduce 3 qt. 1 pt. to a decimal part of a peck.

SOLUTION. 3 qt. 1 pt. = 7 pt.
1 pk. = 16 pt.
Therefore, 3 qt. 1 pt. = $\frac{7}{16}$ pk.
$\frac{7}{16}$ pk. = .4375 pk.

Written Work

Reduce to units of higher denominations:

1. 367 pints (dry measure).
2. 133 in.
3. 47 ft.
4. 12 yd.
5. 216 oz. (avoir.).
6. 31 pt (liquid measure).
7. 2,369 sq. in.
8. Reduce 4 sq. ft. 68 sq. in. to a decimal part of a square rod.
9. Reduce 1 qt. 2 pt. to a decimal part of a gallon.
10. Reduce 7 oz. to a decimal part of a pound (avoir.).
11. Reduce $\frac{2}{3}$ yd. to a decimal part of a rod.
12. Reduce $2\frac{1}{2}$ ft. to a decimal part of a yard.
13. Reduce 47 sq. rd. to a decimal part of an acre.
14. Reduce $8\frac{1}{2}$ in. to a decimal part of a foot.
15. Reduce $2\frac{1}{2}$ qt. to a decimal part of a gallon.
16. Reduce 4 ft. 7 in. to a decimal part of a rod.

95. Addition of Denominate Numbers.

Example. Find the sum of 2 yd. 2 ft. 9 in. and 4 yd. 1 ft. 7 in.

SOLUTION.

2 yd.	2 ft.	9 in.
4 "	1 "	7 "
6 yd.	3 ft.	16 in.

or

7 "	1 "	4 "

State a method for adding denominate numbers.

Written Work

Add:

1.

2 yr.	5 mo.	18 da.
5 "	8 "	19 "
6 "	11 "	13 "

2.

5 gal.	3 qt.	1 pt.
4 "	2 "	1 "
5 "	1 "	1 "

3.

2 lb.	5 oz.	17 pwt.
3 "	8 "	19 "
4 "	9 "	12 "

4.

5 A.	126 sq. rd.	
16 "	249 "	"
13 "	168 "	"

5. 7 mo. 12 da.
5 " 5 "

6. 12 mi. 4 rd.
8 " 9 "

7. A rectangular field is 18 rd. 2 yd. 2 ft. long, and 13 rd. 1 yd. $2\frac{1}{2}$ ft. wide. What length of wire will be required for a fence 6 wires high?

8. A dealer bought 17 gal. 3 qt. of milk from each of two men, and 29 gal. 3 qt. 1 pt. from a third man. What was the entire cost at 24 cents a gallon?

96. Subtraction of Denominate Numbers.

If it is required to subtract 1 mi. 42 rd. 13 ft. from 3 mi. 25 rd. 14 ft., we write the numbers as follows:

3 mi.	25 rd.	14 ft.
1 "	42 "	13 "

When, as in this problem, the number of units of some denomination of the minuend is smaller than the number of corresponding units of the subtrahend, combine one unit of the next larger denomination with the number of units of the minuend. The problem thus becomes:

2 mi.	345 rd.	14 ft.
1 "	42 "	13 "
1 "	303 "	1 "

Written Work

Subtract as indicated:

1. 13 bu. 3 pk. 2 qt.
6 " 2 " 7 "

2. 7 A. 5 sq. rd. 6 sq. ft.
3 " 7 " " 2 " "

3. 27 rd. 4 yd. 1 ft.
13 " 5 " 2 "

4. 14 gal. 2 qt. 1 pt.
5 " 3 "

5. Daniel Webster was born Jan. 18, 1782, and lived to the age of 70 yr. 9 mo. 6 da. On what date did he die?

6. A man owned a field containing $1\frac{3}{5}$ A. From it he sold two lots each having an area of $11\frac{1}{2}$ sq. rd. What was the area of the part remaining?

7. From a cask which contained 31 gal. 3 qt. 1 pt. of vinegar, 19 gal. 3 pt. were drawn out. How much remained?

97. Multiplication Involving Denominate Numbers.

Example. Multiply 3 yd. 2 ft. 7 in. by 4.

SOLUTION.

	3 yd.	2 ft.	7 in.
			4
	12 yd.	8 ft.	28 in.
or	15 yd.	1 ft.	4 in.

Written Work

Multiply as indicated:

1. 5 gal. 3 qt. 1 pt. by 7.
2. 7 lb. 5 oz. by 12.
3. 5 cu. yd. 19 cu. ft. 364 cu. in. by 13.
4. 4 mi. 19 rd. 3 yd. by 18.

98. Division Involving Denominate Numbers.

Examples. 1. Divide 4,568 inches by 8.

SOLUTION.

4568 in. ÷ 8 = 571 in.
571 in. = 2 rd. 8 yd. 2 ft. 7 in.

2. Divide 17 gal. 2 qt. 1 pt. by 3.

SOLUTION.

17 gal. 2 qt. 1 pt. = 141 pt.
141 pt. ÷ 3 = 47 pt.
47 pt. = 5 gal. 3 qt. 1 pt.

3. How many times is 4 gal. 2 qt. contained in 1 barrel?

SOLUTION.

4 gal. 2 qt. = 18 qt.
1 bbl. = 126 qt.
126 qt. contains 18 qt. 7 times.

Explain the method used in this illustration.

Written Work

1. Find out at what time the sun will rise and set tomorrow at the place where you live and compute the time from sunrise to sunset.
2. How many times are 4 yd. 2 ft. 7 in. contained in 29 yd. 6 in.?

3. Reduce to the decimal part of a mile: 5 rd.; $7\frac{1}{2}$ rd.; 5 ft.; 13 yd; 5 ft. 9 in.

4. An automobile traveled 1 mi. in $3\frac{1}{2}$ min. What was the rate per hour?

5. If a man sells 17 loads of wheat, each containing 53 bu. 3 pk., at $97\frac{1}{2}$ cents a bushel, how much should he receive?

6. Fifteen cans hold an average of 10 gal. 2 qt. 1 pt. How much do they all hold?

7. Change .427 sq. mi. to units of lower denominations.

8. A cubic foot of water weighs 62.5 lb., cast iron weighs about 7.2 as much as water; what is the weight of $2\frac{3}{4}$ cu. ft. of cast iron?

9. Determine whether the window space in your school room is as much as $\frac{1}{6}$ of the floor space. It should be at least $\frac{1}{6}$ in a well-lighted room.

10. A train leaves a city at 4:10 P.M., and reaches a certain station $87\frac{1}{2}$ mi. distant at 6:50 P.M. Allowing 12 min. for station stops, what is the rate of travel per hour?

11. There are twenty-four students in a class and each needs $2\frac{1}{2}$ cups of hot water for a cooking lesson. How much water is needed to supply all the class at one time, allowing 4 cups to a quart?

General Exercises — Denominate Numbers

1. Lots in a certain section of a city are worth 32 cents a square foot. What is the value of a plot containing .6 acres?

2. A bin contains 2,360 cubic feet. What is its capacity in bushels when level full?

3. A coal dealer bought 1,890 tons of coal by the long ton and sold it all by the short ton. When he sold the coal it had shrunk in weight 40 pounds per long ton. How many short tons of coal did the dealer sell from this purchase?

4. From a barrel which contained 50 gallons of lubricating oil a dealer sold the following quantities: 4 gal. 2 qt., 5 gal. 1 qt., 6 gal. 3 qt. What quantity is left in the barrel?

5. A projectile leaves a certain gun with a velocity of 2,060 feet per second. At this rate how far would it travel in 42 seconds?

6. In a contest an aviator flew at the rate of 130 miles an hour. What was the time per mile? How many feet did he travel per second?

7. How many tons of steel rails will be required to lay a mile of railroad if the rails weigh 63 lb. per yard?

8. Abraham Lincoln was born February 12, 1809. How many years, months, and days ago was that?

9. How many times will a wheel with a circumference of 14 feet revolve in going one mile?

10. A dairyman delivered the following quantities of milk to a hotel during a week. Compute the total quantity.

	GAL.	QT.	PT.
Mon.	19	2	1
Tues.	28	1	
Wed.	27	3	1
Thurs.	19	1	1
Fri.	26	3	
Sat.	27	2	1
Sun.	28	3	1

11. A man can row across a lake, a distance of $2\frac{3}{5}$ mi. in 55 min. At this rate, how long will it take him to row one mile? How far could he row in 1 hour, at this rate?

12. If a cubic foot of ice weighs 57 lb. 8 oz., what will be the weight in tons of 840 cubic feet of ice?

13. Estimate the following distances then, check your estimates by measurement: 100 ft., 10 rd., 50 ft., 30 yd.

14. What place is approximately a mile from your school?

15. Certain flower seeds are to be planted 4 inches apart. How many such seeds should be planted in a row 3 yd. 2 ft. 9 in. long?

16. When the wind is blowing with a pressure of .07 lb. per sq. in. what is the total pressure on a surface of 920 sq. ft. 30 sq. in.?

17. How many cubic feet of air will be required to supply 75 cu. yd. of fresh air per hour for each student in your class?

18. A boy sold $\frac{1}{2}$ bu. of raspberries in pint boxes at 15 cents a box. How much did he receive for them?

19. How many years, months, and days ago was George Washington born?

20. Why is the date, July 4, 1776, of great interest to us? How many years, months, and days have passed since that date?

21. If a watch gains 4 seconds an hour, how much will it gain in two days?

22. At the beginning of a trip the indicator on a man's automobile registered 1,784.7 mi. At the end of the trip it registered 1,949.1 mi.

What was the distance traveled per hour if the trip was made in 6 hours?

23. Milk is usually shipped in 10 gal. cans to stations in cities where it is bottled and distributed to customers. How many quarts in a shipment of 5 such cans?

24. A farmer living near a lake cut his own ice. If he cut 165 cakes averaging 70 lb. each, how much per ton did the ice cost him, the total cost being $9.40?

CHAPTER XI

THE METRIC SYSTEM

THE **metric system** of weights and measures originated in France about 1800. Since that time its use has become universal in scientific measurements, and it is established as the commercial system in a large part of the civilized world. The United States made the metric system "lawful" in 1866, and obligatory in Porto Rico and the Philippine Islands about 1900. The advantage of the metric system lies in the fact that it is based on a decimal scale. The only multiplier or divisor used is 10 or a power of 10. Reductions are made by merely changing the position of the decimal point.

99. Definitions of Terms. The unit of **length** is the **meter.** It is approximately 39.37 inches.

The unit of **capacity** is the **liter** (lēter), which is equivalent to a cube having an edge one-tenth of a meter long.

The unit of **weight** is the **gram,** which is the weight of a cube of distilled water having an edge of $\frac{1}{100}$ of a meter.

100. Fractions and Multiples of Units.

Fractional parts of these units are expressed by Latin prefixes, as follows:

milli- = one thousandth.
centi- = one hundredth.
deci- = one tenth.

Multiples of the units are expressed by Greek prefixes, as follows:

deka- = ten.
hekto- = one hundred.
kilo- = one thousand.
myria- = ten thousand.

Thus, a millimeter is .001 of a meter.
a milligram is .001 of a gram.
a centigram is .01 of a gram.
a dekaliter is 10 liters.

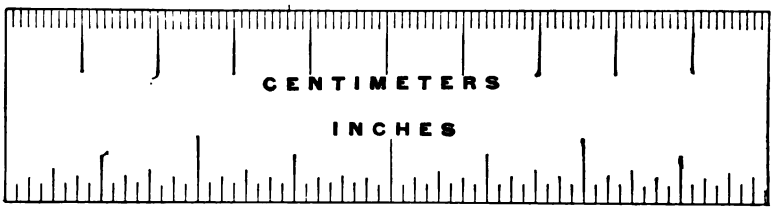
The prefixes and their meanings should be learned.

In the tables which follow the most commonly used measures are printed in heavy type.

101. Measures of Length.

10 **millimeters** (mm.)	= 1 **centimeter** (cm.).
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 **meter** (m.)
10 meters	= 1 dekameter (Dm.).
10 dekameters	= 1 hektometer (Hm.).
10 hektometers	= 1 **kilometer** (Km.).
10 kilometers	= 1 myriameter (Mm.)

The following rule is 10 centimeters, or 1 decimeter, long. The smallest subdivisions are millimeters.



The meter is used for measuring certain merchandise and for other purposes where the foot and yard would be used in the English system. It is also used in engineering and constructions. The kilometer is used for measuring distances which would be expressed in the English system in miles.

The centimeter is used instead of the inch in certain measurements such as in expressing the size of paper and books. The millimeter is used for fine measurements in machine construction and similar work.

Examples. 1. Reduce 473 m. to Km.

SOLUTION. 473 m. = .473 Km.

2. Reduce .54 Km. to m.

SOLUTION. .54 Km. = 540 m.

Oral Work

1. Reduce 734 m. to Hm.; Km.; dm.
2. Reduce .042 Km. to cm.; Hm.; mm.

3. Reduce 5,427 cm. to m.; Km.
4. Reduce 4.72 Km. to dm.
5. Reduce .034 m. to Km.
6. Reduce .0842 Km. to m.

The advantage of the metric system is well illustrated in the measurement of distances. A distance of 1 kilometer, 6 dekameters, 4 decimeters, 8 centimeters, is expressed as 1,060.48 meters, and is read "ten hundred sixty meters and forty-eight centimeters, just as we would say "ten hundred sixty dollars and forty-eight cents."

102. Measures of Area.

The table of area is formed by squaring the units of length, as in the English system.

100 sq. millimeters (sq. mm.)	= 1 **sq. centimeter** (sq. cm.).
100 sq. centimeters	= 1 sq. decimeter (sq. dm.).
100 sq. decimeters	= 1 **sq. meter** (sq. m.).
100 sq. meters	= 1 sq. dekameter (sq. Dm.).
100 sq. dekameters	= 1 sq. hektometer (sq. Hm.).
100 sq. hektometers	= 1 **sq. kilometer** (sq. Km.).

The abbreviations of the system are not uniform, $mm.^2$ is frequently used instead of sq. mm.; etc.

103. Land Measure.

The **are** is ten meters square. It has a side about 33 feet long.

The **hektare**, containing 100 ares, is about $2\frac{1}{2}$ acres. The hektare is the unit of land measure. A quarter section (160 acres) is almost exactly 64 hektares.

Examples. 1. Reduce .4367 sq. m. to sq. Hm.

SOLUTION. 4,367 sq. m. = .4367 sq. Hm.

2. Reduce .0547 sq. Dm. to sq. dm.

SOLUTION. .0547 sq. Dm. = 547 sq. dm.

Oral Work

1. Reduce 427 sq. m. to sq. Dm; sq. Hm.
2. Reduce 37 sq. cm. to sq. m.; sq. mm.
3. Reduce .0354 sq. Hm. to sq. m.; sq. Dm.
4. Reduce .1 sq. m. to sq. cm.; sq. mm.

104. Measures of Volume.

The cubic meter is the unit of volume. It is sometimes called the **stere.**

1,000 cu. millimeters (cu. mm.) = 1 **cu. centimeter** (cu. cm.)
1,000 cu. centimeters = 1 cu. decimeter (cu. dm.)
1,000 cu. decimeters = 1 **cu. meter** (cu. m.)
1,000 cu. meters = 1 cu. dekameter (cu. Dm.)
1,000 cu. dekameters = 1 cu. hektometer (cu. Hm.)
1,000 cu. hektometers = 1 cu. kilometer (cu. Km.)

The abbreviations $mm.^3$, $cm.^3$, etc., are sometimes used.

Examples. 1. Reduce 4,386 cu. m. to cu. Dm.

SOLUTION. 4,386 cu. m. = 4.386 cu. Dm.

2. Reduce .0427 cu. cm. to cu. mm.

SOLUTION. .0427 cu. cm. = 42.7 cu. mm

Oral Work

1. Reduce 37,864 cu. m. to cu. Dm.
2. Reduce .04276 cu. cm. to cu. mm.
3. Reduce 742,863 cu. dm. to cu. Dm.
4. Reduce .0476 cu. m. to cu. cm.

105. Measures of Capacity.

The unit of capacity is the liter, which is a cubic decimeter. It equals 1.05668 liquid quarts or 0.9081 dry quart.

10 milliliters (ml.) = 1 centiliter (cl.)
10 centiliters = 1 deciliter (dl.)
10 deciliters = 1 **liter** (l.)
10 liters = 1 dekaliter (Dl.)
10 dekaliters = 1 **hektoliter** (Hl.)
10 hektoliters = 1 kiloliter (Kl.)

The liter serves the same purpose in measuring capacities as the quart and gallon. The hektoliter is used for measuring quantities commonly measured by the bushel in countries where the English system is used.

Oral Work

1. Reduce 427 l. to Dl.; Hl.; cl.
2. Reduce .043 Dl. to dl.; cl.
3. Reduce .04278 Kl. to l.; dl.; Hl.

106. Measures of Weight.

The gram is the unit of weight. A liter of water weighs a kilogram, or 1000 grams.

10 milligrams (mg.)	= 1	centigram (cg.)
10 centigrams	= 1	decigram (dg.)
10 decigrams	= 1	gram (g.)
10 grams	= 1	dekagram (Dg.)
10 dekagrams	= 1	hektogram (Hg.)
10 hektograms	= 1	kilogram (Kg.)

Oral Work

1. Reduce 4,736 g. to Kg.; cg.; mg.
2. Reduce .03 cg. to mg.; g.; Dg.
3. Reduce .07428 Kg. to g.; dg.; cg.

The kilogram and the " half kilo " are the common measures used in trade.

The metric ton or tonneau (1,000 kilograms) is used for larger weights.

107. Comparative Table of Weights and Measures.

The following table of comparisons will be useful in reducing measurements stated in the English system to equivalent metric measures, and vice versa. It is not to be memorized.

1 inch	= 2.54001 centimeters.
1 foot	= .304801 meter.
1 yard	= .914402 meter.
1 mile	= 1.60935 kilometers.
1 square inch	= 6.452 square centimeters.
1 square foot	= .09290 square meter.
1 square yard	= .8361 square meter.
1 square mile	= 2.59 square kilometers.
1 cubic inch	= 16.3872 cubic centimeters.

1 cubic foot	= .02832 cubic meter.
1 cubic yard	= .7646 cubic meter.
1 acre	= .4047 hectare.
1 liquid quart	= .94636 liter.
1 liquid gallon	= 3.78543 liters.
1 dry quart	= 1.1012 liters.
1 peck	= 8.80982 liters.
1 bushel	= .35239 hectoliter.
1 grain	= .06480 gram.
1 ounce (avoir.)	= 28.3495 grams.
1 ounce (Troy)	= 31.10348 grains.
1 pound (avoir.)	= .45359 kilogram.
1 pound (Troy)	= .37324 kilogram.
1 millimeter	= .03937 inch.
1 centimeter	= .3937 inch.
1 meter	= 39.37 inches.
1 meter	= 3.28083 feet.
1 meter	= 1.093611 yards.
1 kilometer	= .62137 mile.
1 square millimeter	= .00155 square inch.
1 square centimeter	= .155 square inch.
1 square meter	= 1.196 square yards.
1 square kilometer	= .3861 square mile.
1 cubic centimeter	= .061 cubic inch.
1 cubic meter	= 1.3079 cubic yards.
1 hectare	= 2.471 acres.
1 milliliter	= .03381 liquid ounce or .2705 apothecaries' dram.
1 liter	= 1.05668 liquid quarts.
1 liter	= .9081 dry quart.
1 hectoliter	= 2.83774 bushels.
1 gram	= 15.4324 grains.
1 kilogram	= 2.20462 pounds (avoir.).

Exercise

State the result in each of the following in metric units. (Use a meter stick in making the measurements.)

1. Measure the edges of the cover of this book.
2. How many square centimeters in the surface of the cover?

3. How tall are you?

4. Find the dimensions of the top of your desk.

5. What is the length of your classroom? What is its width? What is its height?

6. What is the area of the floor?

(Other measurements may be suggested by the teacher.)

Using the table of comparative weights and measures, make the following reductions:

7. 1 ft. 8 in., to meters.

8. $3\frac{1}{2}$ meters to feet and inches.

9. 6 sq. ft. to square meters.

10. 14 square meters to square yards.

11. 36 cubic inches to cubic centimeters.

12. 130 cubic centimeters to cubic inches.

13. A French dealer ordered 5,500 kilograms of wheat. How many bushels should the broker buy?

14. A certain horse weighs 715 kilograms. How many pounds does he weigh?

15. The distance from Boston to Chicago by rail is 999 miles. How many kilometers?

16. What is the cost of 16 yards of cloth at $1.35 per meter?

17. A barrel (31.5 gal.) will contain how many liters?

18. How long would a tank 4 feet wide and 3 feet high have to be to contain 6 cubic meters? (State your result to the nearest foot.)

19. The Eiffel tower in Paris is about 300 m. high. State this height in feet.

20. Olive oil weighs .92 as much as an equal volume of water. What is the weight of 1 liter of olive oil?

21. If a stream flows at the rate of $1\frac{1}{2}$ Km. per hour, what is its rate of flow per second? Express the result in centimeters.

22. A liter of mercury weighs 13.596 Kg. How many $mm.^3$ of mercury weigh 3 g.?

23. Fifty-four miles are how many kilometers, to the nearest unit?

24. 428 ft. are how many meters, to the nearest unit?

25. Sound travels 332 m. per second, how long will it take it to travel 4.5 Km.?

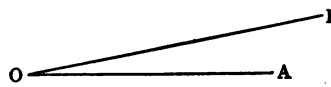
CHAPTER XII

PRACTICAL MEASUREMENTS

108. Mensuration in Business. In many business transactions the measurement of lengths, areas, and volumes is involved. A knowledge of mensuration is necessary in many lines of business.

Plane Figures

109. Rectilinear Figures. An **angle** is the opening between two lines which meet.

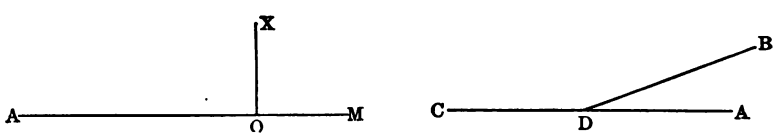


Thus, the angle AOB is formed by the lines AO and OB, which meet (intersect) at O.

The lines which intersect are called the **arms** of the angle and their point of intersection is called the **vertex** of the angle.

Thus, AO and OB are the arms of the angle and O is the vertex.

A **right angle** is an angle formed when one straight line meets another straight line so as to make the adjacent angles equal. The lines forming the angles are said to be **perpendicular** to each other.



Thus, the angles MOX and XOA are right angles and the lines AM and OX are perpendicular to each other.

An **acute** angle is an angle less than a right angle.

An **obtuse** angle is an angle greater than a right angle.

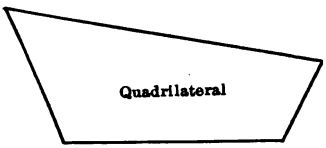
Thus, the angle ADB is acute and the angle BDC is obtuse.

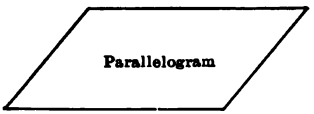
A **surface** has length and breadth but no thickness.

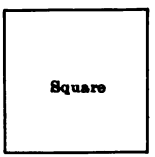
A **plane**, or **plane surface**, is a level surface such as that of still water.

A **plane figure** is a figure all of whose points lie in the same plane.

A **quadrilateral** is a plane figure bounded by four straight lines.
A **parallelogram** is a quadrilateral whose opposite sides are parallel.
A **rectangle** is a parallelogram whose angles are right angles.
A **square** is a rectangle whose sides are all equal.



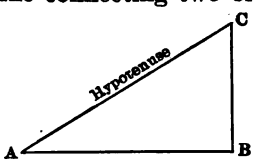




A **diagonal** of a quadrilateral is the straight line connecting two of its opposite vertices.

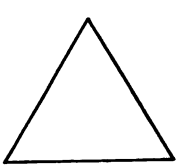
A **triangle** is a plane figure bounded by three straight lines.

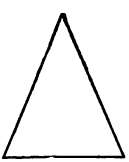
A **right triangle** is one that has a right angle.

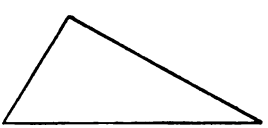


The **hypotenuse** of a right triangle is the side opposite the right angle.

A triangle which has three sides equal is called **equilateral.** If two sides are equal it is called **isosceles.** If it has no equal sides it is called **scalene.**


Equilateral


Isosceles


Scalene

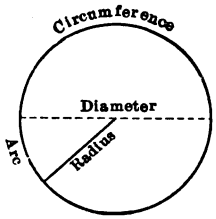
The **perimeter** of a plane figure is the sum of its sides.

A **circle** is a plane figure bounded by a curved line called the circumference, every point of which is equally distant from a point within called the center.

The **diameter** of a circle is a straight line passing through the center and terminated by the circumference.

The **radius** of a circle is the straight line from the center of the circle to the circumference. It is equal to one half of the diameter.

An **arc** of a circle is any part of the circumference.



The **base** of a plane figure is the side on which it is supposed to stand.

The **altitude** of a plane figure is the perpendicular distance from the highest point above the base to the base or to the base produced. An area is always expressed in terms of some square unit, such as square inch, square foot, or square yard.

To find the area of any figure is to find the number of square units it contains.

A square whose side is one unit is said to have an area of one square unit. Thus, a square whose side is one inch has an area of one square inch.

Oral Work

1. Into how many squares 1 in. on a side can you divide a rectangle 5 in. long and 1 in. wide?
2. Into how many squares 1 in. on a side can you divide a rectangle 5 in. long and 3 in. wide?
3. How many square feet in the area of a rectangle 7 ft. long and 3 ft. wide?

Areas of Plane Figures

The number of square units in the area of a rectangle is equal to the number of units in the length times the number of units in the width.

Find the areas of the rectangles having the following dimensions:

4. 12 ft. by 8 ft.
5. $3\frac{1}{2}$ rd. by 6 rd.
6. 25 rd. by 25 rd.
7. 11 in. by 15 in.
8. 15 in. by 15 in.
9. 35 rd. by 35 rd.
10. 16 ft. by 12 ft.
11. 3.5 ft. by 3.5 ft.

Written Work

1. How many acres are there in a rectangular field 130 rd. long and 80 rd. wide?
2. Find the cost of painting a rectangular floor 30 ft. long and 18 ft. wide at 80 cents a square yard.

3. A rectangular floor 24 ft. long and 22 ft. wide is to be covered with tiles 8 in. square. How many tiles will be required?

4. A football field is 120 yards long and $53\frac{1}{3}$ yards wide. How many acres does it contain?

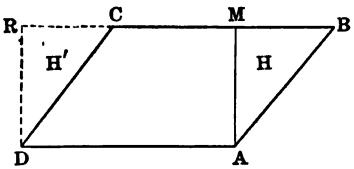
5. A tennis court is 78 ft. long and 36 ft. wide. What part of an acre does it contain?

6. A field 96 rd. by 130 rd. produces 2,340 bu. of wheat. What is the average yield per acre?

7. Find the area of a square field whose perimeter is 128 rd.

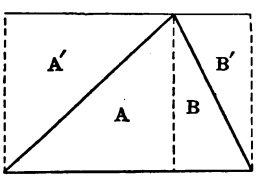
8. A rectangular field is 42 rd. long and 26 rd. wide. Find the cost of fencing it at \$1.20 a rod.

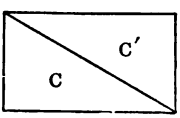
9. Find the cost of painting the four walls of a room 14 ft. long, 10 ft. 6 in. wide, and 9 ft. high at 80 cents per square yard, no allowance being made for openings.



The parallelogram $ABCD$ may be shown to be equivalent to the rectangle $AMRD$ by cutting off the area H and placing it in the position H'.

It is evident that the rule for finding the area of a parallelogram is the same as for finding the area of a rectangle. State the rule.





A triangle may be shown to be equivalent to one half the area of a rectangle with the same base and altitude.

The number of square units in the area of a triangle is equal to one half the number of units in the base times the number of units in the altitude.

Oral Work

State the area of the triangles whose bases and altitudes are as follows:

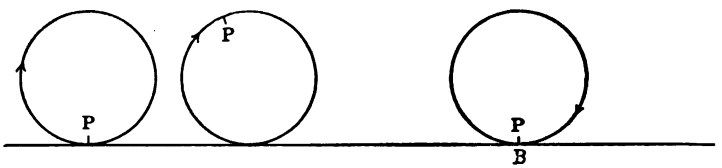
	Base	Altitude		Base	Altitude
1.	10 ft.	6 ft.	2.	12 rd.	7 rd.
3.	4.2 rd.	20 rd.	4.	$10\frac{1}{2}$ ft.	12 ft.
5.	18 yd.	11 yd.	6.	1 ft. 4 in.	9 in.
7.	3 yd. 1 ft.	$2\frac{1}{2}$ ft.	8.	4 ft. 6 in.	20 in.

110. Circumference and Area of a Circle. Find the length of the circumferences of several circles of different sizes. This may be done as follows:

Mark a point on the circumference and roll the circle along a level surface. Determine the distance between the places where the point comes in contact with the level surface.

In the figure below, the length of the line PB is equal to the length of the circumference of the circle.

After the circumferences of several circles of different sizes have been measured, find the ratio of the length of the circumference of each circle to the length of its diameter. Find the average of these ratios. If the measurements were made with care you will find that, whatever the radius of the circle, the length of the circumference divided by the length of the diameter gives a quotient slightly larger than



three. It is shown in geometry that the ratio is always the same and that it is approximately 3.1416 or $\frac{22}{7}$. This constant ratio is called π (pronounced pī). Length of circumference ÷ length of diameter = π.

This is usually expressed as follows: $\frac{C}{D} = \pi$, or $C = \pi D$.

If the diameter of a circle is known, the circumferences may be found by multiplying the diameter by the value of π.

If the circumference is known, the diameter may be found by dividing the circumference by the value of π.

NOTE. For most practical purposes and for all problems in this text, $\frac{22}{7}$ may be used as the value of π.

Written Work

Find the circumferences of the circles with the following diameters:

1. 16 ft.	**2.** 32 in.	**3.** 18 yd.	**4.** 3 ft. 4 in.
5. 9 ft. 2 in.	**6.** 110 yd.	**7.** 2 rd.	**8.** 6 ft. 8 in.
9. 2 rd. 1 yd.	**10.** 7 ft. 2.5 in.	**11.** 3.5 ft.	**12.** 2.8 ft.

Find the diameters of circles with the following circumferences:

13. 110 yd.	**14.** 44 ft.	**15.** 5,000 ft.
16. 5 ft. 6 in.	**17.** 1 mi.	**18.** 48 mi.

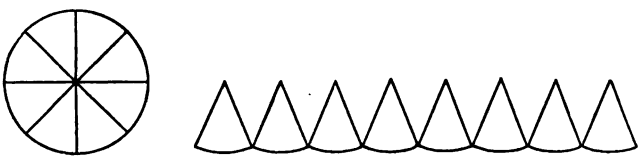
19. An automobile wheel has a diameter of 34 in. How many times will the wheel turn in going 1 mile?

20. The inner circumference of a circular race track is one mile. The track is 90 ft. wide. How long is the outer circumference?

If a circle be divided as shown in the figure below, it is evident that the circle is composed of figures which are approximately triangles. The area of the circle is equal to the sum of the areas of the approximate triangles.

The altitude of each of the triangles is equal to the radius of the circle, and the sum of the bases of the triangles is equal to the circumference of the circle.

We may conclude, therefore, *that the number of units in the area of the circle is equal to one half the number of units in the circumference, times the number of units in the radius.*



This may be expressed as follows: area of circle $= \frac{C \times R}{2}$, where C represents the number of units in the circumference and R the number of units in the radius.

Since $C = \pi D$, or $2\pi R$, the area is equal to $\frac{2\pi R \times R}{2}$, or πR^2.

Hence, *to find the area of a circle, square the radius and multiply the result by* π.

NOTE. The parts into which the circle is divided are not exact triangles but it is proved in geometry that the area of the circle is the same as that of a triangle having a base equal to the circumference and an altitude equal to the radius.

Written Work

1. Find the area of a circle the radius of which is 6 ft.

2. Find the area of a circle the radius of which is 12 ft. Compare with the preceding area.

3. Find the cost of painting a circular area having a diameter of $7\frac{1}{2}$ ft. at 75 cents per square yard.

4. Find the area inclosed by a circular running track with an inner circumference of $\frac{3}{8}$ mi.

5. Find the area of a circular window 6 ft. in diameter.

6. Find the cost of refinishing the top of a circular table which has a diameter of 4 ft. 4 in., at 25 cents per square foot.

Compute the areas of the following:

7. A triangle having a base of 11 in. and an altitude of 9 in.

8. A parallelogram having a base of 12 in. and an altitude of 3.5 in.

9. A circle having a radius of 3 ft. 7 in.

10. A circle having a circumference of 14 ft. 3 in.

11. A rectangle having an altitude of 7.5 ft., and a base of 3.45 ft.

12. A triangle having an altitude of 4 rd. 3 yd., and a base of 12.3 rd.

13. A square which has a perimeter of 14.72 in.

14. What must you know and what must you do to find the area of each of the following: circle; square; triangle; parallelogram; rectangle?

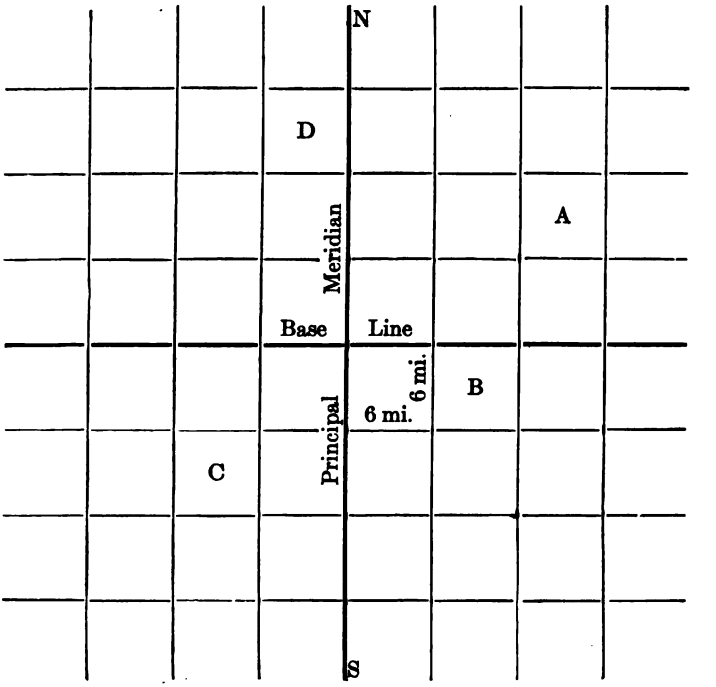
Land Measure

111. Description of Farm Lands.

When the land of the Central and Western states was surveyed the following methods were employed:

a. Imaginary lines running north and south were established and called *principal meridians.* These principal meridians were numbered from 1 to 24. The first runs between Ohio and Indiana and the last through Oregon.

b. Imaginary lines running east and west were established, and called *base lines.* Intersections of meridians and base lines are indicated by stone markers.



c. Lines were then run at intervals of six miles parallel to the principal meridians, and others at intervals of six miles parallel to the base lines.

d. The land was thus divided into tracts 6 miles square. Each such area is called a *township.* The townships are described by their relation to the principal meridians and to the base line.

A row of townships extending north and south is called a *range.* The ranges are designated as east or west.

The township marked *A* is described in real estate records thus:

Twp. 2 N, R 3 E of the —— * principal meridian.

Twp. 2 N means that it is in the second row of townships north of the base line; while R 3 E means that it is in the third row (range) east of the specified principal meridian

Description of *C:* Twp. 2 S, R 2 W.

Describe townships marked *D* and *B.*

What is meant by Twp. 3 N, R 1 W? Twp. 3 S, R 2 E?

* Here is stated the number of the meridian.

e. Each township is divided into 36 sections, each one mile square. Each section, therefore, contains 640 acres. The sections are numbered as shown in the following plot.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

Assuming that this is a plot of township *A* in the figure on page 127, the 25th section would be described as follows:

Section 25, Twp. 2 N, R 3 E of the —— principal meridian.

Fractions of a section are often sold, and the following method of description is used.

The section is divided into quarters.

M is described as the northwest $\frac{1}{4}$ of Section 25, Twp. 2 N, R 3 E.

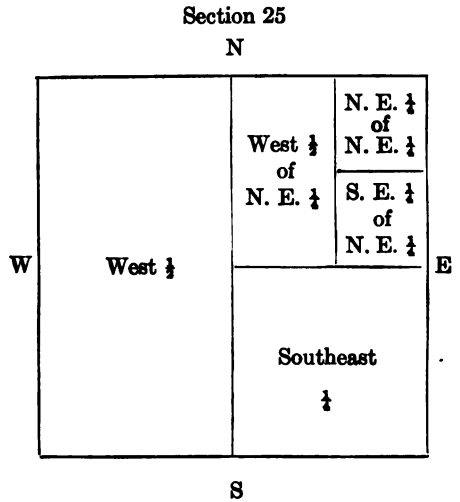
H is described as the southeast $\frac{1}{4}$ of Section 25, Twp. 2 N, R 3 E.

Section 25

North West M	North East 160 Acres Q
South West O	South East H

If M and Q were sold together, they would be described as the north $\frac{1}{2}$ of Section 25, Twp. 2 N, R 3 E.

The following diagram shows a section divided into various farms, with their descriptions.



Written Work

Since a section is a mile square, each side of the section is 320 rd. in length. State the dimensions of each of the farms shown in the above diagram; thus,

The west $\frac{1}{2}$ of Section 25 is 160 rods east and west by 320 rods north and south and contains 320 acres.

112. Measurements of Farm Lands. The unit of farm land measure is the acre, which contains 160 square rods.

Thus, a piece of land 10 rods wide and 16 rods long contains an acre, or a strip of land 1 rod wide and 160 rods long contains an acre.

Oral Work

1. Locate farms on plot of Section 25 having the following dimensions. How many acres in each?

a. 160 rods × 80 rods.
b. 160 rods × 160 rods.
c. 320 rods × 160 rods.
d. 80 rods × 80 rods.

2. The owner of the southwest $\frac{1}{4}$ sells a strip 20 rods wide along the south side of his farm. How much does he receive at \$150 per acre? Describe the property sold.

3. The owner of the S. E. $\frac{1}{4}$ of the N. E. $\frac{1}{4}$ sells the north half of his farm for \$125 per acre.

How much land did he sell? Describe it. What was the total selling price?

Square Root and its Applications

113. Extracting the Square Root. The **square** of a number is the product obtained by using the number twice as a factor. Thus, since $3 \times 3 = 9$; 9 is the square of 3.

The **square root** of a number is one of the two equal factors of the number. Thus, the square of 4 is 16. The square root of 16 is 4.

The square root of a number may be indicated by writing the number under the **radical sign** or by placing "$\frac{1}{2}$" above and to the right of the number. Thus, both $\sqrt{144}$ and $144^{\frac{1}{2}}$ indicate the square root of 144.

Oral Work

What is the square root of each of the following: 36, 64, 81, 169, 225, 25, 49, 1, 256, 400, 100, 10,000?

How many digits in the square of each of the following: 1, 9, 10, 99, 100, 999, 1,000, 9,999?

How does the number of digits in the square of a number compare with the number of digits in the number?

The square of 53 may be found as follows:

$$\begin{array}{r} 50 + 3 \\ 50 + 3 \\ \hline (50 \times 3) + 3^2 \\ 50^2 + (50 \times 3) \\ \hline 50^2 + 2(50 \times 3) + 3^2 \end{array} \qquad \begin{array}{rr} 50^2 & = 2{,}500 \\ 2(50 \times 3) & = \quad 300 \\ 3^2 & = \quad\ 9 \\ \hline & 2{,}809 \end{array}$$

By squaring several numbers, as illustrated above, you will understand that *the square of a number is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.*

This principle may be applied in finding the square root of a number whose square root cannot be found by inspection.

The square of a number contains twice as many digits, or one less than twice as many digits, as the original number. Hence, if an integer be separated into groups or periods of two digits each, from right to left, there will be as many digits in the square root as there are groups of digits in the original number.

Example. Find the square root of 576.

SOLUTION. Beginning at the right, separate the digits into groups of two each. The greatest square in 5 is 4, and the square root of 4 is 2. Two is therefore the tens digit of the root.

Find the remainder and annex the next group (76).

The result is 176. We have taken the square of the tens from the number, hence the remainder (176) must contain twice the product of the tens by the units plus the square of the units. Twice two tens is four tens or 40. 40 is contained 4 times in 176. Hence, 4 is the units' digit of the root. Twice the tens, times the units, plus the square of the units, is the same as the sum of twice the tens and the units, times the units. Therefore, add 4 units to the 40 and multiply the result by 4. The result is then 176. Therefore, the square root of 576 is 24.

```
  2 4
 ---
 5 76
 4
 ---
40|176
 4|176
 ---
44|
```

To find the square root of a number:

Begin at the units and separate the number into groups of two digits each. Find the greatest square in the left-hand group, and write its square root for the first digit of the required root. Subtract the square of the root digit from the left-hand group and then annex the second group for a dividend.

Annex a zero to the part of the root already found and multiply the result by two. Divide the dividend by this product. Place the quotient as the next figure of the root. Add the quotient to the divisor and multiply the result by the digit of the root just found.

Continue in like manner until all of the groups have been used. The result will be the square root. (It may not be the exact square root, as indicated in the note on page 132.)

If the number contains a decimal fraction, begin at the decimal point and separate the number into groups of two digits each way from the decimal point. If the last group on the right of the decimal point has but one figure, annex a zero. Each decimal group must contain two figures.

To find the square root of a common fraction whose numerator and denominator are not perfect squares, first reduce the common fraction to a decimal, then extract the root.

Examples. 1. Extract the square root of 16,641.

SOLUTION.

$$\begin{array}{r|l} & 1\ 2\ 9 \\ \hline 1 & 16\ 64\ 41 \\ 1 & \\ \hline 20 & 66 \\ 2 & 44 \\ \hline 22 & 2241 \\ & \\ \hline 240 & \\ 9 & \\ \hline 249 & 2241 \end{array}$$

2. Extract the square root of 18.0625.

SOLUTION.

$$\begin{array}{r|l} & 4.\ 2\ 5 \\ \hline & 18.\ 06\ 25 \\ & 16 \\ \hline 80 & 206 \\ 2 & 164 \\ \hline 82 & 4225 \\ \hline 840 & \\ 5 & \\ \hline 845 & 4225 \end{array}$$

Written Work

Find the square root of:

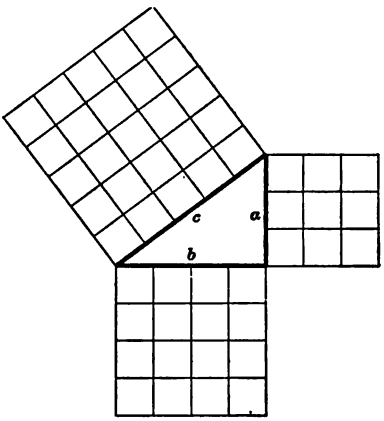
1. 3,025.	**2.** 9,025.	**3.** 89.25.	**4.** .1764.
5. 52.9984.	**6.** .005476.	**7.** 5.	**8.** $\frac{3}{4}$.
9. .0256.	**10.** 3.	**11.** $\frac{2}{5}$.	**12.** 7.

NOTE. If a number is not a perfect square, its approximate square root may be found by placing the decimal point in its proper place and annexing zeros to the right of the last digit. It is usually not necessary to find a root to more than two or three decimal places.

114. The Square and the Right Triangle. Since the number of units in the area of a square is found by squaring the number of units

in one side, it follows that the square root of the number of units in the area is the number of units in one side.

It is proved in geometry that *the square on the hypotenuse of a right triangle is equal to the sum of the squares on the other two sides.* This is illustrated in the figure.



The length of the hypotenuse is equal to the square root of the sum of the squares on the other two sides. To find the length of either side, extract the square root of the difference between the square on the hypotenuse and on the other side.

Written Exercise

1. A square field contains 9025 sq. rd. Find the length of one side.

2. The base lines on a ball field are each 90 ft. in length, and are at right angles to each other. What is the distance from home plate to second base?

3. Is it correct for a carpenter to "square a corner" by means of a right triangle whose sides are 6, 8, and 10 ft. respectively?

4. A ladder 19 ft. long placed on level ground reaches 13 ft. up a wall. How far is the foot of the ladder from the wall?

5. The rafters of a building are 25 ft. long. The ridge of the roof is 18 feet above a line joining the foot of the rafters. How wide is the building?

6. A rectangular corner lot is 90 ft. long and 60 ft. wide. A path runs diagonally across it. What distance is saved by using the path?

7. A guy rope 80 ft. long is attached to a flagpole at a point 54 ft. above the ground. How far from the foot of the pole may the rope be made to touch the ground when stretched taut?

115. To find the area of a triangle when the length of each side is known.

When the altitude of a triangle is not known but the length of the sides is known, the area may be found as in the following:

Problem. Find the area of a triangle the sides of which are 12 ft., 14 ft., and 22 ft.

SOLUTION.

$$12 + 14 + 22 = 48$$
$$\tfrac{1}{2} \text{ of } 48 = 24$$
$$24 - 12 = 12$$
$$24 - 14 = 10.$$
$$24 - 22 = 2.$$
$$24 \times 12 \times 10 \times 2 = 5{,}760.$$

$\sqrt{5{,}760} = 75.88$, the number of square feet in the area.

Rule.

From one half the sum of the three sides subtract each side separately. Find the product of the half sum and of the three remainders. Extract the square root of this product. The result is the number of units in the area.

Written Work

1. Find the area of an equilateral triangle the sides of which are 8 inches.

2. The perimeter of an isosceles triangle is 28 inches. One of the equal sides is 9 inches. Find the area.

3. Compare the area of an equilateral triangle with a side of 4 inches with that of one with a side of 8 inches.

4. Find the value at \$160 an acre of a triangular field having sides of 60 rd., 78 rd., and 84 rd. respectively.

ROOFING

116. Roofing is generally measured by the *square* of 100 square feet.

Shingles average 4 in. in width, and are usually laid 4 in., 4½ in., 5 in., or 5½ in. to the weather. A bunch of shingles contains 250. Part of a bunch is not sold.

The following table shows the usual estimate per square.

EXPOSED TO WEATHER	NUMBER PER SQUARE
4 in.	1,000
4½ in.	900
5 in.	800
5½ in.	700

Slate varies in size. It is usually 16″ by 24″, or 6″ by 12″.

Written Work

1. When shingles are laid 4″ to the weather, how many 4-in. shingles cover a square foot? How many to the square foot when they are 5″ to the weather?

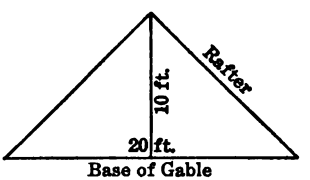
2. A roof is 42 ft. long and 28 ft. wide on each side. How many bunches of shingles will be required to cover it, if the shingles are laid 5 in. to the weather?

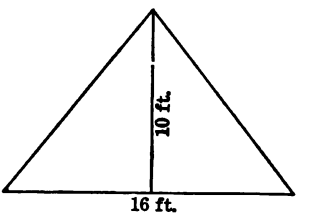
3. At $4.50 a square what will be the cost of tinning a roof 22 ft. by 18 ft.?

4. A roof is 40 ft. long and 30 ft. wide on each side. How many shingles, laid 5 in. to the weather, will be required to cover it?

5. How many bundles of shingles, laid 5 in. to the weather, will be required for a roof 28 ft. by 21 ft.?

The pitch of a roof is found by dividing the rise in the rafters by the width of the base of the gable. If the rafters rise 10 ft. and the base of





the gable is 40 ft., the pitch of the roof is $\frac{1}{4}$. A pitch of $\frac{5}{8}$ is called the Gothic pitch.

Written Work

Find the pitch of the roof in each of the following:

	Width of Base	Height
1.	45 ft.	20 ft.
	36 ft.	18 ft.
	64 ft.	16 ft.
	20 ft.	5 ft.
	24 ft.	15 ft.

2. Find the height of the gable.

Width of Base	Pitch of Roof
50 ft.	$\frac{1}{2}$
20 ft.	$\frac{1}{4}$
32 ft.	Gothic

Flooring

117. Flooring is usually measured by the **square** or by the **thousand square feet.** There is some waste when lumber is "tongued" and "grooved." Dealers measure the width before the lumber is matched. The amount of waste depends upon the width of the lumber. It is customary to allow $\frac{1}{4}$ for flooring 3 in. or more in width and $\frac{1}{3}$ for flooring less than 3 in. in width.

Illustration. How many square feet of 4-in. flooring will be required for a room 28 ft. by 24 ft.?

$28 \times 24 = 672$, the number of square feet to be covered.
$1\frac{1}{4} \times 672$ sq. ft. $= 840$ sq. ft., the quantity required.

Written Work

1. A room is 24 ft. by 36 ft. Find the cost of the $2\frac{1}{2}$-in. flooring at \$50 per thousand square feet.
2. Compute the cost of flooring a hall 72 ft. by 50 ft., with 4-in. lumber at \$30 a thousand, incidentals \$28.40 extra.
3. How many feet of 4-in. flooring will be required for a hall 60 ft. by 42 ft.?
4. What will the 4-in. flooring for a hall 60 ft. by 42 ft. cost at \$36 per thousand?

Plastering, Papering, Painting, Carpeting

118. Plastering is usually measured by the square yard. There is no uniform rule regarding allowances to be made for openings.

To find the area of the walls, multiply the perimeter by the height.

The dimensions of a room are usually stated in the following order: length, breadth, height.

Thus, the dimensions of a room 18 ft. long, 14 ft. wide, and 9 ft. 6 in. high would be indicated as follows: 18 ft. $\times$ 14 ft. $\times$ 9 ft. 6 in. or $18' \times 14' \times 9'\ 6''$.

Wall paper is generally 18 inches wide. It is sold by the single roll of 8 yards or by the double roll of 16 yards. One cannot buy less than a single roll. There is no uniform procedure in making allowance for openings. Some paper hangers deduct the total width of all openings from the perimeter of the room, and cover the spaces about the openings with the parts of strips left from the rolls.

Painting is usually measured by the *square yard.* It is not customary to make any allowance for openings.

Carpet is sold by the linear yard regardless of its width. Ingrain carpets are usually 1 yard wide, other carpets are usually $\frac{3}{4}$ yard wide. Fractional lengths, but not fractional widths, may be bought. Allowance must sometimes be made for matching designs.

Written Work

The six rooms of a house have the following dimensions and openings: Living room, 20′ × 16′, 3 windows 3′ × 6′ 6″, and two doors 3′ 6″ × 7′ 6″. Dining room, 15′ × 13′ 6″, 2 windows 4′ × 6′, and two doors 3′ 6″ × 7′. Two bedrooms, each 14′ 6″ × 12′ 6″, 2 windows 4′ × 6′ 6″, and 2 doors 3′ 6″ × 7′. Kitchen, 15′ × 15′ 6″, 3 windows 4′ × 7′, and 2 doors 3′ 6″ × 7′. Bathroom, 10′ 6″ × 8′ 6″, 1 window 3′ × 5′, 1 door 3′ 6″ × 7′. All rooms are 9 ft. high and the baseboards are 1 ft. wide.

1. Find the cost of plastering the walls and ceiling of each room at 35 cents a square yard, allowing for one half the openings.
2. Find the cost of papering the walls of the living room and the dining room at $1.10 a double roll, making one half allowance for openings.
3. Find the cost of calcimining the ceilings of these two rooms at 60 cents a square yard.
4. Find the cost of papering the walls of the two bedrooms at 90 cents a double roll, making full allowance for openings.
5. How many square yards of linoleum will be required to cover the floors of the bathroom and the kitchen?
6. Find the cost of painting the baseboards in the six rooms, at 40 cents a square foot, allowance being made for all openings.
7. How many gallons of paint will be required for two coats on a building 32 ft. by 24 ft. by 18 ft., if the first coat takes one gallon for 40 sq. yd. and the second one gallon for 48 sq. yd.?

Solids

119. Rectangular Solids. A **solid** has three dimensions: length, breadth, and thickness.

A **rectangular solid** is bounded by six rectangular surfaces. Such a solid is called a **prism.**

The bases of a prism are equal and parallel.

A **cube** is a rectangular solid having six square faces.

A cube 1 unit long, 1 unit wide, and 1 unit thick contains 1 cubic unit. Volumes are always measured in terms of some cubic unit.

How many cubic inches in a block 1 in. long, 1 in. wide, and 1 in. thick? In a prism 5 in. long, 1 in. wide, and 1 in. thick? in a prism 5 in. long, 3 in. wide, and 1 in. thick? in a prism 5 in. long, 3 in. wide, and 2 in. thick?

It is evident that *the number of cubic units in the volume of a rectangular solid is equal to the product of the number of units in its three dimensions.*

Written Work

Compute the volumes of the following:

1. A cube with an edge of 35 in.
2. A rectangular solid $7' \times 8' \times 5'\ 6''$.
3. A piece of ice $2\frac{1}{2}' \times 1\frac{2}{3}' \times 9''$.
4. A piece of timber 22 ft. long, $1\frac{1}{2}$ ft. wide, and $1\frac{1}{2}$ in. thick.
5. How many cubic feet of earth must be removed in digging a cellar $20' \times 18' \times 7'$?
6. Compute the cost of a stone wall 24 ft. long, 3.5 ft. wide, and 4 ft. high at \$3.65 a cubic yard.
7. A swimming tank is 44′ by 28′ by 9′. How many cubic feet of water does it contain when filled to a depth of 4′? 8′?
8. A ditch for some sewer pipe is to be made 1,200 yd. long, 3 ft. wide, and 7 ft. deep. Compute the cost of making the excavation at 28 cents per cubic yard.
9. Compute the number of cubic feet of air space allowed each student in your schoolroom.
10. Compare the volumes of two rectangular pieces of ice having the same length and breadth, the thickness of one being twice that of the other.
11. Compute the volume of a rectangular bin $18' \times 16' \times 7'$.
12. Find the capacity in bushels of a grain elevator $60' \times 42' \times 24'$. (A bushel is equivalent to $1\frac{1}{4}$ cu. ft.)

120. A **cord** of wood or stone is a pile 8 ft. long, 4 ft. wide, and 4 ft. high. It contains 128 cu. ft. In certain localities a cord of wood means a pile 8 ft. long, and 4 ft. high, the price varying with the length of the stick.

Written Work

1. A pile of four-foot wood ten feet high and forty-two feet long was cut from a six-acre tract. What was the average number of cords per acre?

2. How many cords of wood in a pile 30 ft. by 4 ft. by 9 ft.?

3. How many cords of 4 ft. wood in a pile 16 ft. by 8 ft. by 12 ft.?

121. A **cylinder** is a solid bounded by two bases which are equal parallel circles and by a uniformly curved surface.

The curved surface of a cylinder is called its **lateral surface.**

If the lateral surface of a cylinder be exactly covered with paper and the paper then removed, the paper will be seen to have the form of a rectangle the base of which is the length of the circumference of the cylinder and the altitude is the height of the cylinder.

Hence, *the number of units in the lateral area of a cylinder equals the product of the number of units in the circumference by the number of units in the altitude. To find the entire surface of the cylinder the areas of the two circular bases must be added to the lateral area.*

It is shown in geometry that the number of units in the volume of a cylinder equals the product of the number of units in the area of the base and the number of units in the altitude.

Written Work

1. Compute the entire surface of a cylindrical cup having a diameter of 3 in. and an altitude of 7 in.

Determine the cubic contents of this cup.

2. A cylindrical cistern having a radius of $4\frac{1}{2}'$ is filled with water to a depth of 9′. How many cubic feet of water are there in the cistern?

3. A cylindrical tank 22 ft. in diameter is filled with water to the depth of 18 ft. What is the weight of the water (1 cubic foot of water weighs 62.5 lb.)?

4. A man contracted to dig a cistern 7 ft. in diameter, 24 ft. deep, at the rate of $1.20 per cu. yd. Find the entire cost.

5. Find the cost of cementing this cistern at $1.80 per square yard.

6. How many square yards of sheet iron will be required for a smokestack 3 ft. in diameter and 24 ft. high, allowing 1 in. for the lap? What will be the cost at 14 cents per square foot?

7. If a steam roller is 5 ft. high and 7 ft. long, how many square feet of ground will it roll at each revolution?

8. At $7\frac{1}{2}$ gal. to the cubic foot what will be the capacity of a cylindrical tank having a base of 6 ft. and an altitude of 14 ft.?

9. A farmer has a roller $2\frac{1}{2}$ ft. in diameter and 6 ft. wide. Allowing 4 in. for lapping, how far must the team travel in rolling a rectangular field 80 rd. long and 56 rd. wide?

10. At 60 cents a square yard find the cost of polishing a cylindrical monument 14 ft. high and 4 ft. in diameter.

122. The Sphere. It is shown in geometry that *the number of units in the surface of a sphere is equal to* $4\ \pi$ *times the square of the number of units in the radius; and the number of units in the volume of a sphere is equal to* $\frac{4}{3}\ \pi$ *times the cube of the number of units in the radius.*

Written Work

Find the surfaces and volumes of spheres of radii:

1. 3″.	2. 7″.	3. 2.5″.	4. 13′.	5. 24.2″.
6. 8.24′.	7. 6.3″.	8. 14.1′.	9. 8.7′.	10. 12.3″.

11. A ball with a radius of 18 in. is to be gilded. How many square inches of gilding will be required?

123. Capacity. Grain is usually estimated by the *stricken bushel* of 2,150.42 cu. in.; fruits, vegetables, coal, and corn on the cob by the *heaped bushel* of 2,747.71 cu. in.

Since a stricken bushel is 2,150.42 cu. in., and there are 1,728 cu. in. in 1 cu. ft., therefore, 1 cu. ft. contains $\frac{1728}{2150.42}$ bushels or approximately .8 of a bushel.

To find the number of bushels of grain a bin will hold, multiply its capacity in cubic feet by .8.

A cubic foot of space will contain $\frac{1728}{2747.71}$ heaped bushels, or .63 heaped bushels.

To find the number of heaped bushels that a bin will hold, multiply its capacity in cubic feet by .63.

Written Work

1. How many bushels of potatoes will just fill a bin $18' \times 7' \times 5'$?
2. How many bushels of wheat will just fill a bin $30' \times 6' \times 6'$?

3. A wagon bed is 10′ long, 4′ wide, and $2\frac{1}{2}$′ deep. How many bushels of wheat will it hold?

4. Mr. Williams wishes to build a bin that will store 250 bushels of wheat. He expects to make the bin 24′ long and $6\frac{1}{2}$′ wide. What must be the depth?

5. A car 34′ long and 7′ wide is filled with oats to a depth of $4\frac{1}{2}$′. How many bushels of oats are in the car?

A gallon is equal to 231 cu. in. A cubic foot of space, therefore, contains $\frac{1728}{231}$ gallons, or 7.48 gallons.

To find the number of gallons when the cubic contents may be found, multiply the capacity in cubic feet by 7.48.

(In practice 1 cu. ft. is considered equal in capacity to $7\frac{1}{2}$ gallons.)

Written Work

Find the capacity in gallons:

1. A cistern 12′ in diameter and 18′ deep.
2. A tank 12′ long, $4\frac{1}{2}$′ wide, and 5′ deep.
3. A cistern 14′ in diameter and 20.5′ deep.
4. What is the capacity in gallons of a cylindrical tank 18.5′ in diameter and 32′ deep?
5. A cylindrical standpipe 60′ high has a diameter of 14′. How many gallons of water will it hold?

124. Board Measure. The **board foot** is the unit of lumber measure. It is a board 1 ft. long, 1 ft. wide, and 1 in. thick. The volume of a board foot is therefore 144 cu. in. (except when the lumber is less than 1 in. in thickness). Boards having a thickness of less than 1 in. are considered as having a thickness of an inch.

A board 16 ft. long, 12 in. wide, and 1 in., or less, in thickness contains 16 board feet. If the thickness were $1\frac{1}{2}$ in., the board would contain 24 board feet.

In measuring boards that taper uniformly the average width is used.

The width of boards, except in expensive lumber, is generally taken as the next smaller half inch.

When no thickness is mentioned the thickness is understood to be 1 inch.

It should be evident that *the number of board feet is the product of the width in inches, by the thickness in inches, by the length in feet, by the number of pieces, divided by* 12.

Illustration 1. How many board feet in 24 scantlings, $2'' \times 4'' \times 16'$?

$$\frac{24 \times 2 \times 4 \times 16}{12} = 256, \text{ the number of board feet.}$$

Illustration 2. How many board feet in 18 pieces, each $4'' \times 9'' \times 16'$.

$$\frac{18 \times 4 \times 9 \times 16}{12} = 864, \text{ the number of board feet.}$$

Oral Work

Compute the number of board feet in each of the following:

1. A board 12 ft. long, 1 ft. wide, and 1 in. thick
2. A board 7 ft. long, 2 in. wide, and 1 in. thick.
3. Two boards each 10 ft. long, 4 in. wide, and 1 in. thick.
4. Two boards each $14' \times 4'' \times 12''$.
5. Twenty boards each $10' \times 3'' \times \frac{3}{4}''$.
6. One hundred boards each $14' \times 4'' \times 2''$.
7. Twenty-five boards each $9' \times 2'' \times 1''$.
8. Three hundred boards each $15' \times 4'' \times \frac{3}{4}''$.
9. Fifty boards each $12' \times 6'' \times 2''$.

Dealers sometimes use a table in determining the number of board feet in a given piece of timber. A portion of such a table is given below.

Size in Inches	Length in Feet				
	10	12	14	16	18
2×4	$6\frac{2}{3}$	8	$9\frac{1}{3}$	$10\frac{2}{3}$	12
2×6	10	12	14	16	18
2×8	$13\frac{1}{3}$	16	$18\frac{2}{3}$	$21\frac{1}{3}$	24
2×10	$16\frac{2}{3}$	20	$23\frac{1}{3}$	$26\frac{2}{3}$	30
2×14	$23\frac{1}{3}$	24	$32\frac{2}{3}$	$37\frac{1}{3}$	42
$2\frac{1}{2} \times 12$	25	30	35	40	45
$2\frac{1}{2} \times 14$	$29\frac{1}{6}$	35	$40\frac{5}{6}$	$46\frac{2}{3}$	$52\frac{1}{2}$
3×6	15	18	21	24	27
3×8	20	24	28	32	36

Written Work

Determine the number of board feet in:

1. 40 pieces of white pine $2'' \times 10''$, $18'$.
2. 130 joists $2'' \times 8''$, $16'$.
3. 36 beams $4'' \times 8''$, $20'$.
4. 20 planks $2\frac{1}{2}'' \times 8''$, $18'$.
5. 80 joists $4'' \times 4''$, $20'$.
6. At \$15 a thousand, find the cost of the following:

14 joists,	$2'' \times 6''$, $18'$.
24 joists,	$2'' \times 10''$, $32'$.
80 boards,	$\frac{3}{4}'' \times 4''$, $16'$
30 scantlings,	$2'' \times 4''$, $16'$
20 beams,	$8'' \times 9''$, $16'$

Miscellaneous Exercises in Practical Measurements

1. A circular pond whose radius is 80 ft. is surrounded by a driveway 16 ft. wide. What is the area of the driveway?

2. If a gallon of a certain paint is enough for two coats on a surface of 150 sq. ft., how many gallons must a farmer order to paint the sides and ends of a rectangular shed 32 ft. long, 18 ft. wide and 14 ft. high?

3. The diameter of the piston in a certain automobile is $3\frac{3}{4}$ in. What is the area?

4. The **surface speed** of a grindstone is the number of feet that a point on the circumference travels in a minute.

a. What is the surface speed of a 9 in. emery wheel revolving 2,000 times per minute?

b. What is the surface speed of a 24 in. grindstone revolving 68 times per minute?

5. What is the capacity in gallons of a cylindrical tank whose inner diameter is 28 in. and whose length is 8 ft.?

6. What is the radiating surface of 120 ft. of pipe the outer diameter of which is 2 in.?

7. If a cubic foot of steel weighs 490 lb., what is the weight of a solid steel plate $\frac{1}{2}$ in. thick and $3\frac{1}{2}$ ft. in circumference?

8. How many gallons of gasoline can be stored in a cylindrical tank having an inner diameter of 34 in. and a height of 48 in.?

9. A wagon wheel is 44 in. in diameter. How long a piece of iron must be cut to make the tire, allowing $\frac{1}{2}$ in. in welding?

10. A cellar is 36 ft. long, 28 ft. wide, and $7\frac{1}{2}$ ft. deep. How many loads of earth were removed in its construction?

11. Water runs through a 2 ft. sewer pipe at a speed of 45 ft. a minute. How many gallons can the pipe discharge in an hour?

12. The drive wheel of a locomotive is 7 ft. 3 in. in diameter. How many revolutions does it make in going a mile?

13. Measure one of the rooms in your home and determine the quantity of wall paper required for walls and ceiling.

14. Compute the cost of painting a house 60 ft. long and 32 ft. wide, if it measures 22 ft. from foundation to eaves and the two gable ends are 10 ft. high. The rate is $4.20 per square.

15. A contractor is preparing a street for paving. If a block is 300 ft. in length and the width from curb to curb is 30 ft., how many cu. yd. of earth must be removed if the depth of the excavation is to average 12 in.?

16. A woman makes some tea towels from a remnant of toweling containing $5\frac{1}{2}$ yd. Each towel is $\frac{7}{8}$ yd. long. How many towels of this length can be made from the remnant and how much will be left over?

17. Theodore Roosevelt was born October 27, 1858, and died January 6, 1919; to what age did he live?

18. How many square yards of concrete are required for the floor of a cylindrical silo the inside diameter of which is 14 ft.?

19. If shelled corn weighs 56 lb. per bushel, how many bushels does a car contain if the total weight of the corn is 51,520 lb.?

20. If the volume of a rectangular solid is known, and also its length and width, how may its depth be found?

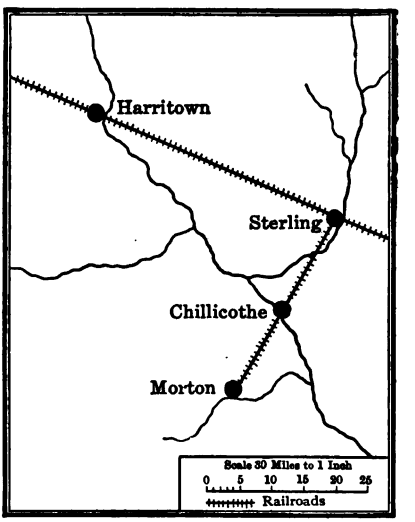
21. A farmer owns a rectangular water tank $6' \times 4' \times 4'$. Water flows in through a pipe 2 in. in diameter at the rate of $1\frac{3}{4}$ ft. per second. How long will be required to fill the tank? How long would be required if the diameter of the pipe were doubled, the rate of flow remaining the same?

22. A third baseman throws a baseball to the first baseman. If both men are on their bases, what is the length of the throw, the baseball diamond being a square 90 ft. on a side?

CHAPTER XIII

MAPS, DRAWINGS, AND GRAPHS

NEWSPAPERS, magazines, and trade journals make frequent use of drawings and graphs in order to make clear the relations between magnitudes. It is often possible to make values and comparisons more clearly understood in this way than by merely printing a table of statistics. Drawings may be divided into three classes: maps, plans, and other dimension drawings and graphs.



MAPS

125. Scale. Distances on maps are usually indicated by a scale similar to the one in the lower right hand corner of the accompanying illustration.

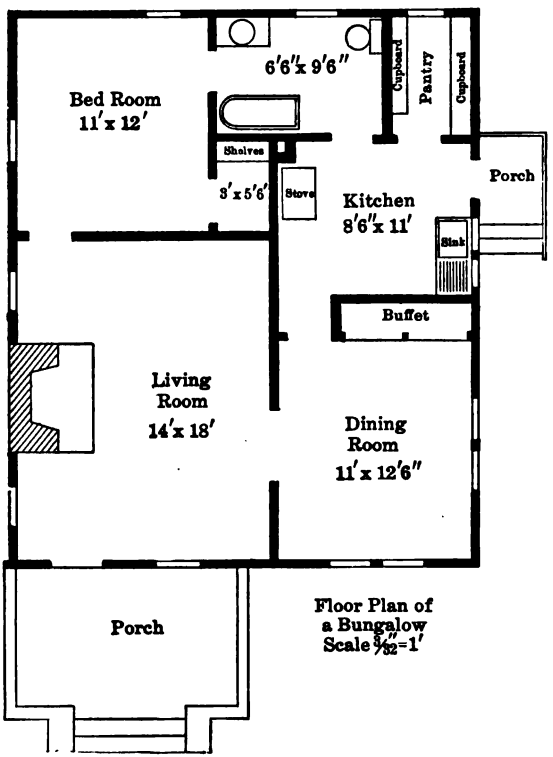
Oral Work

1. The map on this page is drawn to the scale 1″ = ? miles.
2. $\frac{1}{8}$″ on this map represents what true distance?
3. What is the true distance between each of the towns on the railroad? Measure distances from centers of towns.
4. An electric railroad is in process of construction between Morton and Harritown. It is surveyed in a direct line. How much shorter will this road be than the one via Sterling?
5. It is planned to locate a power house halfway between Sterling and Harritown. Indicate its position on the map and determine its distance from the two towns. How far will it be from the nearest point on the new electric line?

Dimension Drawings

126. Purpose. Dimension drawings are used to show quickly and conveniently the shape and dimensions of fields, rooms, buildings, furniture, machinery, etc.

Drawings similar to the illustration are of great value. An architect designs a building and the contractor builds it in accordance with the plans shown in the drawings. Tools and machinery are usually constructed in accordance with drawings prepared by draftsmen and engineers.



127. Drawing to Scale. The ability to read drawings and graphic representations intelligently depends somewhat upon a knowledge of how to draw figures to scale.

The values represented by the lines in drawings and graphs can be determined easily because the lines are drawn to scale; that is, a certain distance on the drawing represents a specified amount of the quantity represented.

In the floor plan on this page one foot of true distance is represented by $\frac{3}{32}$ of an inch plotted distance on the drawing. All lines in the drawing correspond to this scale.

128. Types of Scales. The scale may be stated in various ways, the most common of which are:

a. A fraction.

For example, a scale of $\frac{1}{4}$ means that the true distances or values are four times as great as the plotted distances or values.

b. Statement of equivalent dimensions or values.

For example, 1″ = 1′. (Read 1 in. = 1 ft.) One inch of the drawing represents one foot of the real object. In the graph showing immigration into the United States, page 149, the scale is 1″ = 440,000.

c. A diagram or scaled rule accompanying the drawing. This is the method commonly used in map drawing.

d. Graphs may be drawn on ruled paper, the distance between the ruled lines representing a specified value.

129. Determining the Scale. When a drawing or a graph is to be made, it is first necessary to determine the scale. The scale depends, in general, upon three things:

a. The size of the paper to be used.
b. The largest dimension to be shown on the graph.
c. Convenience in showing fractional parts of the scale.

Problem. A room is 15′ 6″ long by 10′ 3″ wide. What scale should be used to show a diagram of this room on a sheet of paper 8″ by 10″, leaving a margin of at least one inch around the drawing?

SOLUTION. The largest dimension of the room is 15′ 6″, or 186″.

The largest dimension of the paper is 10 inches. After deducting 2 inches for margins, there is a space of 8″ available for the drawing.

The largest scale possible, therefore, is 8″ represents 186″.

This is equivalent to a scale of 1″ = $23\frac{1}{4}$″. (How was this determined?)

For convenience in showing fractional parts of a foot, it will be better to use the scale of 1″ = 24″.

Written Work

1. A scale of 1″ = 1′ is equivalent to what fractional scale?

2. A scale of 1″ = 5″ is equivalent to what fractional scale?

3. A scale of 1″ = 5′ is equivalent to what fractional scale?

4. If the scale is $\frac{1}{12}$, what distance on the drawing would represent 6′ 3″? 4′ 9″?

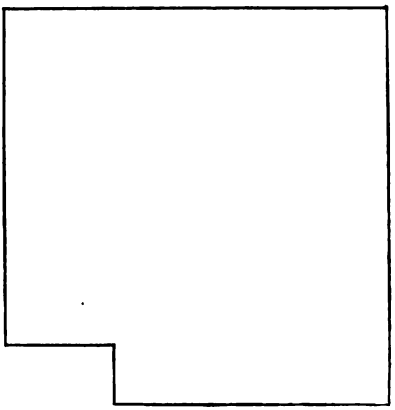
5. If the scale is $\frac{1}{300}$, what plotted distance would represent 750 feet? 750 inches?

6. If the scale is 1″ = 1,000 bushels, what distance on a graph would represent 5,125 bushels? 8,500 bushels? 1,750 bushels?

7. The following floor plan is drawn to a scale of $\frac{1}{12}$.

One inch of plotted distance on the floor plan is equivalent to how many feet of true distance?

8. One foot of true distance on the floor plan is represented by what part of an inch on the drawing?



9. What are the true dimensions of this room?

10. Assume that this drawing is made to the scale, $\frac{1}{8}$ in. = 1 ft. ($\frac{1}{8}'' = 1'$). What are the true dimensions of the room?

11. With the scale $1'' = 2'$, what length lines would represent the dimensions of a room $15'\ 6'' \times 10'\ 3''$?

12. Use this scale to make a drawing of a room $12'\ 6''$ by $9'\ 9''$.

13. A field is 120 rods by 80 rods. Scale, $1'' = 1$ rod. How large a paper is needed to diagram the field, leaving a margin of 2 inches all around? Would this be a reasonable scale to use?

14. If a scale 1 inch = 20 rods is used, what size paper is necessary in order to diagram this field and leave a margin of one inch on all sides?

15. What is the largest scale that could be used to plot this field on a paper the size of this page, leaving margins of at least $\frac{3}{4}''$?

16. Draw a diagram of your schoolroom, entering the dimensions as illustrated in the drawing on page 146.

17. Draw a diagram of a room in your home. Do not enter the dimensions but state the fractional scale used, and draw all lines very carefully in accordance with this scale.

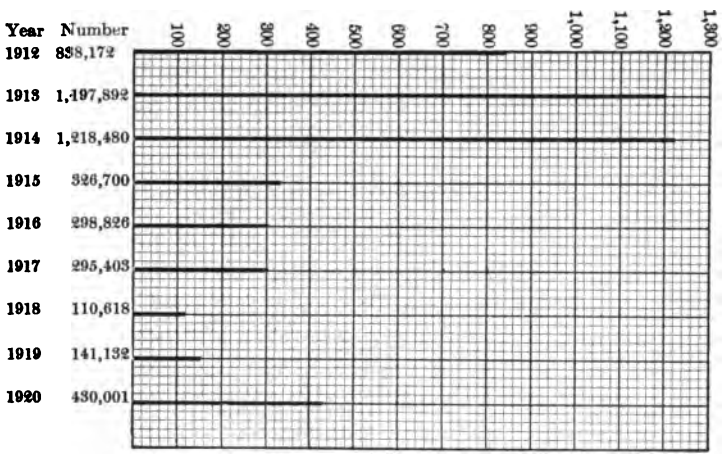
NOTE TO TEACHER. When these drawings are brought to class, they may be exchanged, and the students required to compute the dimensions in accordance with the scale stated.

GRAPHS

130. Forms of Graphs. Since graphs are used to display many kinds of statistics, it is to be expected that the graphs will be drawn in a variety of forms. The following illustrations are intended to suggest the uses which may be made of different types of bar, line, and circle graphs.

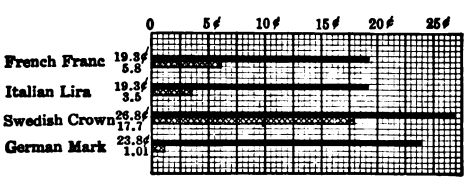
131. Comparison by Straight Lines. The following graph illustrates the use of straight lines for the comparison of quantities, and is based on the following immigration statistics:

Year (Ending June 30)	Number
1912	838,172
1913	1,197,892
1914	1,218,480
1915	326,700
1916	298,826
1917	295,403
1918	110,618
1919	141,132
1920	430,001



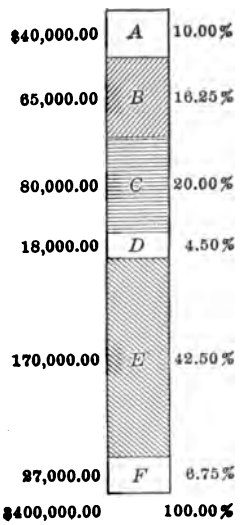
IMMIGRATION INTO THE UNITED STATES 1912 TO 1920

132. Cross-Hatching. More extensive comparisons are possible if cross-hatched bars are used. In the following graph, the solid black bars represent the par value in cents of certain foreign coins; the cross-hatched bars represent their lowest exchange value during the year 1920.



133. Component Parts Shown by Cross-Hatching. A bar may be cross-hatched in various ways to show different parts of a total, in the manner described below.

TOTAL SALES OF A DEPARTMENT STORE

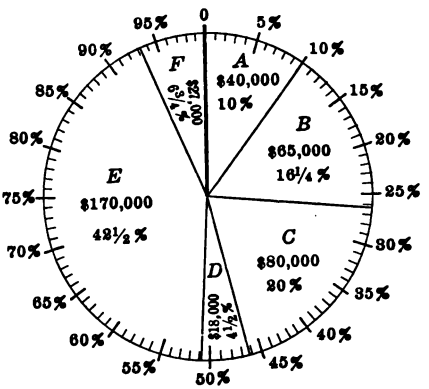


DEPARTMENT	SALES	PER CENT
A	$ 40,000.00	10.00
B	65,000.00	16.25
C	80,000.00	20.00
D	18,000.00	4.50
E	170,000.00	42.50
F	27,000.00	6.75
Total	$400,000.00	100.00

134. Component Parts Shown by Circle. The same facts may be shown by a circle graph, but this form is more difficult to use and is not as easily read as a bar.

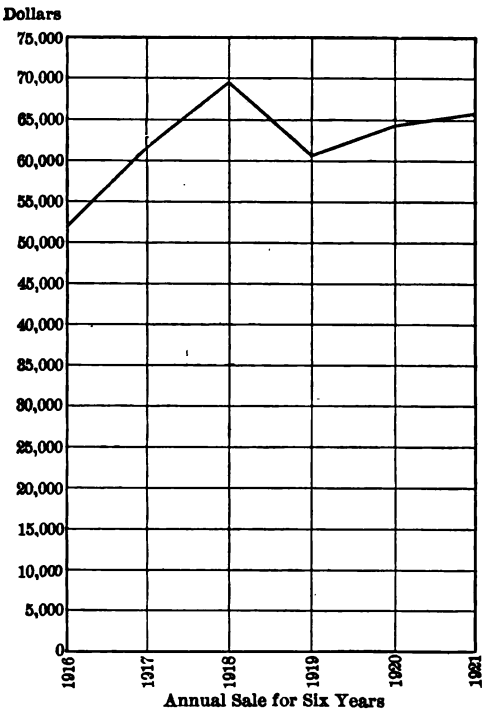
COMPARISON OF SALES IN SIX DEPARTMENTS

This type of graph may be easily prepared by obtaining paper with the circle printed thereon, the circumference being divided by graduations on a percentage basis. If such paper can not be obtained, a protractor may sometimes be obtained scaled in per cents. If necessary, an ordinary protractor may be used. The protractor shown in the illustration at the right is divided on the basis of degrees of arc. Since there are 360° in a circle, 100 % is represented by 360°; 10 % by 36°; 16.25 % by 58.5°; etc.



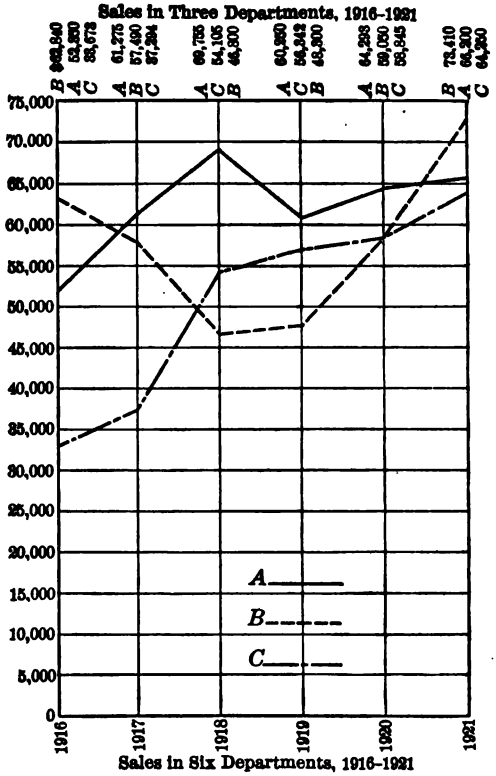
135. Single Curve Showing Quantities. Plotted curves are especially adapted to showing changes in quantities at different dates or during a series of periods. The paper should be ruled both horizontally and vertically. The horizontal lines should represent a quantity scale,

and the vertical lines should represent dates or time periods. To draw such a curve, dots are placed on the vertical lines at the proper points to represent the desired quantities, and the dots are connected by a line called a curve.



136. Several Curves in One Graph. By using different colors of ink or different forms of lines, it is possible to plot several curves together, as in the following illustration, which shows the sales in three departments of a store during six successive years. The graph is prepared from the following statistics:

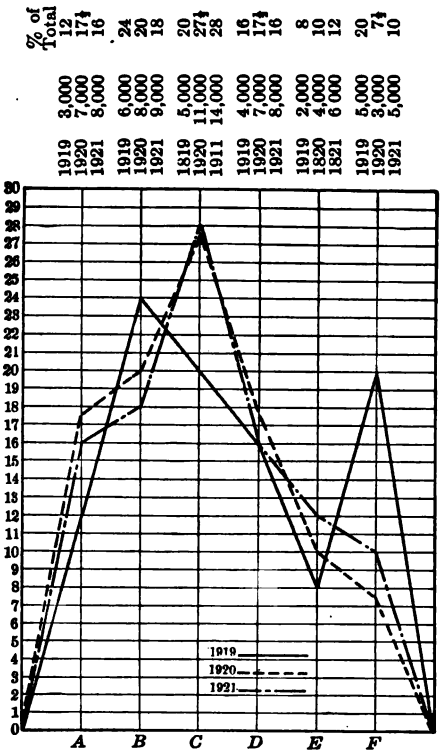
Year	Dept. A	Dept. B	Dept. C
1916	$52,350	$62,840	$33,673
1917	61,275	57,490	37,294
1918	69,755	46,800	54,105
1919	60,250	48,300	56,342
1920	64,293	59,050	58,845
1921	66,200	73,410	64,250



137. Comparative Percentage Curves. Several curves may be plotted in one graph as shown in the following illustration.

SALES IN SIX DEPARTMENTS

DEPARTMENT	1919		1920		1921	
	$	%	$	%	$	%
A	3,000	12	7,000	17.5	8,000	16
B	6,000	24	8,000	20	9,000	18
C	5,000	20	11,000	27.5	14,000	28
D	4,000	16	7,000	17.5	8,000	16
E	2,000	8	4,000	10	6,000	12
F	5,000	20	3,000	7.5	5,000	10
	25,000	100	40,000	100	50,000	100



Per Cent of Total Annual Sales in each of Six Departments

Written Work

Prepare graphs from the following statistics.

1. The figures in the following table are known as "index numbers." The basis is 100, and represents the average for the years 1909 to 1913. Then if the value of land in 1919 is represented by 202, farm land more than doubled in value.

Year	Land Values	Farm Wages	Crop Prices	Articles Farmers Buy
1919	202	207	221	212
1918	167	172	206	202
1917	153	142	188	153
1916	136	114	117	125
1915	123	105	101	112
1914	111	104	100	103

2. Bradstreet reports the following analysis of the reasons why business men fail, and the per cent of total failures due to each cause.

Incompetence	38.2%
Inexperience	5.6
Lack of capital	30.3
Unwise credits	1.3
Fraud	7.0
Failures of others	1.7
Extravagance	1.1
Neglect	1.7
Competition	1.1
Specific conditions	11.3
Speculation	.7
	100.0

3. The following statistics are taken from Lloyd's annual returns showing the merchant vessels of over 100 gross tons launched during a recent year.

Country	Number	Tons
United Kingdom	612	1,620,442
Canada	201	271,264
Denmark	46	37,766
France	34	32,633
Holland	100	137,086
Italy	32	82,713
Japan	133	611,883
Norway	82	57,578
Spain	41	52,609
Sweden	53	50,971
United States	1,051	4,075,385

4. Five departments of a wholesale store produced profits in one year as follows:

Department	Profits
I	$2,165.00
II	3,297.00
III	794.00
IV	1,719.00
V	4,625.00

Show the profits of each department and the per cent of the total made by each department.

5. Show by a graph the data for the attendance at your school for a school year. (The principal can supply the data.)

6. Graph the temperature for a certain hour for several successive days.

7. Make a graph showing your progress in speed and accuracy in addition, subtraction, multiplication, and division for a given set of examples.

8. Devise a convenient form for showing the following data graphically.

Great ship canals of the world:

Canal	Length (miles)	Width (feet)
Cape Cod	8	150
Corinth (Greece)	4	72
Kronstadt-Petrograd (Russia)	16	220
Elbe and Trave (Germany)	41	72
Kiel (Germany)	61	150
Manchester	35.5	120
New Orleans Industrial	6	150
Panama	50.5	300
Suez (Egypt)	90	108
Welland (Canada)	26.75	100

9. Graph the following:

Population of the world in 1920:

Africa	142,872,000
Asia	830,108,000
Europe	484,118,000
North America	145,441,000
Oceania	63,998,000
South America	63,754,000

CHAPTER XIV

PERCENTAGE

A KNOWLEDGE of percentage is necessary in many lines of business. The topic does not involve any mathematical principles with which the student is not familiar.

138. Relation of Percentage to Common and Decimal Fractions. Percentage is the process of computing by hundredths. The symbol % stands for per cent.

Thus, $15\,\%$ means $\frac{15}{100}$ or .15.

$8\,\%$ means $\frac{8}{100}$ or .08.

$300\,\%$ means $\frac{300}{100}$ or 3.00.

$\frac{1}{2}\,\%$ means $\frac{\frac{1}{2}}{100}$ or $.00\frac{1}{2}$ or .005.

$\frac{2}{3}\,\%$ means $\frac{\frac{2}{3}}{100}$ or $.00\frac{2}{3}$.

$7\frac{1}{2}\,\%$ means $\frac{7\frac{1}{2}}{100}$ or $.07\frac{1}{2}$ or .075.

From the illustrations above it is clear that common fractions, decimal fractions, and per cents are different forms of expression, and that any per cent may be expressed as either a common or a decimal fraction.

Oral Work

Express each of the following as a common fraction and as a decimal fraction:

1. $7\,\%$	**2.** $4\,\%$	**3.** $19\,\%$	**4.** $24\,\%$
5. $2\frac{1}{3}\,\%$	**6.** $\frac{1}{4}\,\%$	**7.** $\frac{1}{10}\,\%$	**8.** $200\,\%$
9. $184\,\%$	**10.** $5\frac{1}{8}\,\%$	**11.** $\frac{1}{100}\,\%$	**12.** $16\frac{2}{3}\,\%$

Express with the symbol %:

13. .74	**14.** .03.	**15.** .07$\frac{1}{4}$	**16.** 2.00
17. .12$\frac{1}{2}$	**18.** .001	**19.** 1.00	**20.** .0045
21. .5	**22.** .005	**23.** .16$\frac{2}{3}$	**24.** .14$\frac{2}{7}$
25. $\frac{1}{2}$	**26.** $\frac{1}{4}$	**27.** $\frac{1}{5}$	**28.** $\frac{3}{2}$
29. $\frac{3}{4}$	**30.** $\frac{3}{5}$	**31.** $\frac{4}{5}$	**32.** $\frac{1}{3}$

33. 40 % of a quantity is how many hundredths of it? What fractional part of it?

34. 100 % of a number is how many times the number?

35. 300 % of a number is how many times the number?

36. 140 % of a number is how many times the number?

37. 2 times a number is what per cent of the number?

38. 5 times a number is what per cent of the number?

39. What fractional part of a number is:

20 % of it?	12 % of it?	13 % of it?
80 % of it?	75 % of it?	$\frac{1}{5}$ % of it?
100 % of it?	$\frac{1}{2}$ % of it?	12$\frac{1}{2}$ % of it?
37$\frac{1}{2}$ % of it?	87$\frac{1}{2}$ % of it?	50 % of it?
6$\frac{2}{3}$ % of it?	62$\frac{1}{2}$ % of it?	66$\frac{2}{3}$ % of it?

40. Express each of the following as a decimal fraction and with the symbol %.

$\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{3}$, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{2}{5}$, $\frac{1}{4}$, $\frac{3}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{1}{10}$, $\frac{5}{12}$, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$

139. Terms Used in Percentage.

Base. The number of which a per cent is to be taken is called the **base.**

Rate. The per cent of the base to be taken is called the **rate.**

Percentage. The result obtained by taking a certain per cent of the base is called the **percentage.** Thus,

$$64 \times .25 = 16;$$

or **base × rate = percentage.**

Fundamental Processes of Percentage

Computations in percentage are based on the following:

base × rate = percentage.

64 × .25 = 16.

Since the percentage is the product of the base and rate, it is evident that,

a. The percentage divided by the base equals the rate. For example: $16 \div 64 = .25 = 25\%$.

b. The percentage divided by the rate equals the base. For example: $16 \div 25\% = 16 \div .25 = 64$.

From these statements it is evident that if any two of the terms are known, the other one can be found.

In the chapter on Decimal Fractions you learned how to multiply and divide by any number expressed decimally. Percentage involves the mathematical principles of decimal fractions. The exercises in decimal fractions involved computations with tenths, hundredths, thousandths, etc. In percentage, computation is by hundredths.

140. All mathematical computations involved in percentage may be grouped under three headings:

a. To find a given per cent of a number.
b. To find what per cent one number is of another.
c. To find a number when a certain per cent of it is known.

141. To find a given per cent of a number; that is, to find the percentage.

Problems. 1. Find 1% of 428.

SOLUTION. $1\% = .01$ $.01$ of $428 = 4.28$.

2. Find 7% of 378.

SOLUTION. $.07 \times 378 = 26.46$.

3. Find 14% of \$360.

SOLUTION.

$$\begin{array}{r} \$\ 360 \\ .14 \\ \hline 1440 \\ 360 \\ \hline \$50.40 \end{array}$$

To find the percentage, multiply the base by the rate.

Oral Work

1. Find 1% of each of the following: 18; 146; 198; 7; .03; 876; .045.

2. Find 5% of each of the following: 80; 60; 25; 42.

3. Find 10% of, $18.20; of 478 mi.; of 32 yd.; of 648 A.

4. How many places should be pointed off in finding each of the following:

17% of 4762?	17% of 476.2?	17% of 47.62?
1.7% of 4762?	.17% of 4762?	17% of .4762?

5. Find 10% of 400; 560; 8.46.

6. Find 3% of 40; 60; 15.

7. Find 100% of 15; 40; 3.

8. Find 20% of 80; 70; 60.

9. Find 50% of 40; 2; 10.

10. Find 30% of 30; 40; 80.

11. Find $12\frac{1}{2}$% of 480; 96; 640.

12. Find $37\frac{1}{2}$% of 24; 80; 72.

13. Find 200% of 9; 32; $18\frac{1}{2}$.

14. Find $14\frac{2}{7}$% of 21; 35; 350.

15. Find 25% of 12; 17; 96.

16. Find $\frac{1}{2}$% of 120; 34; 18.

17. Find $\frac{1}{2}$% of 30; 50; 24.

Written Work

Find:

1. 17% of 472.

2. 92% of $537.

3. $16\frac{2}{3}$% of $420.

4. 3% of 48,729.

5. 200% of 37 ft.

6. 1.8% of 42,690.

7. 17.5% of 479,362.

8. 250% of $142.

9. 128% of 346 yd.

10. 5% of 83.6.

11. $7\frac{1}{2}$% of 42.736.

12. $\frac{1}{2}$% of 8476.

13. 32% of $4.76.

14. 19% of .084.

15. 25% of 264 sq. mi.

16. $37\frac{1}{2}$% of 24 A.

17. $12\frac{1}{2}$% of $488.

18. $87\frac{1}{2}$% of 648 mi.

19. $33\frac{1}{3}$% of $\frac{1}{3}$.

20. 20% of $\frac{7}{8}$.

21. $6\frac{2}{3}$% of 32 cu. ft.

22. 40% of 85 sq. rd.

23. $62\frac{1}{2}$% of 40 A.

24. $33\frac{1}{3}$% of $36.

25. $66\frac{2}{3}$% of 120 ft.

26. $12\frac{1}{2}$% of 64 yd.

27. A school has an enrollment of 450 students. How many students are absent when 4% are absent?

28. What is $200 increased by 4% of itself?

29. In 1910 the population of a certain city was 52,840. In 1920 the population was 15% larger than in 1910. What was the population in 1920?

30. A boy made a journey of 48 miles. He rode 85% of the distance in an automobile and walked the remaining distance. How far did he walk?

31. An architect's fees for designing a house are 4% of its cost. The cost is $7360. What is the fee?

32. A man whose salary is \$3,200 spends 19 % of it for rent. How much does he spend for rent?

33. A clerk receives 2 % of the amount of his sales. If his sales for Monday were \$160.40, how much did he receive?

34. The report of a school with an attendance of 400 showed the per cent of tardiness to be $1\frac{1}{2}$. How many were tardy?

35. A man earns \$1,400 a year and saves 18 % of it. How long will it take him to save \$756?

142. To find what per cent one number is of another; that is, to find the rate.

Method. Since the percentage is the product of the base and the rate, and since the product may be divided by one factor to find the other factor, we have the following:

percentage ÷ base = rate.

Problems. 1. 6 is what per cent of 12?

SOLUTION. $\frac{6}{12} = \frac{1}{2} = .50$ or 50 %.

2. A farmer had 72 sheep and sold 18 of them. What per cent of his flock did he sell?

SOLUTION. $18 \div 72 = .25$ or 25 %.

3. George had one brother and three sisters. What per cent of the children of the family were boys?

SOLUTION. There were 5 children in the family.
$\frac{2}{5}$ of the children were boys.
$2 \div 5 = .4$ or 40 %.

Oral Work

1. What per cent of 8 is 4?
2. What per cent of 8 is 2?
3. What per cent of 24 is 3?
4. What per cent of 12 is 18?
5. What per cent of 9 is 3?
6. What per cent of 25 is 5?
7. What per cent of 25 is 20?
8. What per cent of 15 is 30?
9. What per cent of 10 is 8?
10. What per cent of 50 is 20?

11. What per cent of 60 is 40?

12. 15 is what per cent of 20?

13. 20 is what per cent of 40?

14. 18 is what per cent of 24?

15. 1 ft. is what per cent of 1 yd?

16. 20 bu. are what per cent of 60 bu.?

17. 4 mo. are what per cent of 1 yr.?

18. 3 da. are what per cent of 1 wk.?

19. If you sleep 8 hr. out of 24, what per cent of the time do you sleep?

20. If a man earns $24 a week and saves $4, what per cent of his earnings does he save?

21. An agent sold goods amounting to $1,800 and received $200 for his services. What per cent did he receive on the sales?

Written Work

1. A baseball team played 60 games and won 42. What per cent of the games played did the team win?

2. In a school of 250 students 45 were seniors. What per cent of the students were seniors?

3. In a school auditorium there are 650 seats. Four hundred eighty of the seats are reserved for the students. What per cent of the seats are not reserved?

4. A building worth $14,500 is insured for $10,150. For what per cent of its value is it insured?

5. In a spelling lesson a boy spelled all but 2 of the 30 words correctly. What per cent of the words did he spell correctly?

6. A corporation has a capital stock of $164,000. Dunham owns $41,000 of the stock. What per cent of the stock does he own?

7. A business man invested $3,000. His income from the investment the first year was $420. What per cent did he earn?

8. In a certain school there are 224 boys and 312 girls. What per cent of the total enrollment is boys? girls?

9. Eighteen of the boys in this school failed to pass in at least one study. What per cent of the boys failed?

10. Twenty-three of the girls failed. What per cent of the girls failed?

11. What per cent of the entire school failed?

12. The total population of the United States in 1910 was 91,972,266. The number of foreign-born inhabitants was 13,345,545. What per cent of the total population was foreign born?

13. The largest number of foreign-born people were Germans. They numbered 2,501,181. What per cent of the entire population was of German birth?

14. What per cent of the foreign-born population was of German birth?

15. The forests of the United States originally contained about 5,200 billion board feet of timber. Cutting, clearing, and fire have reduced the stand to about 2,500 billion board feet. What per cent of the timber remains?

16. The following table is similar to one used by a clothing store to determine the tailoring firm which furnishes suits having the best sale. Record is kept of the number of suits purchased from each firm, and of the number of these suits sold.

Find what per cent of the suits purchased from each of the manufacturers was sold, and enter all statistics on a ruled form similar to the model.

Of what value is this table of statistics to the merchant?

PER CENT OF SALES FROM STOCK PURCHASED FROM CLOTHING MANUFACTURERS

MANUFACTURER	NUMBER BOUGHT	NUMBER SOLD	PER CENT SOLD
TAYLOR, BRIGHT & CO	368	293	
WELLS & BAILEY	493	316	
F.H BAINUM	639	582	
DUDLEY & COLE	523	416	
WILLIAMS, ELDRIDGE & CO.	863	719	
M.J HARTMAN & SONS	1392	1202	
W.C DENNISTON & CO.	1020	905	

17. The following table shows one of the ways in which merchants determine which clerk is selling the most goods.

Rule a form similar to the model; enter on the blank the facts which are given in the model; find the total sales made by all of the salesmen. Find what per cent of the total sales was made by each salesman. Enter all of these facts on the blank.

TABLE SHOWING PER CENT OF MONTH'S SALES MADE BY VARIOUS SALESMEN

SALESMAN'S NAME	SALES		PER CENT OF TOTAL
J B. DENNY	$ 1264	90	
O.F. ALBRIGHT	1913	52	
C.F. SLATER	1732	69	
WILLIAM WARNER	2213	72	
C.B. WILEY	1963	45	
H. WALKER	1627	32	
FRANK McGEE	1692	18	
F.L. MONTGOMERY	2138	45	
C.H. HENRY	1728	46	
TOTAL			

18. The following tabulation is a record of a week's sales in four departments of a store.

Rule a blank similar to the model.

Enter the statistics on the blank.

Find the total sales for each day made in all departments; enter these totals in the "total" column at the right.

Find the total sales made during the week in each department; enter these totals on the line at the bottom marked "total."

Find (in two ways) the total sales made in all of the departments for the entire week. Enter in the space marked Grand Total.

Find what per cent of the grand total sales was made each day; enter these per cents in the column at the right.

Find what per cent of the grand total sales was made in each department. Enter these per cents on the line at the bottom marked per cent.

One of the qualities which a business man desires in his employees is the ability to follow directions. This exercise will give you an opportunity to prove your ability along this line. Get a clear understanding of the instructions, and follow them by preparing the exercise without asking for any further directions.

A Week's Sales in a Department Store

Day	Dept. I	Dept. II	Dept. III	Dept. IV	Total	Per Cent
Monday	$362 50	$562 83	$862 94	$126 39	$	
Tuesday	415 75	475 92	732 83	143 62		
Wednesday . . .	396 21	415 60	769 42	129 38		
Thursday . . .	472 96	516 29	640 20	145 17		
Friday	387 29	429 36	721 32	96 27		
Saturday	493 89	562 64	816 25	163 92		
Total					Grand Total	
Per Cent . . .						

143. To find a number when a certain per cent of it is known; that is, to find the base.

Method. Since the *percentage* is the product of two factors, the base and the rate, and since any product can be divided by one of its factors to find the other factor, it is evident that,

percentage ÷ rate = base.

Therefore, *to find the base, divide the percentage by the rate expressed as a decimal.*

Problems. 1. 18 is 25 % of what number?

Solution. 18 is the percentage, 25 % is the rate; the base is not known.
18 ÷ .25 = the required number, or base.
18 ÷ .25 = 72.

2. 45 % of the number of students in a school are boys. If there are 135 boys, how many students are in the school?

Solution. 135 ÷ .45 = 300, the total number of students.

This may be explained as follows:

Since 45 % of the number of students = 135
1 % of the number of students = $\frac{1}{45}$ of 135 = 3
and 100 % of the number of students = 100 × 3 = 300.

Oral Work

Find the number of which.

1. 40 is 20 %.	2. 12 is 4 %.	3. 10 is 5 %.
4. 86 is 2 %.	5. \$45 is 15 %.	6. 40 is 80 %.
7. 12 is $\frac{1}{2}$ %.	8. 42 is 100 %.	9. 60 is 200 %.
10. 12 is $33\frac{1}{3}$ %.	11. 35 is 7 %.	12. 19 is $\frac{1}{2}$ %.
13. 8 is $\frac{2}{5}$ %.	14. 2,700 is $66\frac{2}{3}$ %.	15. 24 is $\frac{1}{3}$ %.

16. A boy lost 40 cents, which was 10 % of what he had. How much did he have?

17. A man sold 32 acres of land, which was 80 % of all that he owned. How many acres did he own?

Written Work

1. In a certain school 5 % of the pupils are absent and 475 pupils are present. What is the enrollment?

2. By saving 28 % of his salary a man saved \$2,240 in 5 years. What was his salary?

3. 18.75 % of a class failed. If 26 passed, how many were in the class?

4. The distance from New York to San Francisco via the Panama Canal is 5,278 miles. This distance is 60.44 % less than the distance via Cape Horn. What is the distance via Cape Horn?

5. A boy paid \$3 for a pair of shoes and had 70 % of his money left. How much money had he at first?

6. Four partners engage in business. The first invests \$4,000; the second invests \$3,000; each of the others invests 25 % of the total capital. What is the total capital?

7. The Washington Monument is 555 ft. and the Eiffel Tower is 984 ft. high. The height of the monument is what per cent of the height of the tower?

8. The average velocity of wind is 18 mi. per hour and that of sound is 1,090 ft. per second. The velocity of wind is what per cent of the velocity of sound?

9. The mean annual rainfall in Denver is 14 inches, and that in New York City is 44.8 inches. Each is what per cent of the other?

10. The length of the Hudson River is 280 miles; the length of the Ohio is 950 miles; and the length of the Mississippi is 3,160 miles. The length of each is what per cent of the length of the others?

11. The record for the 100-yard dash is $9\frac{3}{5}$ sec. It was formerly $9\frac{4}{5}$ sec. By what per cent was the record reduced?

12. In 1776 there were 13 states in the Union. There are now 48 states. What has been the per cent of increase?

13. A certain basketball player can shoot on an average 90 % of his free chances. If there are 20 fouls called on his opponents in a game, how many points should he make on fouls?

Percentage of Increase and Decrease

Percentage is frequently employed to find the relation between numbers, or to find how much larger or smaller one number is than another. This does not involve any new mathematical principles.

Problem. What number increased by 20% of itself is 360?

Solution. 100 % of the number = the number,
20 % of the number = the increase,
120 % of the number = 360,
1 % of the number = 360 ÷ 120 = 3,
100 % of the number = 100 × 3 = 300,

or we may say: 1.20 times the number = 360, therefore the number = $\frac{360}{1.20}$ = 300.

Oral Work

(Use equivalent fractions if more convenient.)

What number increased by

1. 20 % of itself is 240?
2. 25 % of itself is 250?
3. $12\frac{1}{2}$ % of itself is 360?
4. 200 % of itself is 1,800?
5. $14\frac{2}{7}$ % of itself is 88?
6. 75 % of itself is 14?
7. 50 % of itself is 600?
8. 5 % of itself is 420?
9. 300 % of itself is 2,400?
10. $37\frac{1}{2}$ % of itself is 330?
11. 1 % of itself is 202?
12. 7 % of itself is 428?
13. $6\frac{2}{3}$ % of itself is 4,800?
14. $66\frac{2}{3}$ % of itself is 40?
15. 60 % of itself is 64?
16. 100 % of itself is 600?
17. How much is \$40 increased by 20 % of itself?
18. How much is 60 increased by 10 % of itself?
19. How much is \$42 increased by 10 % of itself?
20. How much is 3 increased by 100 % of itself?
21. How much is 24 bu. increased by $37\frac{1}{2}$ % of itself?
22. How much is 12 yd. increased by $33\frac{1}{3}$ % of itself?
23. How much is 10 mi. increased by 7 % of itself?
24. How much is \$18 increased by 200 % of itself?
25. How much is 32 acres increased by $12\frac{1}{2}$ % of itself?

144. Per Cent of Increase.

To find the increase, *multiply the base by the per cent of increase.*

Problem. Mr. Smith, whose salary was $1,200.00, received a 5% increase. How much was the increase?

SOLUTION.

$$\begin{array}{rl} \$\,1{,}200 & \text{Base} \\ .05 & \\ \hline \$60.00 & \text{Increase} \end{array}$$

To find the per cent of increase, *divide the increase by the base.*

Problem. Mr. Smith, whose salary was $1,200.00, received an increase of $60.00. What was the per cent of increase?

SOLUTION. $60.00 ÷ $1,200.00 = .05, or 5%.

To find the base.

a. Given the increase and per cent of increase.

Problem. Mr. Smith received a 5% increase in salary. The in crease was $60.00. What was his original salary?

SOLUTION. $60.00 ÷ .05 = $1,200.00.
Increase ÷ per cent of increase = base.

b. Given the increase and the amount.

Problem. Mr. Smith's salary was increased $60.00, after which he received $1260.00. What was the per cent of increase?

SOLUTION. $1,260 − $60 = $1,200, the base.
$60 ÷ $1,200 = .05.
Hence, the per cent of increase is 5%.

c. Given the per cent of increase and the amount.

Problem. After receiving a 5% increase, Mr. Smith's salary was $1,260.00. What was his original salary and what was the increase?

SOLUTION. His original salary was 100% of itself. His increase was 5% of the original salary. Therefore, the new salary was 105% of the old one.

$1,260 ÷ 1.05 = $1,200.

Written Work

1. A merchant's sales during 1921 were $6,238.92. By advertising, he increased his sales $2,485.35 the following year. What was the per cent of increase, and what was the amount of the sales in 1922?

2. The deposits of a certain bank on June 30, 1921, were $365,894.56. On June 30, 1922, the deposits had increased to $396,268.55. What was the increase and the per cent of increase?

3. A factory produced 240 articles of a certain kind per day. By installing new machinery, the output was increased 12½ %. How many articles were produced per day after the new machinery was installed?

4. Williams paid $16.87 more taxes this year than he did last year; an increase of 7.8 %. What was the amount of his taxes each year?

5. The payroll of the Maynard Manufacturing Company for the week ending April 4, 1921, was $428.62. The payroll for the week ending April 3, 1922, was 13 % larger. What was the pay roll for the week in 1922, and what was the increase?

6. A merchant built an addition to his store which increased the floor space 28 %. After the addition was completed, the floor space was 8,960 square feet. What was the number of square feet of floor space in the original store, and what was the amount of the addition?

145. Per Cent of Decrease.

To find the per cent of decrease, *divide the decrease by the base.*

Example. Mr. Smith's coal bill in 1920 was $80.00. In 1921 his bill was $12.00 less. What was the per cent of decrease?

SOLUTION. Decrease ÷ base = per cent of decrease.
Thus, $12.00 ÷ $80.00 = 15 %.
The decrease was 15 %.

Written Work

1. The proprietor of a retail grocery employed five clerks at a total annual expense of $5,000.00 for wages. In order to reduce the expense, he released one clerk whose wages were $60.00 per month. How much did he decrease the cost of the clerk hire per year? What was the per cent of decrease? What was the annual cost of the clerk hire after one clerk was dismissed?

2. A paper mill which burned 2,400 tons of soft coal per year remodeled its furnaces and boilers and now burns 2,000 tons of coal per year. How many tons less does this mill consume per year than formerly, and what is the per cent of decrease?

3. Previous to 1921, Mr. Waterbury owned the only drug store in the town of X. When a new drug store was opened, the competition decreased Mr. Waterbury's annual sales $1,067.60, or 17 %. What was

the amount of Mr. Waterbury's sales the year before the new store opened?

4. In 1920 Mr. Snowden, a grocer, lost $327.00 from bad debts. A credit-rating association was formed in the town; the merchants informed each other of persons who did not pay their bills; and the following year Mr. Snowden decreased his loss from bad debts 18%. How much did the credit-rating association save Mr. Snowden, and what was his loss from bad debts in 1921?

5. Robbins moved his store into a new building where the rent was $5.00 per month less than the rent of the building which he formerly occupied. The rent in the new location was $125.00 per month. What was the per cent saved on rent?

6. Barnes said to his partner, "We cut down our delivery expenses 30% when we sold the horses and bought an auto truck. Our annual cost for making deliveries after we made the change was only $1,645.00. When we delivered with horses, the cost per year was $———, so we are saving $———."

7. The following table shows one of the uses which business men make of per cent of increase and decrease.

COMPARATIVE SALES SHEET

DEPARTMENT NUMBER	SALES, FRIDAY, JUNE 1, 1921	SALES, FRIDAY, JUNE 2, 1922	INCREASE OR DECREASE	PER CENT INCREASE OR DECREASE
1	$ 826 95	$ 914 32	$	
2	1,034 78	1,231 64		
3	1,237 62	1,196 14		
4	2,643 80	2,843 27		
5	1,413 80	1,376 29		
6	962 40	1,235 96		
7	2,642 16	2,927 92		
8	1,964 39	2,129 80		
9	1,636 48	1,596 27		
10	1,213 42	1,723 96		
11	1,394 29	1,146 92		
12	415 75	496 25		
13	3,460 00	3,246 29		
14	2,690 70	2,889 00		
15	878 25	794 60		
Total				

This table compares the sales on the first Friday in June, 1921, with the sales on the first Friday in June, 1922. The merchant assumes that trade conditions were about the same on the two days. A table of this kind is prepared, comparing the business for the day with the business of the corresponding day of the previous year.

Prepare a table similar to the model, and enter the facts given. Show increases and per cents of increase in black ink; decreases and per cents of decrease in red ink.

What was the increase in business in department 1? What was the per cent of increase?

Complete the table.

8. The following table shows the value of irrigating arid land. For example, the government records show that, in the regions where rainfall is insufficient, the potato crop on an acre of irrigated land was recently worth $15.37 more than the crop grown on an acre of unirrigated land. This is an increase of 34.4 % due to irrigation. What is the average value per acre of the potato crops grown on unirrigated land and on irrigated land?

Complete the table.

COMPARATIVE VALUE OF CROPS GROWN ON IRRIGATED AND UNIRRIGATED LAND

CROP	EXCESS OF AVERAGE VALUE OF CROP PER ACRE GROWN ON IRRIGATED LAND, OVER THAT GROWN ON UNIRRIGATED LAND		AVERAGE VALUE OF CROP PER ACRE ON UNIRRIGATED LAND	AVERAGE VALUE OF CROP PER ACRE ON IRRIGATED LAND
Potatoes	$15 37	34.4 %		
Sugar Beets . .	5 39	10.4		
Wheat	8 65	58.6		
Alfalfa	5 97	35.2		
Oats	7 36	63.2		
Barley	6 51	55.1		
Corn	3 51	24.0		
Timothy and Clover, mixed . . .	3 63	27.6		
Timothy alone. .	3 08	24.1		

146. Per Cent of Maximum. It is sometimes desired to compare a certain number with a larger number. For example, a certain packing plant is equipped to slaughter and pack a maximum of 2,000 hogs daily. On October 22, this plant packed 1,832 hogs.

The day's pack was $\frac{1833}{2000}$ of the maximum capacity, or 91.6%. State a rule for finding per cent of maximum.

MISCELLANEOUS PROBLEMS — PERCENTAGE

Written Work

1. Compute the per cent of increase in the population of each of the following cities for each of the ten year periods.

Cities	1900	1910	1920	Cities	1900	1910	1920
New York ..	3,437,202	4,766,883	5,620,048	Pittsburg	462,801	553,905	588,343
Chicago	1,698,575	2,185,283	2,701,705	Los Angeles..	104,266	319,198	576,673
Philadelphia.	1,293,697	1,549,008	1,823,779	San Francisco.	342,782	416,912	506,676
Detroit	290,277	465,766	993,678	Buffalo.....	352,387	423,715	506,775
Cleveland ..	384,111	560,663	796,841	Milwaukee ...	285,315	373,857	457,147
St. Louis....	575,238	687,029	772,897	Washington ..	278,718	331,069	437,571
Boston	560,892	570,585	748,060	Newark	248,849	347,469	414,524
Baltimore ..	508,957	558,485	733,826	Cincinnati ...	331,866	363,591	401,247

2. The total value of a man's property is $28,400. Of this amount 45% is invested in city real estate, 12½% in government bonds, and the remainder in a farm. Compute the amount invested in each.

3. The following table gives the number of farms in the United States and in several of the states in 1920 and the percentage of increase since 1910. Compute the number of farms in each in 1910.

Number of Farms in 1920, and Percentage of Increase Since 1910

	Number	Per Cent
United States	98,496	1.5
Arizona	1,589	17.2
California	29,493	33.4
Colorado	13,821	29.9
Idaho	11,302	36.7
Montana	31,227	119.1
Nevada	475	17.7
Oregon	4,686	10.3
Utah	3,988	18.4
Washington	10,096	18.0
Wyoming	4,624	42.1

147. Per Cent of Average. Percentage is also used to show the relation between different numbers by comparing them with their average.

Written Work

The following table shows the monthly sales of a number of clerks:

Clerk No.	Sales	Per Cent	Clerk No.	Sales	Per Cent
201	$1,412.32		205	$1,836.24	
202	1,218.16		206	1,216.26	
203	967.32		207	1,375.85	
204	1,046.89		208	1,493.85	

Find the average sales.

The sales of Clerk No. 201 are what per cent of the average?

Complete the table.

What is the value of such a table for the manager of a business?

Miscellaneous Exercises

1. If a merchant sells all goods at an advance of 10% of the cost, what will be the selling price of an article which cost $3.40? What will be the profit on this article? If this merchant's annual sales are $24,126.35, what is his gross profit?

He marked all goods at 25% increase on the cost the following year and his sales dropped to $15,625.90. What was the cost of the goods sold, and the gross profit?

2. In the following table is recorded the number of games won and lost by the teams in a certain school. What per cent of the games played did each team win?

Team	Games won	Games lost
Invincibles . . .	18	7
Tigers	15	10
Midgets	9	16
Lions	12	13
Giants	7	18

3. Compute the per cent of increase in population from 1910 to 1920 for each of the largest five cities in your state.

4. Six salesmen reported the annual sales and expenses given in the table below. Compute the per cent which each man's expenses were of his sales. Compute also the average sales and average expenses.

	Total Sales	Total Expenses
George Robbins. . .	$18,428.50	$2,148.65
Arthur Smith . . .	16,302.90	1,943.90
Robert Adams . . .	21,408.50	2,290.50
Martin Evans . . .	11,390.00	2,380.40
Carl Moore	17,365.40	2,406.80
Joseph Carter . . .	9,438.80	1,841.50

5. When a man's daily wages are increased from $4.80 to $5.60, what is the per cent of increase? When his daily wages are reduced from $5.60 to $4.80, what is the per cent of decrease?

6. The purity of gold is measured in carats, or twenty-fourths, 18 carats meaning $\frac{18}{24}$ pure gold. What is the per cent of pure gold in a ring that is 12 carats fine? What is the per cent of pure gold in a watch chain that is 18 carats fine?

7. What was the per cent of advance in price when sugar increased from 8 cents to 27 cents a pound?

8. What is the difference in pounds between $\frac{3}{4}$ of a ton and $\frac{3}{4}$% of a ton?

9. A certain kind of cloth loses $1\frac{7}{8}$% of its length when it is sponged. What will be the length of a piece of this cloth after it is sponged if the length before sponging is 8 yards?

10. An ice dealer finds that on the average the ice which is placed in his ice houses loses 15% of its weight before it is sold. How much ice should he store in order to have 38,250 tons to sell?

11. A certain poultry dealer finds that chickens lose 27% in weight when dressed. He has an order from a hotel for 219 pounds of dressed chickens. How many pounds of live chickens will be required to fill the order?

12. A dairyman decides to test two cows. One cow gives 32 pounds of milk a day, which tests 3.4% of butter fat; another gives 30.5 pounds, testing 3.7% of butter fat. Butter fat being worth 38 cents a pound, which cow yields the larger daily return and how much larger?

13. A poultry man finds by experience that on the average 84% of the eggs which he places in an incubator hatch out chicks. If he wishes to obtain about 700 chicks, how many eggs must be placed in the incubators?

14. The following table gives a summary of the government appropriations for the fiscal year 1920. Compute the per cent of the total appropriated for each.

RECAPITULATION OF APPROPRIATIONS FOR THE FISCAL YEAR 1920

Legislative	$ 18,899,687.03
Executive Proper	221,080.00
Department of State	12,313,743.16
Department of War	861,465,699.64
Department of the Treasury	1,423,921,594.35
Department of the Navy	637,544,113.54
Department of the Interior	268,398,697.13
Department of Justice	17,229,282.73
Post Office Department	471,750,370.26
Department of Agriculture	143,187,541.50
Department of Commerce	34,532,720.03
Department of Labor	8,278,062.03
Independent Establishments	1,969,664,423.98
District of Columbia	17,721,049.94
Miscellaneous Objects	1,134,260,047.67
Total	

15. It is estimated that for every $10 spent by the average wage earner's family in a certain city, $3.78 is spent for food; $1.60 for clothing; $1.49 for rent; $.60 for light and fuel; $.44 for furnishings; and $2.09 for miscellaneous items. Compute the per cent of the total spent for each purpose.

16. In the following table is recorded the total imports and exports of the United States for certain years. Compute the per cent of increase or decrease for each year, using the year 1914 as a basis of comparison.

FOREIGN COMMERCE OF UNITED STATES

YEAR ENDED	EXPORTS	IMPORTS
June 30		
1880	$ 835,638,658	$ 667,954,746
1890	857,828,684	789,310,409
1900	1,394,483,082	849,941,184
1907	1,880,851,078	1,434,421,425
1912	2,204,322,409	1,653,264,934
1914	2,364,579,148	1,893,925,657
1916	4,333,482,885	2,197,883,510
1919	7,920,425,990	3,904,364,932
1920	8,228,016,307	5,278,481,490

148. Percentage Analysis of a Business. Mr. E. C. Barton is the proprietor of a wholesale store. At the end of each month he analyzes the records of his business. The following analysis was made June 30, 1922. Results should be approximate to the nearest hundredth of a per cent.

1. The purchases during June, 1921, were $8,146.90. Some of the goods purchased were defective, and were returned. The value of goods returned was $123.60. The returned goods were what per cent of the purchases?

2. Gross sales for June, 1921, were $11,216.29. Goods returned to the store by dissatisfied customers, $97.65. The returned sales were what per cent of the gross sales?

3. Gross sales for June, 1921, were $9,862.15. Returned sales, $101.80. What per cent of the sales were returned in June, 1921?

4. What was the increase in the gross sales, and what was the per cent of increase?

5. Gross Sales – Returned Sales = Net Sales.

What were the net sales for June, 1921? For June, 1922? What was the increase in net sales, and the per cent of increase?

6. The merchandise on hand June 1, 1922, was worth $14,162.45 at selling prices. The net sales for the month were found in problem 5. What per cent of the stock on hand at the beginning of the month was sold during the month?

7. Net Sales – Cost of Goods Sold = Gross Profit.

The gross profit for June, 1921, was $1,688.54. The gross profit was what per cent of the net sales for the month? (For net sales see results to problem 5.)

8. The gross profit for June, 1922, was $2,001.36. The gross profit was what per cent of the net sales for that month?

9. The expenses of the business in June, 1921, were $1,015.60. The expenses had increased in June, 1922, to $1,140.26. What was the per cent of increase in the expenses?

10. Net Sales – Gross Profit = Cost of Goods Sold.

What was the cost of goods sold in June, 1921?

What was the cost of goods sold in June, 1922?

11. Gross Profit – Expenses = Net Profit.

What was the net profit for June, 1921?

What was the net profit for June, 1922?

12. What was the per cent of increase or decrease in the net profit?

13. Net Profit ÷ Net Sales = Per Cent of Net Profit on Sales.
What was the per cent of net profit for June, 1921?
What was the per cent of net profit for June, 1922?

CHAPTER XV

BUYING AND SELLING MERCHANDISE

149. The Invoice. When a merchant sells goods, he usually gives the customer a bill or invoice. The invoice states the following facts:

The date of the sale.

The name and address of the seller.

The name and address of the purchaser.

The terms of the sale.

A detailed list of the articles sold, including the price of each item and the total of the invoice.

If prepaid freight is to be charged to the purchaser, the amount of freight is added to the invoice.

If the invoice has been paid, a statement of the receipt of payment is made by the seller.

Study the following invoice and state as many as possible of the facts enumerated above.

TELEPHONE MAIN, 1340
PRIVATE EXCHANGE

A. B. HUGHES

DEALER IN

STAPLE AND FANCY GROCERIES

211-213-215 Lake Street,

Chicago, Illinois

TERMS: 2% Cash in 10 days, Net 30 days.

June 10, 1922

SOLD TO Scobey & Company, Fayette, Iowa

		2 Cases Tomato Catsup 4 doz.	1.90	7	60		
		6 Cases Elgin Canned Corn 12 doz.	1.80	21	60		
		5 Cases Echo Peas 10 doz.	1.85	18	50	47	70

150. What the Purchaser Does. After the purchaser receives both the goods and the invoice, he inspects the merchandise to see whether it agrees with the items for which he has been charged on the invoice. If stock of the correct kind and quantity has been received, a check mark is entered on the invoice opposite each item in the column at the left.

The invoice is next checked to ascertain:

a. Whether the correct prices have been charged.

b. Whether an error has been made in the extension of the cost of any item, or in the total of the invoice.

151. Credit Memorandum. In case an error is found, notice is sent to the firm from which the goods were purchased. When credit is to be given by the seller on account of an error in the invoice, or for imperfect goods, or for any other cause, the purchaser will usually receive a credit memorandum, notifying him of the amount of credit entered to his account.

CREDIT MEMORANDUM

A. B. HUGHES
CHICAGO.

June 17, 19.

TO Williams & Co.,
Albany, New York

Your account has been credited $1.25
On account of overcharge on invoice of June 10, 19

Written Work

When an invoice has been checked and found to be correct, the extensions and footings are checked as shown in the illustration.

$16	50✓		
23	00✓		
4	80✓	$44	30✓

Check the invoice on page 177. If the multiplications are found to be correct, enter a check mark at the right of each item. If the total is correct, check it also.

Written Work

Find the totals of each of the following sales:

Apply short methods when possible.

1.

			Price
4 doz.	Men's Irish Linen Handkerchiefs	doz.	$6.25
5 doz.	Men's Hemstitched Handkerchiefs	doz.	7.35
14 doz.	Men's White Initial Handkerchiefs	doz.	4.87
9 doz.	Women's Imported Swiss Embroidered Handkerchiefs	doz.	3.75
22 doz.	Children's Linen Handkerchiefs	doz.	$1.66\frac{2}{3}$
7 doz.	Women's Swiss Lawn Handkerchiefs	doz.	.95
13 doz.	Children's Mull Handkerchiefs	doz.	1.05

2.

17 doz.	Men's Leather Gauntlets	doz.	$11.85
11 doz.	Men's Buckskin Gloves, sizes 8 to 10	doz.	12.50
16 pairs	Men's Leather Automobile Mittens	pair	$2.37\frac{1}{2}$
$4\frac{1}{2}$ doz.	Men's Suede Gloves, sizes $7\frac{3}{4}$ to 10	doz.	7.50
$3\frac{1}{2}$ doz.	Men's No. J 264 Kid Gloves	doz.	17.50
$5\frac{1}{2}$ doz.	Imported Kasan Cape Gloves	doz.	19.75

3.

16	Silk Taffeta Umbrellas	each	$ 3.65
28	8 Rib Pure Silk Taffeta Umbrellas	each	6.15
32	Men's Suit Case Umbrellas	each	4.05
12	Extra Size Umbrellas	each	$1.12\frac{1}{2}$

4.

3 doz. pairs	Walton Cotton Blankets	pair	$ 1.15
1 doz. pairs	Cotton Crib Blankets	pair	$.92\frac{1}{2}$
8 doz. pairs	Wool Filled Gray Blankets	pair	5.15
7 doz. pairs	Douglass Wool Finish Plaids	pair	$2.67\frac{1}{2}$
4 doz. pairs	All Wool Plaids	pair	9.00
10 doz. pairs	" Canada Camp " Blankets	pair	$1.87\frac{1}{2}$

5.

20	Assorted Pattern Smyrna Rugs, 36×72	each	$ 5.75
80	Scotch Jute Rugs	each	.95
3 doz.	Cotton Bathroom Rugs, 27×54	each	$1.87\frac{1}{2}$

6.

			Price
3 doz.	Size 6 Children's Fast Black Hose	doz.	$ 1.67½
2 doz.	Size 6½ Children's Fast Black Hose	doz.	1.75
4 doz.	Size 7 Children's Fast Black Hose	doz.	1.83½
5 doz.	Size 7½ Children's Fast Black Hose	doz.	1.57½
6 doz.	Size 8 Children's Fast Black Hose	doz.	1.62½
8 doz.	Size 8½ Children's Fast Black Hose	doz.	1.77½
7 doz.	Size 9 Children's Fast Black Hose	doz.	1.83½
6 doz.	Size 9½ Children's Fast Black Hose	doz.	2.07½
5 doz.	Size 10 Children's Fast Black Hose	doz.	2.12½

7.

20	No. B 645 Silk Woven Waist Patterns (each pattern 3½ yd.)	yd.	$ 1.50
25	No. B 493 Dresden Silk Mull Waist Patterns (3½ yd. pieces)	yd.	1.17½
18 bolts	Irish Linette, 10, 11^2, 12, 10^3, 11^1, 11^3, 12^1, 11^3, 10^3, 12, 10, 10^3, 10^1, 11^2, 12, 11^3, 10^2, 10^3	yd.	.92½
20 bolts	French Gauze Chiffon, 14, 13^3, 12^3, 14^1, 14, 13^2, 13, 12^3, 13, 13^1, 14^2, 13^3, 14^2, 14^1, 13^3, 12^3, 14, 13^3, 13^2, 12^3	yd.	1.45½

NOTE. The small figures mean quarter yards. Thus 11^3 means $11\frac{3}{4}$ yards.

EXPLANATION OF GROCERY ORDERS

The sign # if placed before figures means "number"; if placed after figures it means pounds.

Articles sold in cases [cs.] are usually priced by the dozen. The total number of dozens in the cases is therefore given after the name of the commodity, and the price is the price per dozen.

Some articles sold in barrels and cases are priced by the pound; in such instances the total number of pounds in all the barrels or cases appears immediately before the price.

Sugar is sold by the hundred pounds (cwt.). The net weight of each barrel is given. Find the total number of pounds; point off two places to the left to find the number of hundred pounds, and multiply by the price per hundred.

8.

		Price
7 cases	Acme Peas	14 doz. $1.85
3 boxes	Peona Soap	6.05
7 bbl.	Northern Salt	2.80
15 bbl.	Winter Wheat Flour ⅛ sacks	8.85

12	bbl. H. & E. Granulated Sugar 329, 335, 347, 351, 344, 347, 350, 331, 342, 355, 349, 333		8.20
5	sacks H. & E. Granulated Sugar	500#	8.15
9	cases Acme Peas	18 doz.	1.85
17	cases Acme Corn	34 doz.	2.25
6	boxes Dried Apples	150#	.12¾
2	bbl. 10# sacks Northern Salt		2.65
6	cases Algonac Tomatoes	12 doz.	1.87½

9.

1	cs. 25# Macaroni		$2.80
5#	Cream of Tartar		.60
1	cs. Acme Peas	2 doz.	1.85
5	cases Hawthorne Pears	10 doz.	3.45
11	cases Hawthorne Peaches	22 doz.	3.20
12	cans Pimento		.23
15	cs. H. L. Shredded Pineapple		3.15
10	cases Tall Salmon	40 doz.	2.85
12	cases F. & B. Tomatoes	24 doz.	3.95
85	sacks Yellow Corn Meal	850#	.05
8	cases 25# Macaroni		2.80
5	cases 15# Spaghetti		2.80
7½	doz. pkgs. Seeded Raisins		1.95
125#	Rice		.11
25	quarts Olive Oil		1.35 gal.
18¾	gal. Sweet Pickles		1.29¼
6½	gal. Olives $\frac{70}{80}$		1.45
25	cases 4 oz. Grd. Pepper	125#	.40
6	cans Paprika		2.65 doz.
14	cases G. M. Soap		3.35
12	cases Soda Crackers	93¾#	.13
12	Cracker cans		.50
1	tub Peanut Butter	48½#	.17½
62½	bbl. ⅛ sacks Spring Wheat Flour		8.40
49¾	bbl. ⅛ sacks Winter Wheat Flour		8.85
16	cases #2 Algonac Lima Beans	32 doz.	1.85
14	cases #2 Algonac String Beans	28 doz.	1.75

10.		Price
10 doz.	W/W 6 in. Plates	$1.37½ doz.
13½ doz.	W/W 4 in. Plates	1.12½ "
5 doz.	/S/ 2½ in. Butter Chips Tk.	.33 "
8¼ doz.	#13 Cups	1.25 "
6 doz.	W/W 4 in. Ice Creams	.67½ "
15 doz.	#64 /G/ Nappies	1.67½ "
8 doz.	A.C. ½ Tk. 30's L/F Bowls	1.92 "
¾ doz.	#211 Tea Pots	3.65 "
2½ doz.	#0 /S/ Custard Pots	1.75 "
17 doz.	#1,352 Tumblers	.98 "
12 doz.	#1,296 Finger Bowls	2.33⅓ "
1¾ doz.	#3,234 Oil Bottles	3.50 "
3½ doz.	12–193 Salts	2.25 "
3½ doz.	12–200 Peppers	2.25 "
2⅙ doz.	444 A Pitchers	13.00 "
⅓ doz.	Punch Glasses	1.25 each
12½ doz.	3,404 Dessert Forks	4.25 doz.
12½ doz.	3,405 Dessert Knives	4.20 "
16 doz.	3,413 Tea Spoons	2.95 "
8 doz.	3,406 Dessert Spoons	3.40 "

Rule invoices and enter the following sales, assuming that you are the selling merchant.

11. To W. K. Sears,
Elgin, Illinois.
Sept. 10.

45	No. R 721 Body Brussels Rugs 6 × 9 ft. each	$31.75
32	No. R 731 Body Brussels Rugs 8 × 10 ft. each	37.50
36	No. R 741 Body Brussels Rugs 9 × 12 ft. each	41.25
25	No. R 751 Body Brussels Rugs 11 × 13 ft. each	48.80

12. To C. B. Perkins,
Amherst, Minn.
Sept. 12.

20	H 20 Family Coffee Mills	each	$ 1.95
16 sets	H 33 Sad Irons	set	1.23
12 doz.	H 545 Flour Sifters	doz.	2.17⅞

25 doz. H 463 Carpet Beaters	doz.	1.11
8 doz. H 6,646 Galvanized Clothes Lines, 20 gauge wire,	doz.	1.29⅜
10 doz. H 6,546 Galvanized Clothes Lines, 18 gauge wire,	doz.	1.88½

13. To Owen & Hendersen,
Cedar Rapids, Iowa.
Sept. 13.

35 F 384 Plush Library Chairs	each	$26.30
28 F 376 Reed Rockers	each	6.80
18 F 394 Bedroom Rockers	each	8.15
26 F 465 Roman Chairs	each	13.75
19 F 414 Morris Chairs	each	27.75

CHAPTER XVI

COMMERCIAL DISCOUNTS

Classes of Commercial Discount. Commercial discounts are of three general classes; namely, Cash Discount, Trade Discount, and Quantity Discount.

Cash Discount

152. Purpose of Cash Discount. In order to encourage the prompt settlement of accounts, merchants frequently offer to deduct a certain per cent of the bill if it is paid within a fixed number of days. The deduction for prompt payment is called **cash discount.**

153. Terms. The terms state the discounts offered, and the time when the bill is due and payable without any deduction for discount. The terms are usually indicated in the following manner:

2/10; 1/30; N/60.

The figure at the left of the line indicates the rate of discount offered; the figure at the right of the line indicates the number of days within which payment must be made in order to obtain the discount. Thus, the terms stated above mean:

2% discount from the face of the bill, if paid within 10 days;
1% discount from the face of the bill, if paid after 10 days but before 30 days;

No discount after thirty days and the bill is due in 60 days.
The terms are read as follows: "Two, ten; one, thirty; net, sixty."

154. Definitions. The amount of the purchase before subtracting the discount is called the **list price.**

The remainder after subtracting the discount is called the **net price.**
The **rate of discount** is stated as a per cent of the list price.

155. Cash Discount Illustrated. The terms of the invoice on page 177 were 2/10; N/30.

In order to obtain the discount, Scobey & Co. must pay the bill on or before June 20, 1922.

The bill is due and payment is expected **in full** on July 10, 1922.

No discount is allowed if payment is made between June 20 and July 10.

156. Computing the Discount and the Net Price.

First Method;

$ 95.25	List price
.02	Rate of discount
$1.9050	Discount

$ 95.25	List price
1.91	Discount
$ 93.34	Net price

State a rule for finding discount and net price by this method.

(Note that five mills or more in the discount are considered a cent. Thus, $1.905 is considered $1.91.)

Second Method;

100 %	List price
2 %	Rate of discount
98 %	Per cent of bill to be paid

$95.25	List price
.98	
$93.345	Net price
or $93.34	

State a rule for computing the net price by this method.

157. The Advantages of Cash Discount. Cash discounts may benefit both the purchaser and the seller. Merchants offer cash discounts because they encourage prompt payments, and thus decrease:

a. the loss from bad debts;
b. the cost of collecting accounts;
c. the amount of capital tied up in outstanding accounts.

It is usually good business for the purchaser to "take his discount" (pay the bill before it is due), even though the rate of discount may be small. An illustration will show this fact.

Suppose the terms of the sale are, Cash less 1%; net 30 days. If the purchaser pays cash, he receives 1% for the use of his money for one month. 1% a month is 12% a year; a high rate of interest.

The rates of cash discount are usually small, varying from 1% to 3%.

Small discounts are offered if the bill is due in a short time.

For example, 1/10; N/30.

Larger discounts are offered if the bill is due after a greater length of time.

For example, 3/30; N/4 months.

As a further means of insuring payment of invoices, merchants frequently charge interest on bills which are not paid when due.

Written Work

Find the list price, the discount, and the net price of each of the following purchases, and answer the following questions about each invoice:

a. What is the last day on which payment can be made and the discount secured?

b. When is the invoice due?

1. Invoice dated April 4. Terms 2/10; N/30.

7 cases Acme Peas	14 doz.	$1.85
3 cases Osea Soap		5.45
7 bbl. Northern Salt		2.80
15 bbl. $\frac{1}{8}$ sacks A. D. Flour		8.85

2. Invoice dated October 26. Terms 1/15; N/30.

12 bbl. H. E. Granulated Sugar, 329, 335, 347, 351, 344, 347, 350, 331, 342, 355, 349, 333#	$8.20
5 sacks H. E. Granulated Sugar, 500#	8.15

3. Invoice dated January 5. Terms 2/10; 1/20; N/30.

16 bolts White Dress Linen, 10, 12, 11^1, 12^2, 10^3, 11^3, 10^3, 12, 10^1, 11^2, 12, 11^1, 10^2, 11^3, 12^1, 10^2	yd.	$0.97\frac{1}{4}$
12 bolts Persian Lawn, 24^3, 25^1, 23^2, 25, 24^2, 24^3, 25^3, 24^1, 23^3, 24^3, 25^3, 25	yd.	$.45\frac{3}{4}$
48 bolts French Nainsook, 12-yard pieces	yd.	$.49\frac{1}{4}$

36 bolts Mercerized Lingerie Batiste, 24-yard pieces .	yd.	.43
24 bolts Imported Linen Lawn, 10-yard pieces	yd.	.96¾

NOTE. There will be two possible net prices for this invoice.

4. Rule a form similar to the following table.

Combine the dollars in the column at the left, with the cents in the row near the top, to form the list price. Deduct the discount at the rate shown above the cents, and enter the net price in the table. The two results entered in the table show the method. Thus,

a.	$26.10	the list price	
	.02	the discount	
	$ 0.52	the discount at 2%	
	$26.10		
	.52		
	$25.58	Net price	

b.	$641.67		
	.025		
	$ 16.04	the discount at 2½%	
	$641.67		
	16.04		
	$625.63	Net price	

	Less 2%		Less 1½%		Less 3%		Less 2½%	
		10		45		75		67
$26	00	$25	58					
14	00							
73	00							
19	00							
261	00							
112	00							
317	00							
216	00							
641	00						$625	63

TRADE DISCOUNT

158. Purpose of Trade Discount. In some lines of business merchants sell both at wholesale and retail. They advertise their goods at a certain price, but when they sell to dealers, they frequently deduct a part of this price. The amount deducted is called a **Trade Discount.**

Trade discounts are common in businesses which issue catalogues. When the catalogue is sent to a dealer, a discount sheet is inclosed.

Specimen Discount Sheet

DISCOUNT SHEET

The following discounts are offered on articles listed in catalogue No. A 23.

Pages 1 to 15	20%
Pages 16 to 39	28%
Pages 39 to 67	40%
Pages 68 to 90	Net
Pages 91 to 135	14⅔%

Do not show this discount sheet to your customers.

NOTE. "Pages 68 to 90 Net" means, no trade discount is offered for goods listed on these pages.

Not Responsible for Goods Lost or Damaged in Transit. Claims for Allowance must be made upon Receipt of Goods. Address all Communications to Abercrombie & Co., Chicago

ABERCROMBIE & COMPANY

PUBLISHERS & BOOKSELLERS

245 WABASH AVENUE

CHICAGO, March 3, 1922

SOLD TO W. M. Rickert Co.,
Home,
Iowa

YOUR ORDER NO. 121
TERMS: 2/10: N/30
CONVEYANCE, U. S. Ex.

IN REFERRING TO THIS ORDER MENTION NO. 4336 ENTERED W. B. 3/3/22.

12	Copies H. & J. Second Year English	.85	10	20		
21	Beeman's Algebra	.90	18	90		
			29	10		
	Less 18%		5	24	23	86

Class Discussion

1. Why do merchants offer trade discounts?
2. What is the catalogue price of the H. & J. Second Year English per copy?
3. How much does each copy of the English book actually cost the retail dealer? Ignore cash discount.
4. If Mr. Rickert sells this book for the catalogue price, 85 cents, how much profit does he make on each book?
5. Trade discounts are usually larger than cash discounts. Why do you think this is the case?

159. Computing Trade Discounts. Trade discounts are computed in the same manner as cash discounts.

Problem. What will goods cost a dealer if they are sold to him for \$46, less a trade discount of 25%?

SOLUTION.

\$46.00 List price	\$46.00 List Price
.25 Rate of discount	11.50 Trade discount
\$11.50 Trade discount	\$34.50 Price to dealer

Written Work

Rule and complete the following table. Enter the price to dealer after deducting the trade discount.

RATE OF TRADE DISCOUNT	CATALOGUE PRICE		
	\$27.85	\$126.39	\$96.45
$14\frac{2}{7}$%			
18%			
20%			
22%			
25%			
15%			
17%			
28%			
$16\frac{2}{3}$%			
10%			

160. Determining the Rate of Trade Discount. In many lines of business the market price of the goods is continually fluctuating. This may be due to the changes in the prices of the raw material used in manufacturing, or to other causes. If new catalogues were published every time the prices changed, there would be an enormous expense in printing and mailing. Moreover, the time required for printing might be so great that new changes in prices would make the catalogue out of date before it reached the customers. To avoid this expense and delay the simple device is adopted of sending customers a new discount sheet.

For example, suppose an article made from steel (the market price of which varies at frequent intervals) is quoted in the catalogue at \$8.

When the market price of steel drops and the article can be sold for \$7, a discount of $12\frac{1}{2}\%$ can be offered.

If raw steel should drop in price so that the article could be sold for \$6, a discount of 25% could be offered.

Problem. What rate of discount will change a catalogue price of \$3 to a market price of \$2.50?

SOLUTION.

\$3.00	Catalogue price
2.50	Price at which article is to be sold
\$0.50	Amount to be deducted as discount

\$0.50 ÷ \$3.00 = $16\frac{2}{3}\%$, rate of discount to be offered

State a rule for finding the rate of discount by this method.

When the quotient obtained by the division involves a fractional per cent, the next lower whole number is sometimes taken as the per cent. Thus, a quotient of $15\frac{1}{7}\%$ may be regarded as 15%; a discount of 18.27% may be regarded as 18%.

Problem. An article is listed at \$24.50. The market conditions are such that it should sell for about \$21.30. What rate of discount should be offered?

SOLUTION.

\$24.50	List price
21.30	Market price
\$ 3.20	Amount to be deducted by discount

\$3.20 ÷ \$24.50 = 13.06%.

13% would therefore be offered.

This discount would make the net price \$. ?

Written Work

Rule and complete the following table, showing what rates of discount would be offered to reduce the list prices at the left to the market prices shown at the top of the table.

Express results to the nearest per cent which will not reduce the market prices below those stated in the table

List Prices	Market Prices									
	$8	50	$7	75	$8	00	$7	60	$6	00
$10	00									
12	00									
11	50									
9	80									
10	50									

Quantity Discounts

161. Purpose of Quantity Discounts. In some lines of business, particularly in manufacturing, it is customary to give a larger rate of discount on a large order than on a small one.

A mail order firm advertises:

1% discount on an invoice of $10; 2% on $20; 5% on $35.

What is the purpose of such quantity discounts?

A printer advertised the following rates:

$10 per thousand for the first thousand copies. On an order of more than 1000 copies, a discount of 15%. This discount is offered because the work of typesetting, making up the forms, and making ready the press must all be done, even if but one copy is to be printed. The cost of printing 2,000 copies, therefore, is not twice as much as the cost of printing 1,000 copies.

Written Work

1. The following prices were quoted by a manufacturer of lockers: No. 6,094. Each $6.30.

Discounts:

Orders of from 50 to 100 — Less 8%.
Orders of from 101 to 300 — Less 10%.
Orders of from 301 to 500 — Less 12½%.

Find the total cost and the cost per locker of an order of 75 lockers; 140; 475.

2. A paper manufacturer quoted the following prices:

No. 020 note paper, cut to size 8 by 10, $.55 per ream.

An extra charge of 3 cents per ream will be made for wrapping in packages of 500 sheets. (500 sheets considered 1 ream.)

On orders of 100 reams or more a discount of 4% will be allowed.

Terms, 1/10; N/30.

Find the cost and state what discounts were allowed:

a. An order of 50 reams, unwrapped, payment made 20 days from date of sale.

b. An order of 150 reams, unwrapped, payment made 15 days from date of sale.

c. An order of 150 reams, wrapped, payment made 18 days from date of sale.

d. An order of 240 reams, wrapped, payment made 7 days from date of sale.

e. An order of 60 reams, wrapped, payment made 8 days from date of sale.

Discount Series

162. Definition. When two or more discounts are allowed on an invoice, these discounts are known as a series.

The first discount applies to the list price; the second applies to the remainder after deducting the first discount; the third applies to the remainder after deducting the first two discounts; et cetera.

163. Trade and Cash Discount. The most common form of series consists of two discounts; the first a trade discount and the second a cash discount.

Problem. What is the smallest amount of money that will pay a bill of $236, subject to a trade discount of 20% and a cash discount of 2%?

SOLUTION.

$236.00	List price	$188.80	Trade price
.20	Rate of trade discount	.02	Rate of cash discount
$ 47.20	Trade discount	$3.7760	Cash discount
$236.00	List price	$188.80	Trade price
47.20	Trade discount	3.78	Cash discount
$188.80	Price to the trade	$185.02	Net price

The cash discount is computed on the trade price, *not* on the list price. Therefore, 20% trade discount and 2% cash discount are not the same as a single discount of 22%.

Written Work

Rule and complete the following table. From each list price deduct the trade discount indicated above the list price; and from the trade price thus found, deduct the cash discount at the rate shown at the left.

CASH DISCOUNT	FACTS TO BE FOUND	TRADE DISCOUNT, 28%	TRADE DISCOUNT, 25%	TRADE DISCOUNT, $33\frac{1}{3}$%
2%	List Price	$36 45	$68 40	$239 64
	Trade Discount			
	Trade Price			
	Cash Discount			
	Net Price			
3%	List Price	205 08	333 95	47 30
	Trade Discount			
	Trade Price			
	Cash Discount			
	Net Price			
5%	List Price	35 00	713 20	609 49
	Trade Discount			
	Trade Price			
	Cash Discount			
	Net Price			

164. Series of Trade Discounts. In some lines of business it is customary to offer a series of trade discounts rather than a single rate.

Problem. What is the trade price of an invoice of $175.00 subject to discounts of 25%, 20%, and 10%?

SOLUTION.

$175 00	List price	$175.00	List price
25	First discount rate	43.75	First discount
43.75	First discount	131.25	After first discount
$131.25	After first discount	$131.25	After first discount
.20	Second discount rate	26.25	Second discount
26.25	Second discount	105.00	After second discount
$105.00	After second discount	$105.00	After second discount
.10	Third discount rate	10.50	Third discount
10.50	Third discount	94.50	After third discount

Written Work

Find the net cost of the following bills:

1. $45.00 less 20%, 15%, and 10%.
2. $28.00 less 25%, 10%, and 5%.
3. $70.00 less 30%, 20%, and 2%.
4. $105.50 less 20%, 12½%, and 8%.
5. $39.80 less 15%, 10%, and 4%.
6. $2,806.75 less 28%, 14%, and 2%.
7. $467.70 less 25%, 20%, and 9%.
8. $369.05 less 40%, 20%, and 10%.
9. $294.46 less 33⅓%, 20%, and 12½%.
10. $723.34 less 37½%, 10%, and 20%.

165. Computing a Single Rate Equivalent to a Series. To compute a single rate equivalent to a series:

Subtract each rate from 100%;

Find the product of these remainders;

Subtract this product from 100%.

Problem. What single discount rate is equivalent to a trade discount of 20% and a cash discount of 2%?

SOLUTION.

100% − 20% = 80%
100% − 2% = 98%
80% × 98% = 78.4%
100% − 78.4% = 21.6% The single equivalent rate

Problem. What single rate is equivalent to 25 %, 20 %, and 10 %?

SOLUTION.

$$100\% - 25\% = 75\%$$
$$100\% - 20\% = 80\%$$
$$100\% - 10\% = 90\%$$
$$75\% \times 80\% \times 90\% = 54\%$$
$$100\% - 54\% = 46\%$$ The single equivalent rate

Written Work

The buyers in some large business houses prepare an elaborate table similar to the one which follows. When manufacturers quote prices subject to a discount series, the buyers can tell by a glance at the table what single discount the series equals.

Rule and complete the following table, using the method just explained to find single discounts equivalent to a discount series.

TRADE DISCOUNT	CASH DISCOUNT, 1%	CASH DISCOUNT, 1½%	CASH DISCOUNT, 2%	CASH DISCOUNT, 2½%	CASH DISCOUNT, 3%
18 %					
15					
20					
25					
33⅓					

With the aid of the table, find the net price of the following:

a. An invoice of $215, less 18 % and 2 %.
b. An invoice of $464.20 less 18 % and 3 %.
c. An invoice of $12.40 less 20 % and 1½ %.
d. An invoice of $23.87 less 33⅓ % and 3 %.

Referring to the previous page; determine what single rate is equivalent to each series and compute the net amount of each bill using the single rates.

Written Review

1. An implement dealer purchased the following invoice:

Dated April 17.

4 No. 364 Plows, $38.75, less 20 %.
7 Self Dump Hay Rakes, $20.50, less 18 %.
7 No. 264 Hay Stackers, $41.50, less 15 %.

Terms, 2% for Cash in 20 days. Net 90 days.
On May 2, he paid the bill with a check for $——.

2. Which of the following prices is better for the purchaser:

$45, less 25%, 18%, and 2%;
$60, less 28%, 25%, and 1%?

3. A merchant's discounts were 25% and 15%. A clerk sold an invoice of $72 and gave a single discount of 40%. How much did his error cost his employer?

4. A merchant lists a desk at $45 less 20%. A competitor sells a similar desk for $48 less 33⅓%. In order to meet his competitor's price, the first merchant decides to give an additional discount of —%.

5. Hewett paid an invoice in time to secure a discount of 3%. If the check sent was for $208.55, what was the list price of the invoice?

6. Graff Brothers sent a check to a wholesale house to pay an invoice. The check was for $801.90. What was the list price of the invoice if the discounts taken were 10% and 1%?

7. Compute the net price of an invoice of $100 subject to discounts as follows:

a. 40%
b. 25%, 10%, and 5%
c. 25%, 5%, and 10%
d. 10%, 25%, and 5%
e. 10%, 5%, and 25%
f. 5%, 10%, and 25%

What conclusions can you draw from this problem?

CHAPTER XVII

RECORDING PURCHASES AND SALES

Marking Goods

166. The Purchases Book. Merchants usually keep a record of purchases and sales. There are several different kinds of books used for this purpose. The following illustration shows a common form of the Purchases Book.

PURCHASES BOOK

Date of Invoice	From Whom Purchased	Amount	Terms	Discount term Expires	Cash Discount	Due Date	When & How Paid
1922 March 3	Abercrombie & Co.	23 86	2/10: N/30	Mar 13	48	Apr 2	Mar 12 By Check $23.38

The entries in the purchases book are made from the invoice received at the time of the purchase. The model shows proper record of the purchase made from Abercrombie & Co., as shown by the invoice on page 188.

Notice that $23.86 is the amount of the invoice after deducting the trade discount of 18 %. The cash discount is not deducted until the bill is paid.

The terms are taken from the invoice.

The discount term expires March 13. It is important to have this date recorded in the purchases book, as it is the last date on which payment may be made and the discount secured.

April 2 is the day when the bill is due and payment is expected. No discount is allowed when payment is made between March 13 and April 2.

167. How to find the Date when an Invoice is Due. If the terms are stated in days, count the actual number of days. Thus, 30 days from March 3 is April 2.

If the terms are stated in months, calendar months are counted. Thus, if the terms had been 2/10; N/1 month, the invoice would have been due on April 3.

No entry is made in the "When and How Paid" column until the invoice is paid.

It is not necessary to enter the items in the purchases book, because the invoice can be kept on file to supply this information.

168. The Sales Book. The following illustration shows a page from a commonly used form of sales book.

The entries were made from the invoice on page 188.

March 3, 1922

	W. M. Rickert & Co. Home, Iowa				
	Terms 2/10, N/30				
	12 Capper's Jr. J. Second Year English	.85	10	20	
	21 " Beeman's Algebra	.70	18	90	
			29	10	
	Less 18%		5	24	23 86

169. Loose-leaf Sales Book. Many merchants record their sales in a loose-leaf sales book. At the time the invoice is made, a carbon copy is also made. This requires very little extra labor, and the carbon copies, called "Charge Sheets," having holes punched at the side, can be bound together in a binder.

The loose leaf sales book has several advantages. Both the invoice and the charge sheet, which forms the sales book, can be made on the typewriter at the same time; fraud is prevented because the invoices are numbered, and a clerk cannot sell goods and make an invoice without also making a charge slip. He cannot keep the money and destroy the charge slip because one of the numbered sheets would be missing, and the fraud would be apparent.

The following illustration shows the charge sheet made as a carbon copy of the invoice shown on page 177.

TERMS: 2% Cash in 10 days, Net 30 days.

CHARGE Scobey & Company, Fayette, Iowa

June 10, 1922

2 Cases Tomato Catsup 4 doz.	1.90	7	60		
6 Cases Elgin Canned Corn 12 doz.	1.80	21	60		
5 Cases Echo Peas 10 doz.	1.85	18	50	47	70

Written Review

Rule a Purchases Book and record the following purchases, making proper extensions.

1. From Eaton & Dunham, 213 Main Street, Indianapolis, Indiana.
   Date, November 21, 1922.
   Terms, 1/15; N/60.

6 doz.	Silk Four-in-hand Ties	each	$ .37½
3 doz.	White Lawn Ties	gross	2.94
38 doz.	Assorted Style Amoryth Linen Collars	doz.	1.92
18 doz.	Policeman's Suspenders	doz.	4.37½
9 doz.	Khaki Overalls	doz.	8.97
6 doz.	Boys' Flannelette Waists (ages 4 to 13 yr.)	doz.	2.16¼
10 doz.	Men's Flannel Work Shirts (sizes 14 to 18)	doz.	8.76½
2¼ doz.	Men's Negligee Shirts	doz.	16.40

2. From J. B. Clark, Aurora, Illinois.
   Date, October 27, 1922.
   Terms 2/20; N/3 months.

7 doz.	Assorted Sizes Men's Silk Half Hose	doz.	$4.29
19½ doz.	Mercerized Lisle Hose	doz.	4.78
3 doz.	Men's Black Overgaiters	doz.	6.93
18	Women's Taffeta Silk Waists (sizes 34 to 42)	each	2.17
24	Navy Blue Silk Chiffon Waists	each	2.98
5 doz.	Embroidered Lace Coat Sets	doz.	6.93¾
2 doz.	Amoskeag Gingham Aprons	doz.	4.69½
2¼ doz.	Lace Vests	doz.	4.25

3. From Bishop & McGee, Independence, Iowa.

Date, October 20, 1922.
Terms, 1/5; N/20.

10 doz.	Defiance Food Choppers	each	$2.85
9 doz.	Climax Food Choppers	each	.72
14 doz.	No. 7 size Skillets	each	.22½
30 sets	Enamel-lined Iron Kettles, each set containing		
	3 2 quarts at .21		
	4 4 quarts at .29¾		
	6 6 quarts at .34⅔		
	4 8 quarts at .39¾		
	2 10 quarts at .46⅔		
2¾ doz.	H 921 Waffle Irons	doz.	$7.84⅞
6½ doz.	H 922 Waffle Irons	each	.93
25	H 464 Soapstone Griddles (round)	each	.55
12 doz.	H 465 Soapstone Griddles (oval)	each	.65⅜

4. From A. D. McHaughton, Fairchild, Missouri.

Date, July 14, 1922.
Terms, 3/30; N/90.

5 doz.	H 731 Wire Waste Baskets	doz.	$. 89
15 doz.	H 732 Wire Letter Baskets	doz.	1.23
28	H 881 Spring Seats	each	.88
15 sets	H 393 Cobblers' Outfits	set	.96
19 doz.	H 242 Curry Combs	doz	.67
14	F 272 Upholstered Rockers	each	8.75
25	F 279 Oak Rockers	each	12.65
20	F 212 Turkish Rockers	each	12.75

Rule a Sales Book, and record the following sales, making proper extension.

5. To J. D. Preston, Monmouth, Illinois.

July 7, 1922.
Terms, 1/10; N/40.

47	No. L 601 Royal Worsted Wilton Rugs	8 × 10 ft.	each	$29.25
52	No. L 602 Royal Worsted Wilton Rugs	6 × 7 ft.	each	19.75
38	No. L 603 Royal Worsted Wilton Rugs	9 × 12 ft.	each	32.35
65	No. L 661 Worsted French Wiltons	9 × 12 ft.	each	44.60
50	No. L 662 Plain Color Wiltons	8 × 10 ft.	each	31.75

6. To R. J. Noble, Dubuque, Iowa.

August 8, 1922.
Terms, 1/15; N/60.

25 doz.	Half-bleached Cotton Towels	24 × 54 doz.	$ .98
16 doz.	Linen Monogram Towels	22 × 39 doz.	4.95
20 doz.	Bleached Turkish Bath Towels	23 × 52 doz.	2.37½
14 doz.	No. T 291 Cotton Face Cloths	10 × 13 doz.	.42½

7. To Oscar Hamilton, Reynolds, North Dakota.

July 9, 1922.
Terms, 2/5; N/2 months.

15 doz.	Palmetto Fiber Scrub Brushes	doz.	$ .86
22 doz.	H 221 Kitchen Spoons, 10 inch	doz.	.23
19 doz.	H 241 Kitchen Spoons, 12 inch	doz.	.27
26 doz.	H 251 Kitchen Spoons, 14 inch	doz.	.33
38 doz.	H 333 Kitchen Forks, 13 inch	doz.	.19
36 doz.	H 334 Kitchen Forks, 15 inch	gross	3.20
12 doz.	H 341 Perforated Steel Spoons	doz.	.84⅞
8 doz.	H 691 Kitchen Sets	doz.	8.75
16 doz.	H 692 Kitchen Sets	doz.	3.95

Marking Goods

170. Method of Marking Goods. When stock is placed on the shelves in the salesroom, the cost of each article should be marked either on the goods, on the package which contains them, on tags attached to the goods, or on card lists placed near the goods.

171. Advantage of Marking. When the stock becomes low and the buyer wishes to purchase a new supply, he can compare the price paid for the goods on the shelves with quotations of prices made to him by the salesmen from the wholesale houses. It would also be a convenience to know the cost of an article if it proved to be a slow seller, and the manager determined to sell at a reduced price to unload the stock. One of the chief advantages of cost marking is in taking inventory of stock and finding its value at cost prices. In marking goods, the cost price is taken from the invoice.

172. "Blind Price Lists." It would be unwise to mark the cost in figures, as this would disclose the cost and the profit to purchasers.

It is, therefore, customary for merchants to adopt a set of symbols, called a cipher, or blind price list. Any symbols may be chosen, but they will be more easily written if letters are used, and these letters will be more easily remembered if they form a word or phrase. The word or phrase selected must not contain the same letter twice. Otherwise the same letter will represent two different figures.

The following will illustrate:

a d m o n i s h e r	mah
1 2 3 4 5 6 7 8 9 0	318
F i t z a u b r e y	izy
1 2 3 4 5 6 7 8 9 0	240

173. The Repeater. To further conceal the cost a "repeater" should be adopted. When the same figure is repeated, as in $1.55, the repeater sign is used for the repeated figure. "x" is frequently used as a repeater, but because it is so commonly used, some other repeater would perhaps be preferable. Words or phrases with eleven letters are frequently chosen as keys, one of the letters being used as a repeater.

Using "t" as the repeater, and "admonisher" as the key,

$3.88 would be written *mht.*

Written Work

Using "blacksmith" as the key word, and "d" as the repeater, indicate the following costs:

1. $9.82.	**2.** $ .09.	**3.** $ .25.	**4.** $ 1.64.
5. $1.00.	**6.** $6.20.	**7.** $ .27.	**8.** $ 7.47.
9. $2.45.	**10.** $ .55.	**11.** $1.33.	**12.** $ .39.
13. $8.23.	**14.** $5.00.	**15.** $1.17.	**16.** $12.47.
17. $6.62.	**18.** $2.24.	**19.** $ .36.	**20.** $ 1.44.

Many articles are bought by the dozen and sold by the piece. In marking goods bought in this way it is necessary to divide the cost by 12. This division will be facilitated if the decimal equivalents of the 12ths, from $\frac{1}{12}$ to $\frac{11}{12}$ inclusive, are memorized.

Make such a table of equivalents and memorize it. Thus,

$\frac{1}{12} = .08\frac{1}{3}$; $\frac{7}{12} = .58\frac{1}{3}$.

Problem. What is the cost of one hat at the rate of \$29 a dozen?

SOLUTION. $\frac{1}{12}$ of \$29 = \$$2\frac{5}{12}$ = \$2.41$\frac{2}{3}$.

Oral Work

What is the cost per article when the cost per dozen is:

1. \$27?	2. \$34?	3. \$64?	4. \$11?	5. \$3.40?
6. \$14.30?	7. \$7.50?	8. \$19.50?	9. \$46.60?	10. \$37.40?

174. Showing Cost and Selling Price. When both the cost price and the selling price are shown, it is customary to write the cost price above and the selling price below a line, thus:

Cost of an article, \$6.25.
Selling price, \$8.00.
Key word, Fitzaubrey.
Price mark, $\frac{\text{uia}}{8.00}$.

Key words or phrases are sometimes used to mark the selling price as well as the cost, thus:

Cost key, " p u r c h a s e d i t "
1 2 3 4 5 6 7 8 9 0 "t" is the repeater.

Selling key, " s t u d y i n g h e r "
1 2 3 4 5 6 7 8 9 0 "r" is the repeater.

Cost, \$3.14.
Selling price, \$4.25.

Mark, $\frac{\text{rpc}}{\text{dty}}$.

Written Work

Using the above keys, show markings for the following:

	COST	SELLING PRICE
1.	\$1.15	\$1.50
2.	\$ .65	\$ .90
3.	\$1.30	\$1.75
4.	\$2.18	\$2.50
5.	\$1.25	\$2.60

What per cent of profit would be realized from selling goods marked as follows, using "purchased it" as the cost key and "studying her" as the selling key?

6. $\frac{\text{phi}}{1.75}$. 7. $\frac{\text{ah}}{\text{ge}}$. 8. $\frac{\text{dh}}{\text{sty}}$. 9. $\frac{\text{at}}{\text{ny}}$.

Mark the cost of each of the following articles, using "Now Pay Quick" as the purchase key.

10. \$3.00 per dozen.
11. \$185.00 per hundred.
12. \$11.52 per gross.
13. \$8.64 per case of 4 doz.

Mark both the cost and the selling price of the following. Devise a cost key of your own, and mark the selling price by using the key "importance."

14. Cost \$2.60, marked to gain 22%.
15. Cost \$1.75, marked to gain 18%.
16. Cost \$25.00, marked to gain 20%.

The following is a list of key words and phrases:

Buy for Cash	Our Last Key	Frank Smith	You Mark His
The Big Four	Republican	Big Factory	Now Be Sharp
Black Horse	Charleston	Don't Be Lazy	He Saw It Run
Buckingham	Cumberland	Market Sign	Hard Moneys
Authorizes	Dozen Black	Now Be Quick	What Prices

175. Adding the Buying Expenses to the Cost of the Goods. The cost of the goods when they are finally placed on the merchant's shelves includes not only the wholesale price but also the freight and drayage, import duties paid, and any other expenses of purchasing and holding the goods for sale, such as rent of storage warehouses. These are known as buying expenses. In the statements which the merchant prepares to determine his profits these buying expenses should be added to the cost of the goods. It is a matter of choice whether the buying expenses shall be included in the cost marked on the goods.

If it is desired to include the buying expenses in the marked costs, the simplest method is to estimate the per cent of total buying expenses to total purchases. When records of total purchases and total buying

expenses have been kept for several years, the average per cent of buying expenses may be computed as shown in the following illustration.

To find an estimated rate of buying expenses to add to the cost of purchases in 1922 based on the figures of 1919, 1920, and 1921:

1919,	Buying Expenses,	$ 785.75	Purchases,	$16,240.00
1920,	Buying Expenses,	. 835.50	Purchases,	19,360.00
1921,	Buying Expenses,	923.90	Purchases,	21,365.00
Total	Buying Expenses,	$2,545.15	Purchases,	$56,965.00

$2,545.15 ÷ $56,965.00 = 4.46+ %, the per cent of buying expenses.

In marking costs, 4.5 % of the cost of each article should be added as buying expenses.

NOTE. Probably 5 % might be used for convenience.

Written Work

1. Compute the per cent of buying expenses to be added to the wholesale cost of goods purchased in 1922. Carry the result to the nearest whole per cent.

YEAR	PURCHASES	BUYING EXPENSES
1919	$10,246.80	$534.50
1920	12,726.95	592.90
1921	14,825.75	615.45

2. Find the marked cost of each of the following articles after adding the buying expenses. Use the per cent of buying expenses found in Problem 1.

a. Wholesale cost, $16.20.
b. Wholesale cost, 15.75.
c. Wholesale cost, 2.35.

3. At what price should each of these articles be sold to gain 23 % on the total cost including buying expenses?

4. The purchases made by a store in 1921 were $3,287.20; the buying expenses for the same year were $327.90. On the basis of these figures, what per cent should be added for buying expenses on purchases made in 1922? Approximate result to the nearest per cent.

5. What objection do you see to determining a per cent of buying expenses from the data of only one preceding year?

6. Find the marked cost of each of the following purchases, after adding the per cent of buying expenses. Use the per cent of buying expenses found in Problem 4.

a. Wholesale cost, $ 2.45.
b. Wholesale cost, 1.60.
c. Wholesale cost, 34.50.

7. At what selling price should each of the articles in Problem 6 be marked to gain $16\frac{2}{3}\%$ of the total cost?

CHAPTER XVIII

PAYING FOR GOODS

176. Making Change. The payment of debts between persons in the same community is usually made with either cash or checks. When cash is the medium of payment, it is often necessary to " make change."

Speed and accuracy in making change are desirable. The following method is generally used by experienced tellers and cashiers. Beginning with the amount of the purchase, take from the cash drawer enough coins to bring the total to even dollars, using as few coins as possible, then take out dollars or larger denominations until the total cost of the purchase and change returned equals the payment made.

Problem. A five-dollar bill was given in payment for a purchase of \$.39. How should change be made?

SOLUTION. Take 1 penny,
1 dime,
1 fifty-cent piece,
4 dollars.

As a check on the accuracy of the change, say as you give the customer the money: "39 cents, 40, 50, \$1, \$5.00."

Oral Work

Following the method described above, state what coins and bills should be given to make change for the following purchases, using the largest denominations possible:

	PURCHASE	PAYMENT		PURCHASE	PAYMENT
1.	\$ .07	\$ .50	2.	\$ .21	\$ .50
3.	\$ .56	\$ 1.00	4.	\$ .63	\$ 2.00
5.	\$1.36	\$ 5.00	6.	\$2.79	\$ 5.00
7.	\$3.66	\$ 5.00	8.	\$4.24	\$10.00
9.	\$5.70	\$10.00	10.	\$6.32	\$20.00
11.	\$ .98	\$ 5.00	12.	\$2.77	\$10.00

	Purchase	Payment		Purchase	Payment
13.	$ 3.46	$20.00	14.	$11.87	$15.00
15.	$14.42	$20.00	16.	$ 7.37	$ 8.00
17.	$11.13	$20.00	18.	$ 1.13	$20.00

177. Payments by Check. The payment of all bills by the actual transfer of money would be so inconvenient that the giving of checks has been substituted to a great extent and it is said that about 90% of all bills are now paid by checks.

178. Deposits and Withdrawals. Business men keep the greater part of their cash funds on deposit with banks or trust companies. When money is deposited, a deposit slip similar to the illustration below is filled out by the depositor, showing his name, the date, the items, and the amount of the deposit.

DEPOSITED BY

C. L. Stevenson

IN THE

STATE BANK OF OAK PARK

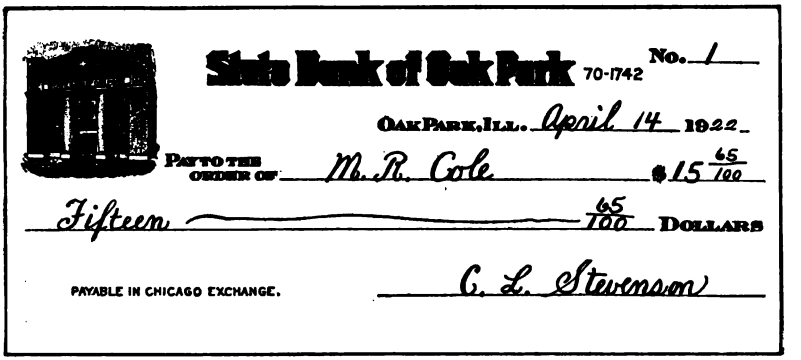
OAK PARK, ILL.

April 10 1922

	DOLLARS	CENTS
GOLD	40	00
SILVER	7	35
BILLS	14	00
Total	61	35

A Deposit Slip

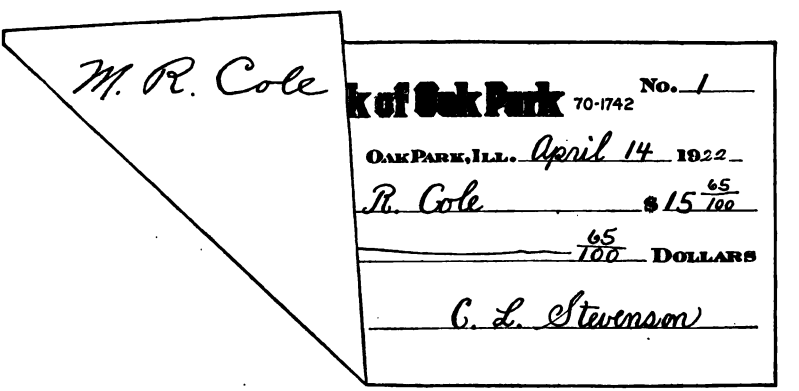
The depositor, wishing to pay a bill, draws a check ordering the bank to pay from the funds on deposit the sum of money stated on the check to the person named thereon. If the depositor wishes to draw cash from his account. he may make the check payable to "Self" or to "Cash."


State Bank of Oak Park 70-1742 No. 1
Oak Park, Ill. April 14 1922
Pay to the order of M. R. Cole $15 65/100
Fifteen 65/100 Dollars
Payable in Chicago Exchange.
C. L. Stevenson

A Check

C. L. Stevenson, the *drawer* of this check, is paying M. R. Cole, the *payee*, $15.65.

Depositors are credited by the bank with their deposits, and are charged with the checks drawn by them. Checks received from other people may be cashed at the bank, or they may be deposited the same as cash. All checks cashed or deposited must be indorsed; that is, they must be signed on the *back* by the person cashing or depositing them. Checks must also be indorsed if they are transferred to another person. Indorsements should be made on the back of the left end of the check.



A Check Indorsed

M. R. Cole's indorsement before cashing the check

Indorsements

179. Blank Indorsement. This form of indorsement is illustrated on the back of the check on page 209. It makes the check payable to bearer.

180. Indorsement in Full. This form is illustrated below.

Pay to the order of
I. B. Banning
M. R. Cole

Checks indorsed in full must be indorsed by the new owner before being cashed.

New Netherland Bank
OF NEW YORK
DEPOSITED BY V. E. Finch
NEW YORK, May 6, 1922

PLEASE LIST EACH CHECK SEPARATELY

	DOLLARS	CENTS
BILLS	16	00
GOLD	25	00
SILVER	12	45
CHECKS Buffalo	4	35
" Newark N.J.	14	23
" Boston	213	25
"		
"		
"		
"		
"		
"		
"		
"		
Total	285	28

Deposit Slip Showing Cash and Check Deposit

If a check, note, or draft is not paid by the party primarily liable each indorser in turn becomes responsible for its payment. An indorser may limit his responsibility by writing the words "without recourse" above his signature. When this is done, the indorsement is said to be *qualified*.

Written Work

1. You are opening an account by depositing to-day in some bank of your city, gold, $5; silver, $2.25; bills, $7; and the three following checks: 1st, drawn by Henry Bailey on the Merchants' National Bank of Fargo, North Dakota, amount $26.30; 2d, drawn by W. A. Owen on the Corn Exchange National Bank of Chicago, amount $8.35; 3d, drawn by Frank Mitchell on Carter's Bank, Eldora, Missouri, amount $12.40.

Rule a deposit slip and enter this deposit properly.

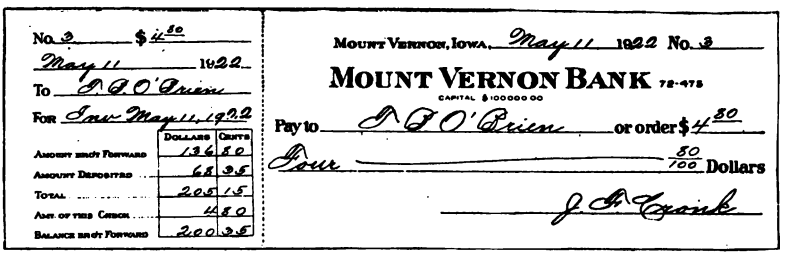
2. On a blank sheet of paper draw a check on the bank in which you have just made this deposit, paying F. G. Benton $23.25.

What is your balance in the bank after drawing this check?

3. Seven days ago you bought goods from F. G. Peterson, amounting to $28.50. The terms of the invoice were 2/10; N/30. To-day you pay the bill by check. Draw the check.

What is your balance in the bank?

181. The Check Book. Blank checks are furnished by the bank, bound in a check book. Checks are usually attached to stubs, on which are recorded, at the time of drawing the check, its number, the date, the name of the payee, the purpose for which it was given, and the amount. On the stub there is also kept a record of the balance remaining in the bank. As shown by the following illustration, a balance of $136.80 was brought over from May 10, and on May 11 a deposit of $68.35 was made and a check for $4.80 was drawn.


No. 2 $ 4 80
May 11 1922
To P. O'Brien
For Inv. May 11, 1922
Dollars Cents
Amount brought Forward 136 80
Amount Deposited 68 35
Total 205 15
Amt. of this Check 4 80
Balance brought Forward 200 35
Mount Vernon, Iowa, May 11 1922 No. 2
MOUNT VERNON BANK 72-475
CAPITAL $100000 00
Pay to P. O'Brien or order $4 80
Four 80/100 Dollars
J. F. Crank

Written Work

1. Using the following information, rule a check and stub similar to the preceding illustration, enter the necessary facts, and draw the check.

Name of bank, Fourth National, your city.

Balance from yesterday, $103.27. Deposit of $19.38.

Check No. 6, drawn by you to-day, payable to G. D. Fitzgerald and Co., in payment of the invoice purchased from them eight days ago. Amount of invoice, $73.28; terms, 2/10; N/60.

Write the stub.

What is the new balance?

182. The Bank's Accounts with its Depositors. All banks keep a depositor's ledger. Usually a page is devoted to the deposits and checks of each depositor.

Williams & Harris

DATE			DEPOSITS		CHECKS		BALANCE		
1922									
Jan.	6		143	65			143	65	
	7				26	25	117	40	
	12		68	60	13	25	172	75	
	16				6	80			
					12	40			
					3	75	149	80	
	21		6	00	43	74			
					102	73	9	33	

AN ACCOUNT IN A DEPOSITORS' LEDGER

Written Work

Rule a page of a depositors' ledger, similar to the preceding illustration, and enter the following deposits and checks made by yourself, computing the daily balances. In computing balances, use the method of adding and subtracting explained in Section 12.

March 2, Deposit $127.80. March 3, Check $26.14. March 4, Check $23.19. March 5, Deposit $114.32; Check $12.19. March 6, Checks $13.14, $12.17. March 10, Deposits $32.76, $22.14; Check $17.14. March 11, Checks $26.19, $127.14. March 13, Deposits $12.19, $76.27; Checks $32.15, $27.19, $13.81. March 16, Deposit $27.94. March 18, Checks $23.42, $27.93.

183. Reconciling the Bank Account. It is the custom of depositors to reconcile the bank account monthly, to determine whether the balance shown by the stub of the check book agrees with the balance shown by the bank's records, and if the balances do not agree to determine the cause of the discrepancy. At the close of each month the bank gives the depositor a statement. This statement is a copy of the bank's account with the depositor, showing the balance at the beginning

of the month, the deposits and other credits, the checks cashed and any other debits, and the balance at the end of the month. The bank also returns the depositor's cancelled checks.

The depositor should now arrange the returned checks in numerical order and place a check mark on each stub corresponding to a returned check. Any stubs which are not marked represent checks which have been drawn but which have not been paid by the bank prior to the close of the month. A list should be made of these checks, showing the number and amount of each.

The deposits shown by the check book stubs should be compared with the credits shown on the bank statement. In many cases the deposits made on the last day of the month will not be put through the bank's books until the first day of the following month because of the bookkeeping involved. Hence the depositor's check book stubs may show a deposit which is not shown on the bank's statement.

If there are any other items which are not included in both the bank's and the depositor's records, these also should be noted. For instance, the bank may have allowed interest on the deposit while the depositor has not entered this on the check book stub; or the bank may have made a charge because the depositor's balance was less than a stipulated sum during the month.

After all of these items have been noted, a reconciliation statement should be made as follows:

RECONCILIATION STATEMENT

September 30, 1922

Balance per Check Book		$457.68
Add Interest Allowed		.48
Adjusted Balance		458.16
Balance per Bank Statement		491.35
Deduct Checks Outstanding:		
No. 2109	$36.21	
No. 2112	19.45	
No. 2114	7.46	63.12
		428.23
Add Deposit, September 30, 1922		29.93
Adjusted Balance		458.16

If the adjusted balances fail to agree the accounts do not reconcile, and the depositor should determine whether he has omitted some item from his records or made some other error, or whether an error has been made by the bank.

Written Work

On June 30, 1922, Walter Dayton's check book showed a balance of $178.65. The bank's statement showed a balance at the same date of $176.30. On comparing the cancelled checks with the stubs, Dayton found that the following checks were outstanding: No. 1315, $12.60; No. 1317, $7.80; No. 1320, $3.60. On June 30 he deposited $25.35 but this deposit did not appear on the bank's statement. The bank made a charge of $1.00 because the balance during the month had been less than $300.00. Prepare a reconciliation of the bank account.

184. The Clearing House. Any bank will cash the checks drawn on other banks, provided the person presenting them is known at the bank or is properly identified. For example, if you are given a check on the First National Bank, you can cash it or deposit it at the Second National Bank. The Second National then collects the amount of the check from the First National.

In the large cities, the checks are collected through a Clearing House. The representatives of the various banks meet at the Clearing House, bringing the checks which they have paid for other banks. Each bank is credited with the amount of the checks which it has paid for other banks, and debited with the amount of the checks which all other banks have paid for it. The Clearing House collects from those having debit balances and pays those having credit balances.

Let us consider three of the banks in a city; the First National, the First State, and the City National. The representatives will meet, each bringing the checks paid by his bank for the other banks the preceding day, and a sheet showing the amount paid for each of the other banks. The statements brought by the representatives of the banks are shown below:

FIRST NATIONAL

PAID FOR	AMOUNT
First State	$16,265.00
City National	12,146.80
Total	$28,411.80

FIRST STATE

PAID FOR	AMOUNT
First National	$ 8,368.28
City National	9,208.48
Total	$17,576.76

CITY NATIONAL

PAID FOR	AMOUNT
First National	$12,875.00
First State	4,738.80
Total	$17,613.80

At the Clearing House a sheet similar to the following is prepared. In the debit column is placed the total value of checks paid *for* each bank; in the credit column is placed the total value of checks paid *by* each bank. The balance columns show the balances payable by or to each bank.

MANAGER'S SHEET

BANK	DEBITS	CREDITS	DEBIT BALANCE	CREDIT BALANCE
First National	$21,243.28	$28,411.80		$7,168.52
First State	21,003.80	17,576.76	$3,427.04	
City National	21,355.28	17,613.80	3,741.48	
Totals	$63,602.36	$63,602.36	$7,168.52	$7,168.52

The First State and the City National each turns over to the Clearing House the amount of its debit balance; the Clearing House pays this amount to the First National. In this way payment of over sixty-three thousand dollars is made by the actual transfer of only a little more than seven thousand dollars.

Written Work

Prepare a Manager's Sheet, and find the debit or credit balance of each bank in the following Clearing House Association. The amount

of the checks paid by each bank is shown by the debit sheets which follow.

HARPER'S STATE BANK

PAID FOR	AMOUNT
Harper's State	
First National	$23,756.27
Traders'	14,259.95
Merchants' National	25,726.87
Farmers' & Mechanics'	9,243.65
First State	12,060.26

FIRST NATIONAL BANK

PAID FOR	AMOUNT
Harper's State	$13,263.37
First National	
Traders'	8,836.39
Merchants' National	14,856.74
Farmers' & Mechanics'	8,345.93
First State	8,325.57

TRADERS' BANK

PAID FOR	AMOUNT
Harper's State	$ 5,273.85
First National	12,573.82
Traders'	
Merchants' National	7,252.55
Farmers' & Mechanics'	11,257.37
First State	10,456.25

MERCHANTS' NATIONAL

PAID FOR	AMOUNT
Harper's State	$ 3,123.64
First National	6,027.30
Traders'	10,936.40
Merchants' National	
Farmers' & Mechanics'	7,238.37
First State	9,235.35

FARMERS' & MECHANICS'

PAID FOR	AMOUNT
Harpers' State	$ 2,346.75
First National	5,236.73
Traders'	4,232.25
Merchants' National	7,727.86
Farmers' & Mechanics' First State	2,005.39

FIRST STATE

PAID FOR	AMOUNT
Harper's State	$ 8,946.27
First National	1,856.23
Traders'	11,472.75
Merchants' National	923.88
Farmers' & Mechanics' First State	2,423.73

Which banks will pay to the Clearing House, and how much will each pay?

Which banks will receive from the Clearing House, and how much will each receive?

185. Exchange on Checks. Checks are frequently used to send money to people in cities other than the one in which the drawer of the check resides. For example, if you have money deposited in the Home National Bank of your city, you may draw a check against this deposit and send it to Mr. Perkins of Omaha. Mr. Perkins will take it to his bank in Omaha, which will pay the check, even though it has no knowledge of you or your deposit in the Home National Bank. The fact that Mr. Perkins is known at his bank, and that his endorsement is an agreement to refund the money if the check proves worthless, is sufficient protection for his bank.

When banks pay out-of-town checks, they incur a certain amount of expense in collecting them. To cover this expense they often require persons for whom they cash checks to pay them "exchange." When exchange is charged, a common rate is $\frac{1}{10}$ of 1 per cent of the face of the check, with a minimum charge of from 10 cents to 25 cents.

Problem. Mr. Black, who lives in Bloomington, Illinois, received two checks in his mail. One was from Wm. Harris, of Lincoln, Ne-

braska, for $365.85; the other from H. B. Felter, of Cincinnati, Ohio, for $28.25. The bank at which Mr. Black cashed these checks charges for exchange $\frac{1}{10}$% of the face of each check, or a minimum of 10 cents. What was the exchange on the checks?

SOLUTION. $\frac{1}{10}$% of $365.85 = $.37
$\frac{1}{10}$% of $28.25 = $.028
Since $.028 is below the minimum, 10 cents exchange is charged.
The total exchange is $.47.

Written Work

Prepare deposit tickets for the following:

1. Deposited by Wm. Frend, of Peoria, Illinois, in the Peoria State Bank, on August 8, 1922:

Gold, $25; silver, $37.50; currency, $17; and checks as follows:
Check drawn by D. P. Snyder of Amboy, Illinois, $65.75.
Check drawn by Hammond & Davis of Lima, Iowa, $236.57.
Check drawn by B. J. Harris of Springfield, Illinois, $235.
The bank charges $\frac{1}{10}$% exchange. Minimum, 15 cents.
Find the total of the items and deduct the charge for collection.

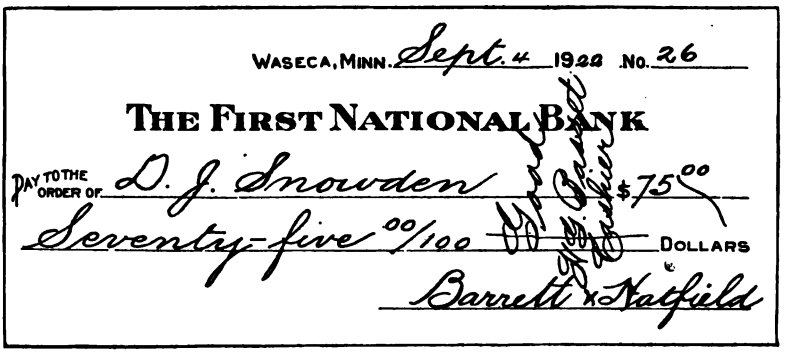
2. Deposited by G. D. Bernham of Hawkeye, Iowa, in the Hawkeye State Bank, on August 21, 1922:

Check drawn by S. Y. Benton of Clarinda, Iowa, $75.88.
Check drawn by D. W. Galbreath of Morris, Illinois, $57.93.
Check drawn by F. J. Ressler of Pasadena, California, $346.49.
Check drawn by D. T. Bailey of Buffalo, New York, $48.95.
The bank charges exchange on the California and New York checks; rate, $\frac{1}{10}$%, minimum 25 cents each.

NOTE. It is the custom of some banks to cash checks on banks in near-by states without charging exchange.

186. Certified Checks. If you wish to assure the person to whom you send a check that you have sufficient money in the bank to pay the check, you may have the check certified. Draw the check in the usual way and take it to your bank. The cashier will write or stamp "Accepted" or "Certified" or "Good" across its face and will sign his name. He will then set aside from your deposit the amount of the check, holding it in the bank's funds until the check is presented for

payment. Certifying the check reduces the receiver's risk of cashing a worthless check, but it does not relieve him from a possible exchange fee.

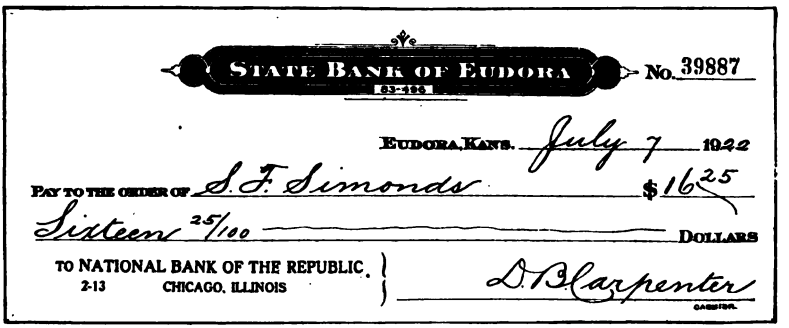

WASECA, MINN. Sept. 4 1922 No. 26
THE FIRST NATIONAL BANK
PAY TO THE ORDER OF D. J. Snowden $75.00
Seventy-five 00/100 DOLLARS
Barrett & Hatfield

A CERTIFIED CHECK

187. Bank Drafts. Banks keep funds on deposit in the banks of the larger cities, and draw checks on these city banks just as you may draw a check on the bank in which you have deposited money. An order drawn by one bank against its deposit in another bank is called a "Bank Draft."

188. Advantages of Bank Drafts. A bank draft is preferable to a check when sending money from one town to another, for two reasons:

a. The draft is drawn by a bank, while the check is drawn by an individual. There is therefore a greater certainty that the bank draft is genuine, and will be paid when presented.


STATE BANK OF EUDORA
No. 39887
EUDORA, KANS. July 7 1922
PAY TO THE ORDER OF S. F. Simonds $16.25
Sixteen 25/100 DOLLARS
TO NATIONAL BANK OF THE REPUBLIC
2-13 CHICAGO, ILLINOIS
D. B. Carpenter

A BANK DRAFT

b. Bank drafts are drawn on banks in large cities because it is only in the large cities or business centers that other banks have any need for deposits. There is less expense incurred in collecting a bank draft drawn on a bank in a large city than in collecting a check drawn on a bank in a small town. Therefore banks seldom charge exchange when cashing bank drafts.

Explanation. D. B. Carpenter is the cashier of the State Bank of Eudora. This bank has funds on deposit in the National Bank of the Republic, Chicago. When the National Bank of the Republic pays this draft, it will deduct the amount from the balance of the Eudora Bank.

S. F. Simonds lived in Eudora and wished to send $16.25 to F. E. Craig of Bloomington, Illinois. When he purchased the draft from his bank, he had it made payable to himself. He might have had it made payable to F. E. Craig, but if he had done so, his (Simonds's) name would not have appeared on the draft, and if it had become separated from the letter accompanying it, Craig might have had difficulty in recalling from whom he received it.

After receiving the draft, Simonds indorsed it as follows:

Pay to the order of
F. E. Craig.
S. F. Simonds,

and sent it to Mr. Craig.

Mr. Craig indorsed it and cashed it at the Corn Belt National Bank of Bloomington.

We will now follow the steps by which the Corn Belt National collected the draft. This bank deposits funds with the Continental National Bank of Chicago, which is called its "correspondent," and it therefore sent this draft, with others paid the same day, to the Continental National Bank as a deposit.

The Continental National sent the draft to the Clearing House, which collected it from the National Bank of the Republic.

After paying the draft, the National Bank of the Republic charged the amount to the State Bank of Eudora.

The preceding statement may be summarized as follows:

State Bank of Eudora sells the draft to

↓

S. F. Simonds, who paid for it either with cash or a check; and sent it to

↓

F. E. Craig, who gave Simonds credit for $16.25; and deposited the draft at

↓

Bloomington Bank, which gave Craig $16.25; and sent the draft to

↓

Continental National, which gave the Bloomington Bank credit for $16.25; and collected the draft through the Clearing House from

↓

National Bank of the Republic, which paid the Continental National $16.25, and charged Eudora Bank $16.25.

INDORSEMENTS ON THE MODEL BANK DRAFT

Pay to the order of
F. E. Craig
S. F. Simonds

Pay to the order of the
Corn Belt National Bank
F. E. Craig

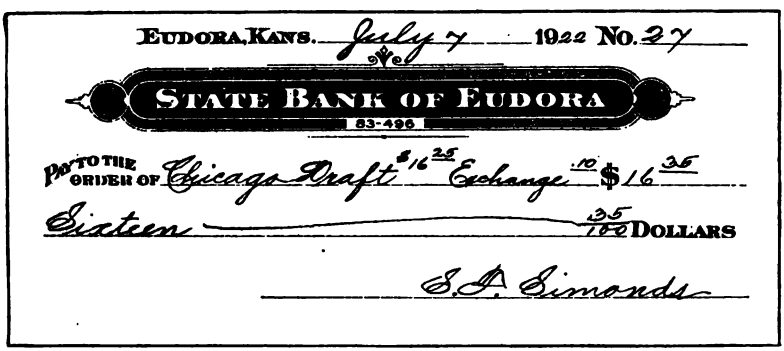
Pay to the order of
Continental National Bank.
All prior indorsements guaranteed
CORN BELT NATIONAL BANK,
BLOOMINGTON, ILL.,
F. A. FREND, Cashier.

Pay to the order of
Any Bank, Banker, or Trust Co.
Prior indorsements guaranteed.
July 12, 1922.
CONTINENTAL NATIONAL BANK,
M. B. JONES, Cashier.

These indorsements indicate the successive owners of the draft. The last two indorsements are made with rubber stamps. The last one is made payable to "any Bank, Banker, or Trust Company," for convenience in collecting through the Clearing House.

189. The Cost of Bank Drafts. When a bank sells a draft, it gives the purchaser the benefit of its deposit relations with a large city bank. For this convenience it may make a charge called "exchange." It will be noted that exchange on a check is paid by the person receiving and cashing it, while the exchange on a draft is paid by the person purchasing and sending it. When exchange is charged, $\frac{1}{10}$% with a minimum of 10 cents is a customary fee.

The following is a copy of the check given by S. F. Simonds to the State Bank of Eudora to pay for the draft purchased. There was a charge of 10 cents for exchange.


EUDORA, KANS. July 7 1922 No. 27

STATE BANK OF EUDORA

83-496

PAY TO THE ORDER OF Chicago Draft $16.25 Exchange .10 $16.35

Sixteen 35/100 DOLLARS

S. F. Simonds

Written Work

1. You owe H. J. Palmer, of Toledo, Ohio, $27.35. You buy a draft, payable to yourself for this amount, from the Merchants' State Bank of your city, of which A. R. Burton is the cashier. The draft is drawn on the Home National Bank of New York City. Exchange, 10 cents. Draft is made payable to you.

Write a draft similar to the one which the bank would give you, and the check which you would give the bank in payment.

Indorse the draft. How soon can Mr. Palmer get his money?

2. Mr. Palmer cashed the draft at the High Street National Bank of Toledo, B. F. Ohren, Cashier. The High Street National sent it to the Bankers' National Bank of New York, G. B. Martin, Cashier.

The Bankers' National collected it through the Clearing House.

Show the indorsements on the draft.

3. Find the cost of a draft sent to pay an invoice of $246.50, less a cash discount of 2%. Exchange, $\frac{1}{10}$% of the face of the draft; minimum, 15 cents.

4. On October 8, you purchased from Benedict & Meredith, Pittsburgh, Pennsylvania, an invoice amounting to $794.86. Terms, 2/15; N/60. On October 16, you paid the invoice by a draft purchased at the Claim Street State Bank of your city, J. D. Haines, Cashier. The draft was drawn on the First National Bank of New York. Exchange, $\frac{1}{10}\%$.

Write the draft which you received, the indorsement, and the check you gave to pay for the draft.

190. Postal Money Orders. A postal money order is an order drawn by one postmaster on the postmaster at some other office, calling for the payment of a stated sum of money to the person named on the order. Postal money orders are commonly used to make payments by mail. They are issued in any amount from $.01 to $100.00. As a means of protection, no order is issued for more than $100.00. If it is desired to send more than this amount, additional orders may be purchased.

Fees for money orders payable in the United States (which includes Hawaii and Porto Rico) and its possessions, comprising the Canal Zone, Guam, the Philippines, as well as in Bermuda, British Guiana, British Honduras, Canada, Cuba, Newfoundland, at the United States Postal Agency in Shanghai (China), in the Bahama Islands, and in certain other islands in the West Indies, are as follows:

For orders	from	$ 0.01	to	$ 2.50	3 cents
	from	2.51	to	5.00	5 cents
	from	5.01	to	10.00	8 cents
	from	10.01	to	20.00	10 cents
	from	20.01	to	30.00	12 cents
	from	30.01	to	40.00	15 cents
	from	40.01	to	50.00	18 cents
	from	50.01	to	60.00	20 cents
	from	60.01	to	75.00	25 cents
	from	75.01	to	100.00	30 cents

Problems. 1. What is the cost of an order for $27.35?

SOLUTION. $27.35 lies between $20.01 and $30.00, and the rate is 12 cents. Total cost of order, $27.35 + .12 = $27.47.

2. What is the cost of sending $267.95 by postal money orders?

SOLUTION. Two orders for $100.00 each, and one order for $67.95, will be purchased.

Fees of two $100.00 orders, 60 cents.
Fee of $67.95 order, 25 cents.
Total fees, 85 cents.
Total cost of orders, $267.95 + .85 = $268.80.

191. Bank Drafts and Postal Money Orders Contrasted. Note the following differences between a bank draft and a postal money order:

A bank draft may be indorsed as many times as desired.
A postal money order may be indorsed only once.
A bank draft may be cashed at *any* bank.
A postal money order must be presented to the post office on which it is drawn, or to a bank which can cash it at that post office.

192. Express Money Orders. Express money orders are similar in many respects to postal money orders. They can be purchased from the agent of the express company, and are payable from the funds of the express company on deposit in various banks specified in the order.

No order is issued for an amount larger than $50. If it is desired to send a larger amount, additional orders must be purchased.

Express Money Order Rates. The rates for express money orders are:

For orders	from	$ .01	to	$ 2.50	5 cents
	from	2.51	to	5.00	7 cents
	from	5.01	to	10.00	10 cents
	from	10.01	to	25.00	12 cents
	from	25.01	to	50.00	15 cents
	from	50.01	to	75.00	18 cents
	from	75.01	to	100.00	20 cents

Problem. What will be the fee for the transfer of $85.60?

SOLUTION. The fee will be 20 cents, one order being issued for $50.00 and another for $35.60.

Problem. What will be the fee for the transfer of $260.75?

SOLUTION. $200.00 at the rate of 20 cents per hundred = 40 cents
60.75 at the rate of . . . . . 18 cents
Total . . . . . 58 cents

193. Telegraph Money Transfers. Money may be transferred by telegraph when there is an urgent necessity for immediate payment. The rates for this service between points in the United States are determined as follows:

To the tolls for a fifteen-word message between the office of deposit and the office of payment add the following charges:

For $25.00 or less	25 cents
25.01 up to $ 50	35 cents
50.01 up to 75	60 cents
75.01 up to 100	85 cents

For amounts above $100 add (to the $100 rate) 25 cents per hundred (or any part of $100) up to $3,000.

For amounts above $3,000 add (to the $3,000 rate) 20 cents per hundred (or any part of $100).

Problems. 1. What is the charge for sending $25 or less to a point where the fifteen-word rate is 65 cents?

SOLUTION.

$.25 Minimum charge
.65 Tolls on 15-word message
$.90 Total charge

2. What is the total charge for transfer of $105 if the tolls on a fifteen-word message are 65 cents?

SOLUTION.

$.85 Charge for first $100
.25 Charge for fraction of second $100
.65 Tolls on 15-word message
$1.75 Total rate

Written Work

1. Assuming that you owe the following bills:

F. G. Young, $ 39.40;
P. S. Sanborn, $112.75;
H. L. Colwell, $416.25, less 3%;

determine the fees for the purchase of either postal or express money orders for transferring money to make payments. Complete the following blank form.

Name	Net Amount of Bills	Fees for Money Orders	Total Cost	Denominations of Orders Received from Ex. Co.	Denominations of Orders Received from P. O.

2. What is the charge for a telegraph money transfer of $23.15 to a point where the fifteen-word message rate is 85 cents?

3. What is the charge for a telegraph money transfer of $52.85 to a point where the fifteen-word message rate is $1.30?

4. What is the total cost of a telegraph money transfer for $215 to a point where the fifteen-word rate is 65 cents?

CHAPTER XIX

COLLECTING BILLS

194. Statements. At periodical intervals, usually on the first day of each month, merchants send statements to their customers. This

STATEMENT

ACCOUNT NO. 5422

WEBSTER & McCLELLAN

16 JEFFERSON STREET

F. B. Turner,
La Grange, Ill.

CHICAGO, June 1, 1922

All bills are due the first of the month after purchase.
This statement is intended to show you the condition of your account on our books. If this statement does not correspond with your accounts please notify us. If correct, please remit.

	To balance, as per former statement	17.65	
May 3	To Mdse, as per invoice	23.78	
May 16		19.65	
May 23		27.84	88.92
	Credits		
May 20	By Mdse returned	2.85	
May 26	By Cash	75.00	77.85
	Balance		11.07

statement shows the date and amount of each purchase, the date and amount of each payment, and the balance due. The items sold are not enumerated in the above statement because the invoices may be consulted to obtain this information.

HENDERSON & BISHOP
143 WEST MONROE AVENUE

R. F. Bailey,
West Union, Iowa.

CHICAGO, July 1, 1922

Interest charged on overdue accounts.
All claims for correction must be made on receipt of statement.

DATE	ITEM	DEBIT	CREDIT	BALANCE
June 1	Balance, May 31			75.60
June 2	Cash		75.00	.60
June 6	Mdse	34.55		35.15
June 12	Mdse	17.26		52.41
June 15	Returned goods		4.35	48.06
June 24	Mdse	5.96		54.02

ANOTHER FORM OF STATEMENT

Written Work

1. Rule a statement similar to the first illustration, and enter the following: Your transactions with S. J. Smith, Fayette, Missouri.

Balance, August 1, $127.86.
Purchases, August 3, $72.85; August 11, $23.89; August 14, $75.23; August 20, $14.76; August 28, $38.97.
Payments, August 2, Cash $126.00; August 13, Returned merchandise, $15.00.
Statement rendered September 1.

2. Rule a statement similar to the second illustration, and enter the following transactions with Beardsley & Russel, Montgomery, Alabama.

Balance, October 1, 1921, $32.11.
Purchases, October 5, $45.89; October 9, $23.87; October 12, $243.87; October 15, $65.00; October 21, $52.57; October 26, $40.50; October 29, $50.25.

195. Commercial Drafts. A commercial draft or bill of exchange is an unconditional order in writing signed by the drawer, ordering the person to whom it is addressed (called the **drawee**) to pay on demand or at a fixed or determinable future time a certain sum in money to the order of a designated person (called the **payee**).

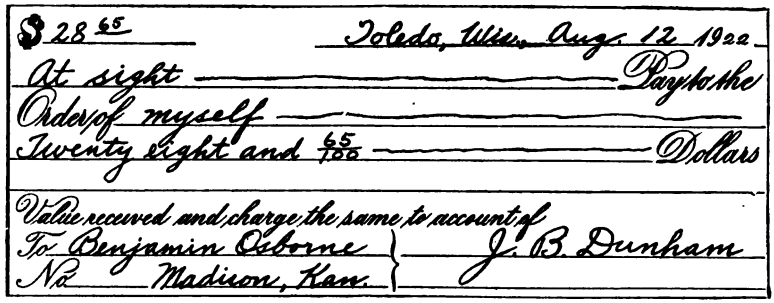
If the draft orders the drawee to pay money to the drawer, the paper is called a **two-party draft.** In such cases the drawer is also the payee. If the payee is a third party, the paper is called a **three-party draft.**

If the draft is payable at sight (that is, on demand), it is called a **sight draft.** If it is payable at a fixed or determinable future time, it is called a **time draft.** A time draft may be drawn payable a certain number of days or months after date; that is, after the date on which it is drawn. It may be drawn payable a certain number of days or months after sight.

Acceptances. An acceptance is a time draft which the drawee has agreed to pay, indicating this agreement by writing across its face "Accepted" together with his signature. If the draft is payable "—— days after sight" or "at —— days' sight," the drawee (who now becomes the acceptor) should also write the date on which he accepts. This is because "after sight" means after acceptance, and it is necessary to know the date of acceptance in order to know the date of maturity. If the draft is payable "after date" the drawee may write the date of acceptance if he desires, but it is not necessary to do so because the draft becomes due a stated time after its date regardless of the date of acceptance. An acceptance which represents a debt which arose out of the purchase of goods is called a **trade acceptance.**

196. Drafts for the Collection of Accounts. A sight draft or a time draft may be used as a means of collecting a book account. Wholesale houses often make terms similar to the following: "Sight draft in ten days, less 2%." At the expiration of ten days from the date of sale, the selling merchant sends a sight draft for the net amount of the bill to the purchasing merchant's bank. The bank presents it to the purchaser for collection.

To illustrate: J. B. Dunham of Toledo, Wisconsin, has sold goods to Benjamin Osborne of Madison, Kansas. In accordance with the terms of the sale, Dunham draws the following draft:


$28.65 Toledo, Wis. Aug. 12 1922
At sight Pay to the
Order of myself
Twenty eight and 65/100 Dollars
NO PROTEST TAKE THIS OFF BEFORE PRESENTING.
Value received and charge the same to account of
To Benjamin Osborne
No Madison, Kan.
J. B. Dunham

Mr. Dunham then indorses the draft as follows:

Pay to the order of
The First State Bank of Toledo
for collection only.
J. B. Dunham,

and gives it to his bank to collect.

The bank, in turn, indorses it as follows:

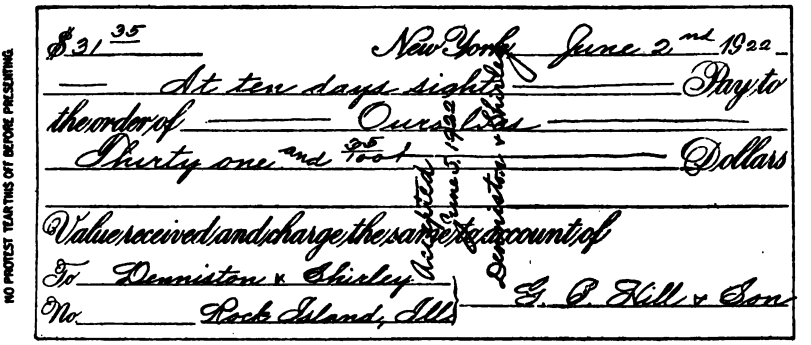
Pay to the order of
The Farmers National Bank of Madison
for collection only.
First State Bank,
Toledo, Wisconsin,
E. J. Baxter, Cashier,

and forwards it to the bank at Madison to collect. The Madison bank presents it to Mr. Osborne for payment.

If Osborne pays the draft, the bank stamps a receipt thereon, and gives the draft to Osborne. The bank keeps a certain amount as a collection fee, and returns the balance to the bank at Toledo, where Mr. Dunham receives the proceeds in cash or as a deposit credit.

If Osborne does not pay the draft, it is returned to the bank at Toledo with a statement of the reason for non-payment.

At the time of selling the goods the wholesale house may require the purchaser to accept a time draft. The wholesale house can then use the trade acceptance as security to borrow money at the bank, or it can sell the acceptance to the bank. At maturity the acceptanceis presented to the acceptor for payment. The following is an illustration of such an acceptance:


NO PROTEST TEAR THIS OFF BEFORE PRESENTING
$31.35
New York June 2nd 1922
At ten days sight Pay to
the order of Ourselves
Thirty one and 35/100 Dollars
Value received and charge the same to account of
To Denniston & Shirley
No Rock Island, Ills
Accepted June 5, 1922
Denniston & Shirley
G. P. Hill & Son

AN ACCEPTED TIME DRAFT

This draft is due thirty days after June 6, or July 6.

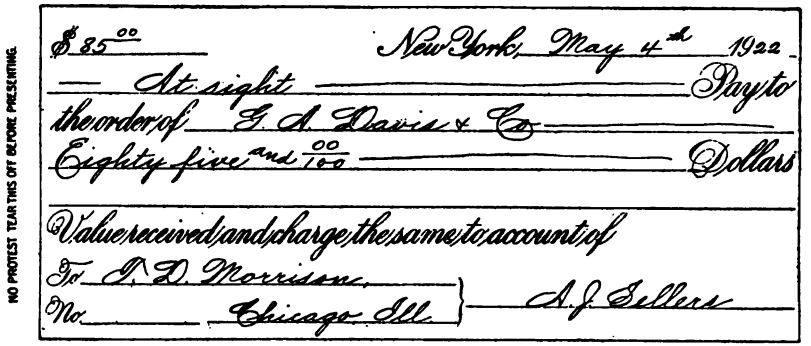
If a creditor has difficulty in collecting an account from a debtor he may draw a draft on him even though the terms at the time of sale made no mention of a draft. The debtor is not obliged to accept it or pay it, but since it will be presented to him by a bank in his own town he may do so in order to keep his reputation good with the bank.

197. Drafts with C. O. D. Shipments by Freight. When a merchant sells goods to be shipped by freight, he receives a bill of lading from the railroad company. This bill of lading is the railroad's receipt for the goods. To make a C. O. D. shipment the seller obtains an Order Bill of Lading from the railroad. The purchaser cannot get the goods from the railroad without surrendering the bill of lading. Therefore, a selling merchant, instead of sending the bill of lading direct to the purchaser, attaches it to a sight draft and sends the draft and the bill of lading to a bank in the purchaser's town.

In order to get the goods the purchaser must have the bill of lading; in order to get the bill of lading he must pay the draft at the bank.

198. Drafts to Avoid the Transfer of Funds. Three-party drafts are used for this purpose, but their use is not as customary as in the past. The following illustration will show the procedure.

T. D. Morrison of Chicago owes A. J. Sellers of New York $85.00. Mr. Sellers owes G. A. Davis & Co. of Chicago $150.00. Mr. Sellers draws the following draft, asking Mr. Morrison to pay G. A. Davis & Co. $85.00.


NO PROTEST TEAR THIS OFF BEFORE PRESENTING
$85.00
New York, May 4th 1922
At sight
Pay to
the order of G. A. Davis & Co.
Eighty five and 00/100 Dollars
Value received and charge the same to account of
To A. D. Morrison,
No. Chicago, Ill.
A. J. Sellers

THREE-PARTY SIGHT DRAFT

Oral Work

What two parties make a payment when this draft is used?

Explain how one transfer of money is saved.

How much will Sellers owe G. A. Davis & Co. after they have received this draft?

Three-party drafts may also be made payable after a stated time by using the terms explained on page 229.

Written Work

1. On July 13, you purchased from A. B. Butler of Minneapolis, Minnesota, an invoice of goods amounting to $123.60; terms, sight draft in 10 days, less 3 %. On July 20, Mr. Butler drew a sight draft on you for the net amount of the bill. He indorsed the draft, and gave it to the Fourth Street Bank of Minneapolis for collection. The Minneapolis bank indorsed it and sent it to the First National Bank of your city. The draft was presented to you by the bank on the 23d of July.

Write the draft with indorsements.

If the First National Bank charged 15 cents for collection, how much did Mr. Butler receive in payment of the draft?

How much did you pay to the bank?

2. You purchased an invoice of goods from Winthrop and Monroe of Toledo, Ohio. Amount of invoice, $365.00. It was your first transaction with them and consequently you were unknown to them. As you wanted the goods immediately and did not wish to delay while inquiries were made regarding your credit, you instructed them to send the goods

by freight and send a sight draft, with bill of lading attached, to the First National Bank of your city. The sight draft, which was dated yesterday, arrived this morning. It was made payable to Winthrop and Monroe and was indorsed to the order of the bank.

Write the draft and show the indorsement made by Winthrop and Monroe.

Write a check in favor of the bank to pay the draft.

What will you receive in exchange for the check?

How will Winthrop and Monroe get their money?

3. On September 3, you purchased an invoice of goods from S. L. Norton of Springfield, Illinois. Terms, thirty-day draft from date of sale, less 1%. Amount of invoice, $75.00. On September 5, you received the draft in the mail; it was dated September 3, and was payable 30 days after date. You immediately accepted the draft and returned it to Mr. Norton.

Write the draft, and show your acceptance.

When will you be expected to pay the draft?

4. You owed S. D. Briggs of Winona, Minnesota, $75.00; Mr. Briggs owed F. R. Tuttle of your city, $136.00. On June 19, Mr. Briggs drew a sight draft on you for the amount of your debt, and sent it to Mr. Tuttle as part payment of his debt. Mr. Tuttle presented it to you. You gave Mr. Tuttle cash in payment of the draft, and received the draft as a receipt.

Write the draft.

How much did Briggs owe Tuttle after the draft was paid?

CHAPTER XX

FOREIGN MONEY AND EXCHANGE

Foreign Money

199. The trade between the United States and foreign countries necessitates changing money values of the United States coinage into that of foreign countries, and also changing values stated in the coinage of foreign countries into that of the United States. In order to provide a standard of comparative values, the Director of the United States Mint at frequent intervals estimates the values of foreign coins, and these values are proclaimed by the Secretary of the Treasury. This proclamation is then used as the standard in computing the value of foreign merchandise imported into this country, until the succeeding proclamation is issued. The data in the table on the following page are copied from a recent proclamation.

200. To reduce a value in foreign coinage to United States money.

Multiply the value of the coin in United States money by the number of the coins.

Problem. Change 2500 francs to dollars.

Solution. 1 franc = \$.193

2500 × \$.193 = \$482.50.

201. To reduce a value in United States money to foreign coinage.

Divide the value in United States money by the value of the foreign unit.

Problem. Change \$86.85 to francs.

Solution. 1 franc = \$.193

\$86.85 ÷ \$.193 = 450, the number of francs.

Written Work

1. The following table gives the value of the United States imports from, and exports to, various European countries in a recent year. Imports are stated in the coinage of the nation from which they came; use the preceding table of values and change the values to United States coinage. Exports are stated in the coinage of this country;

VALUES OF FOREIGN COINS

COUNTRY	LEGAL STANDARD	MONETARY UNIT	PAR VALUE IN TERMS OF U.S. MONEY	REMARKS
Argentine Republic	Gold	Peso	$0.9647	Currency: depreciated paper, convertible at 44 per cent of face value.
Belgium	Gold and silver	Franc	.193	Member of Latin Union; gold is the actual standard.
Canada	Gold	Dollar	1.000	
France	Gold and silver	Franc	.193	
German Empire	Gold	Mark	.238	
Great Britain	Gold	Pound sterling	4.8665	
Greece	Gold and silver	Drachma	.193	Member of Latin Union: gold is the actual standard.
Italy	Gold and silver	Lira	.193	
Japan	Gold	Yen	.498	
Mexico	Gold	Peso	.498	
Netherlands	Gold	Florin	.402	
Newfoundland	Gold	Dollar	1.014	
Norway	Gold	Crown	.268	
Panama	Gold	Balboa	1.000	
Peru	Gold	Libra	4.8665	
Philippine Islands	Gold	Peso	.500	
Roumania	Gold	Leu	.193	
Russia	Gold	Ruble	.515	
Sweden	Gold	Crown	.268	
Switzerland	Gold	Franc	.193	
Turkey	Gold	Piaster	.044	
Uruguay	Gold	Peso	1.034	
Venezuela	Gold	Bolivar	.193	

change these values into the coinage of the nation to which they were shipped.

COUNTRY	IMPORTS FROM	EXPORTS TO
Great Britain	56,085,622 pounds sterling	$564,372,186
Germany	720,085,605 marks	306,959,021
France	645,328,797 francs	135,388,851
Belgium	215,945,179 francs	51,387,618
Netherlands	88,478,696 florins	103,702,859
Italy	248,852,481 liras	65,261,268
Switzerland	124,138,326 francs	855,355
Sweden	35,528,936 crowns	9,451,011
Norway	30,789,992 crowns	8,331,723
Russia	40,110,530 rubles	21,515,660
Turkey	9,852,709 piasters	2,597,239
Greece	19,810,186 drachmas	966,641

Rule a form and enter the statistics as indicated:

Country	Imports from		Exports to	
	Foreign Coinage	Dollars	Foreign Coinage	Dollars

2. Half a dozen pairs of gloves were bought in England for £1 2*s*. What was the cost of 1 pair in United States money?

Foreign Exchange

Most of the methods used for the transfer of money and the collection of accounts between persons in the same country are also available when the persons live in different countries.

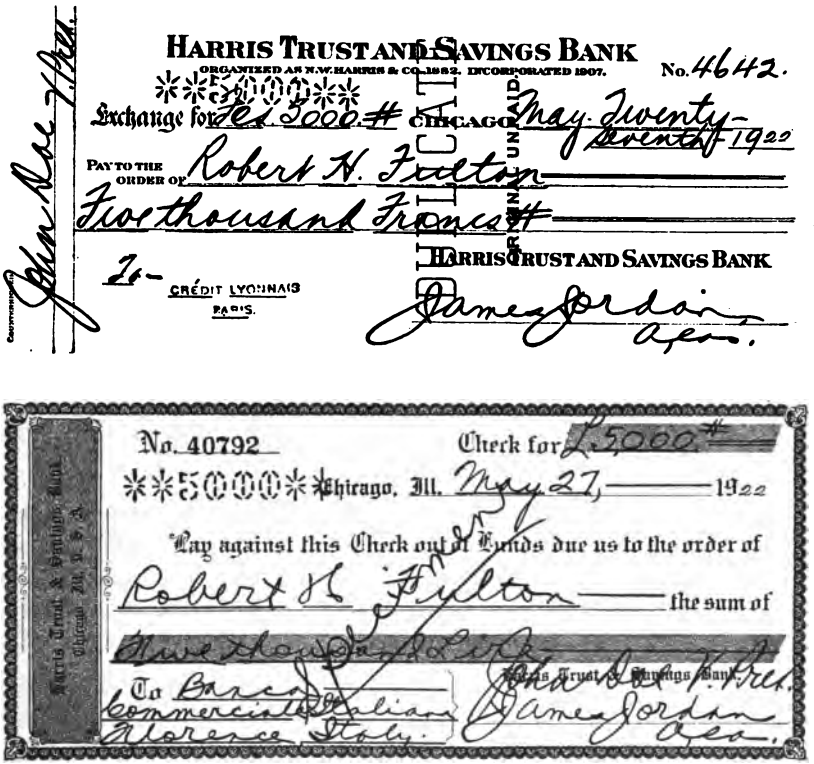
202. Postal Money Orders. Domestic money orders were discussed on page 223. The following table of rates shows the cost of international postal money orders:

For Orders from		For Orders from	
$00.01 to $10.00	10 cents	$50.01 to $60.00	60 cents
10.01 to 20.00	20 cents	60.01 to 70.00	70 cents
20.01 to 30.00	30 cents	70.01 to 80.00	80 cents
30.01 to 40.00	40 cents	80.01 to 90.00	90 cents
40.01 to 50.00	50 cents	90.01 to 100.00	1 dollar

203. Bills of Exchange. Bills of exchange are drafts of a person or a bank in one country on a person or a bank in another country. They are of three kinds: Bankers' Bills, Commercial Bills, and Letters of Credit. Formerly, when the danger of long delays and possible loss in the foreign mails was greater than it is at present, foreign bills of exchange were issued in sets of three; the first to arrive at its destination was paid and the others became worthless automatically. Later,

the number was reduced to two, and it is now customary for only one of the set to be mailed, and the other filed for use in case the first is lost.

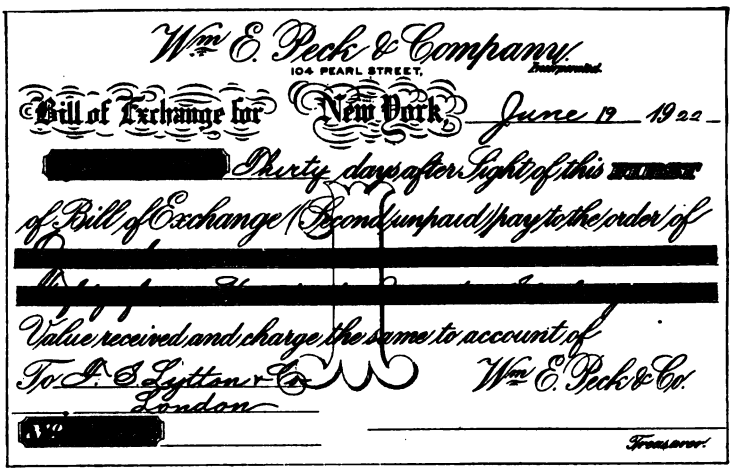
204. Bankers' Bills. A banker's bill of exchange is a draft drawn by a bank in one country on a bank in a foreign country. It corresponds to the bank draft of domestic exchange.


HARRIS TRUST AND SAVINGS BANK
ORGANIZED AS N.W. HARRIS & CO. 1882. INCORPORATED 1907.
No. 4642.
Exchange for Fcs. 5000.-
CHICAGO May twenty-seventh 1922
PAY TO THE ORDER OF Robert H. Fulton
Five thousand Francs
CRÉDIT LYONNAIS PARIS.
HARRIS TRUST AND SAVINGS BANK
James Jordan Agent.
No. 40792
Check for L. 5000.-
Chicago, Ill. May 27, 1922
Pay against this Check out of Funds due us to the order of
Robert H. Fulton
the sum of
Five thousand Lire
To Banca Commerciale Italiana Florence Italy
James Jordan Agent.

BANKER'S BILLS

205. Commercial Bills. A commercial bill of exchange is a draft drawn by a shipper of merchandise on a foreign buyer or his representative. Commercial bills are made payable either at sight or after a certain time. If such a bill is payable in less than thirty days, it is known as a *short bill;* if in thirty days or more, it is known as a *long bill.*

If the bill is accompanied by a bill of lading and other shipping papers, it is known as a *documentary bill;* if no papers accompany it, it is known as a *clean bill.*


Wm. E. Peck & Company
104 PEARL STREET,
Bill of Exchange for New York June 19 1922
[redacted] thirty days after Sight of this [redacted]
of Bill of Exchange (Second unpaid) pay to the order of
[redacted]
Value received and charge the same to account of
To L. S. Lyttan & Co. Wm. E. Peck & Co.
London
[redacted]
Treasurer

COMMERCIAL BILL

A *documentary bill* is accompanied by the bill of lading, the invoice of the goods, and usually by the insurance certificate.

206. Sending Money by Banker's Bill. When it is desired to send money abroad, a banker's bill may be purchased from the bank in much the same manner that a domestic bank draft is purchased. In fact, bankers' bills are frequently called foreign bank drafts. The banker's bill is sent abroad by the purchaser in payment of his debt.

207. Collecting Accounts by Using Commercial Bills. When it is desired to collect an account from a foreign debtor, a merchant may draw a commercial bill and leave it with his bank for collection. The method of collecting the bill is similar to that employed by banks in the collection of domestic sight or time drafts.

208. Securing Immediate Payment for Goods by Using Documentary Bills. Exporters often obtain immediate payment for goods sent abroad by the following method. The goods are delivered to the transportation company and a bill of lading is received by the shipper. The goods are insured against loss in transit, and a certificate of insurance is received from the insurance company. The shipper draws

a bill of exchange on the foreign importer, and attaches the bill of lading and the certificate of insurance to the bill of exchange, thus creating a documentary bill. The three papers are indorsed to the order of a bank which purchases the draft. The exporter thus receives payment for his goods at the time of shipment. If the bill cannot be collected, the goods are taken in payment. If the goods are lost, the certificate of insurance guarantees that the bank will receive payment from the insurance company. The bank, in turn, indorses the documents and sends them to some foreign bank, receiving credit for the bill of exchange. The foreign bank collects the bill from the exporter.

The following summary illustrates the method of collecting a documentary bill of exchange:

American exporter sells the documentary bill to an American bank, and receives cash.
American bank sends the documentary bill to a foreign bank, and receives credit against which it can draw bankers' bills.
Foreign bank collects the documentary bill from the foreign importer.

209. Rates of Exchange. The amounts which a bank will pay for commercial bills and the amounts which it will charge for a banker's bills, depend upon the rate of exchange existing between the countries involved in the transaction.

The **mint par of exchange,** as shown by the table on page 235, is the actual value of the coin of one country stated in terms of the coin of another country. It remains comparatively constant.

The **rate of exchange** is the market value of a bill of exchange. These values or rates of exchange are constantly fluctuating, because the value of a bill of exchange, like the value of other property, varies with the supply and demand.

As we have seen, bankers buy documentary bills to send abroad to create a deposit against which they can draw bankers' bills. At a time when American exporters are shipping large quantities of goods abroad, documentary bills on London will be plentiful. American bankers can easily procure all of these bills they desire in order to keep up their balances in foreign banks. Supply will be relatively much greater than demand, and the price will therefore be low. Exchange on England will be below par, or at a discount.

Par is $4.8665. If the rate of exchange is below par, say at $4.80, the man who sells a documentary bill on London may receive for it

less than its face; while the man who wishes to buy a banker's bill payable in London may buy it for less than its face.

On the other hand, let us assume a condition when American imports greatly exceed exports. While imports are being made in large quantities, American banks will have frequent demands for bankers' bills to send abroad in payment for goods. In order to issue these bills, American banks must maintain a large deposit in the foreign banks. To keep up this large balance, they will require numerous documentary bills payable in London. But since exports are low, the supply of these documentary bills will be small, and bankers will bid against each other to obtain them. This will cause the rate of exchange to go above par. This means that the American exporter can obtain more than $4.8665 per pound sterling for documentary bills on London which he has for sale; while the importer who wishes to send money abroad will have to pay more than $4.8665 per pound sterling for bankers' bills.

210. Quotations of Rates of Exchange. Exchange on foreign countries is stated at the value of the foreign coins in American cents. The following is an extract from the quotations appearing in a daily paper. The current rates quoted are much lower than the par rates because the war disorganized the exchange markets.

London cables	$362\frac{7}{8}$
London checks	$361\frac{7}{8}$
Paris cables	7.80
Paris checks	7.79
Italy	4.55
Antwerp	7.72
Sweden	21.17
Germany	$1.34\frac{1}{4}$
Greece	5.50

Problem. What is the cost of a £600 draft on London, purchased at $361\frac{7}{8}$?

SOLUTION.

$$\begin{array}{rl} \$3.61\frac{7}{8} & \text{Value per pound sterling} \\ 600. & \\ \hline \$2{,}171.25 & \end{array}$$

Problem. How many pounds sterling can be purchased for $10,000.00 at $361\frac{7}{8}$?

SOLUTION: 10,000 ÷ 3.61875 = 2,763.385 Number of pounds
20 (shillings per pound) × .385 = 7.70 Number of shillings
12 (pence per shilling) × .70 = 8.4 Number of pence
£2,763 7*s.* 8*d.* Amount obtained.

211. Travelers' Checks. Travelers' checks are issued by the express companies and by the American Bankers Association. These checks are issued in denominations of United States money, but show on their face their cashable value in the coinage of foreign countries. The purchaser signs the checks when purchased, and again when cashed, as a means of identification. The cost of the checks is the face plus ½% commission.

212. Letters of Credit. A letter of credit is a circular letter addressed to the correspondents of the issuing bank, introducing the holder, certifying that he is authorized to draw a certain sum of money, and requesting that his drafts be honored up to that amount. By depositing cash or securities with a bank, a traveler can obtain a letter of credit. This document is drawn by the bank in which the cash has been deposited. When a bank issues a letter of credit, it thereby authorizes its correspondents (banks in which it has money on deposit) to pay drafts drawn by the traveler up to a specified sum. The bank also furnishes the purchaser a Letter of Identification containing his signature; when the traveler presents a draft to one of the foreign banks specified in the letter of credit, the signature on the draft is compared with that on the letter of indication. If the signatures are identical, the foreign bank pays the draft, charges the amount to the bank issuing the letter, and indorses the amount of the draft on the letter. Thus, the letter of credit shows at all times the balance remaining subject

Circular Letter of Credit.

£50000** No. 0000

HARRIS TRUST AND SAVINGS BANK

ORGANIZED AS N.W. HARRIS & CO., 1882. INCORPORATED 1907.

Chicago May 27, 1922.

Gentlemen:

We hereby beg to introduce to you Mr. Max Gleason Gregg in whose favor we have opened a Credit of £ 5,000** Sterling say Five thousand** Pounds Sterling up to the aggregate of which amount he is authorized to draw demand drafts on the Bank of Scotland, 198 Bishopsgate Str. Within London, each draft to be plainly marked as drawn under Harris Trust and Savings Bank's Letter of Credit No.

We engage that such drafts shall meet with due honor in London if drawn on or before June 1, 1923 and request you to buy the same at the rate at which you purchase demand drafts on London.

The amount of each draft must be endorsed hereon and this letter itself cancelled and attached to the draft which exhausts it. Mr. Gregg has ____ been provided with our Letter of Indication where his signature for identification will be found.

We remain, Gentlemen,
Yours respectfully,
Harris Trust and Savings Bank
By ____

To Messieurs
The Banks & Bankers named on the
accompanying Letter of Indication

COUNTERSIGNED James Gordon, Agent.

PAID IN EXCESS OF STERLING

OBLIGATION. Form 40-B. 12M-2-07.

FOREIGN LETTER OF CREDIT.

Chicago, May 27, 1922

HARRIS TRUST AND SAVINGS BANK.

Dear Sirs: In consideration of your granting to Me *a circular LETTER OF CREDIT on* Bank of Scotland London Eng. *No.* 0 *in favor of* Mr Max Clemon Greigg *for* £5000 * Sterling *say* Five thousand Pounds Stg., *of which* I *acknowledge receipt,* I *hereby engage to pay on presentation, in United States Currency (at the current rate of exchange), any and all sums that may from time to time be drawn under said Letter of Credit, and also a commission of one per cent; or* I *authorize you to charge the same to* My *account with you.*

As collateral security for the prompt payment at maturity of any of the obligations contemplated above I *have deposited with you the following described property:*

£5000 Chicago Railways Co., 1st 5's

which you, or your assigns, are hereby authorized to sell in whole or in part upon the non-performance of this promise or the non-payment at maturity of any of the obligations contemplated above, or at any time or times thereafter (at public or private sale, without advertising the same or demanding payment, or giving notice, and to apply so much of the proceeds thereof to the payment of such indebtedness as may be necessary to pay the same with all interest due thereon, and also to the payment of all expenses attending the sale of said property, returning the over-plus to the undersigned, who shall remain liable for the prompt payment of any deficiency or deficiencies arising on account of such sale.

Yours truly,

Max Clemon Greigg

to draft. When a draft is drawn which exhausts the balance, the letter is surrendered, and sent, together with the final draft, to the issuing bank.

Letters of credit are issued in an amount equivalent to the exchange value of the cash or securities deposited, minus a percentage charged for the accommodation.

HARRIS TRUST AND SAVINGS BANK
ORGANIZED AS E. W. HARRIS & CO., 1882
INCORPORATED 1907
CHICAGO, U. S. A.

May 27, 1922.

To Messieurs The Banks and Bankers Named Herein:

The Bearer of this Letter, Mr Max Clemon Greigg SPECIMEN whose signature for identification is to be found below has been supplied with our Circular Letter of Credit No. 00

HARRIS TRUST AND SAVINGS BANK
BY

BEARER'S SIGNATURE
Max Clemon Greigg

Written Work

1. Find the total cost of a postal money order for $85, payable in Hongkong.

2. Find the total cost of a postal money order for $215.65, payable in Paris.

3. What would be the cost of a London draft for £115, exchange being quoted at 4.8675?

4. A man had $500 which he wished to remit to his mother in Paris. How many francs did she receive, the rate of exchange being 18.5?

5. M. Le Count, traveling in America, presented to a Chicago bank a draft on the Credit Lyonnais, Paris, for 2,000 francs. How much American money did he receive, exchange being quoted at 18.8?

6. The Western Milling Co. sold a Glasgow customer three hundred barrels of flour at 19s. per barrel. The Milling Co. sold its sixty-day time bill, with bill of lading attached, to its banker for 4.60 per pound sterling. What was the amount of the draft in English currency, and how much per barrel United States currency did the company receive for its flour?

7. The Lincoln Grain Co. sold three cars of wheat to a Liverpool miller at 4s. per bushel. The net weight of the cars was: 58,950#, 59,040#, and 64,360#. The bank, as agent for the grain company, sold the draft with bill of lading attached at 4.55. How much did the Grain Company receive?

8. Refer to the financial page of the daily paper for exchange rates and compute the cost of the following:

(*a*) London cable for $15,000.00.
(*b*) Draft on Paris for $6,500.00.
(*c*) Draft on Berlin for $8,000.00.

CHAPTER XXI

ACCOUNTS

An account is an orderly record of the transactions pertaining to any one person or thing.

213. The Cash Account. In keeping a record of cash,

All cash receipts are entered on the left, or debit side;

All cash payments are entered on the right, or credit side;

The difference between the total receipts and total payments is the balance of cash on hand.

The following transactions are recorded in the model cash account in the illustration.

April 6, 1922, C. D. Smith invested $5,000 in business; April 7, he paid $1,640 for goods; April 10, he paid his store rent, $75; April 11, he received $275 for merchandise; April 13, he received $36 from John Appleton; April 14, he paid F. G. Barton $128; April 18, William Hobart paid him $60 on account; April 28, he paid D. F. Hilton $30 on account.

Cash

1922				1922			
April	6	C. D. Smith	5000—	April	7	Mdse. Purchase	1640—
	10	Mdse. Sale	275—		10	Store Rent	75—
	13	John Appleton	36—		14	F. G. Barton	128—
	16	William Hobart	60—		28	D. F. Hilton	30—
					30	Balance	3498—
			5371—				5371—
May	1	Balance	3498—				

214. Personal Accounts. Transactions with persons are recorded in personal accounts. Persons to whom goods are sold are debited for all goods sold to them on account; they are credited for payments made by them on account.

Persons from whom goods are bought, are credited for goods purchased from them on account; they are debited for payments made to them on account.

The two illustrations which follow will show the two common rulings for personal accounts.

May 2, 1922, purchased from Harold Booth, merchandise, $260. May 8, purchased merchandise from him, $37.90; May 10, paid him for the invoice of May 1, less 1% discount, Cash $257.40, Discount $2.60; May 17, paid him for the invoice of May 8, less 1% discount, Cash $36.52, Discount $.38; May 20, purchased merchandise, $271.25.

Harold Booth

1922				1922			
May	10	Dis. 2.60 Cash 257.40	260.—	May	1	Mdse	260.—
	17	Dis. .38 Cash 36.52	37.90		8	Mdse	37.90
					20	Mdse	271.25

June 1, 1922, sold to B. A. Newcomber, goods amounting to $345.70; June 5, sold to Newcomber merchandise amounting to $38.45; June 12, received payment for invoice of June 1, less 2%, Cash $338.79, Discount $6.91; June 13, sold Newcomber an invoice amounting to $83.92; June 14, received payment for the invoice of the 5th, less 1%, Cash $38.07, Discount $.38.

B. A. Newcomber

Date		Sale	Cash	Discount	Balance
1922 June	1	345.70			345.70
"	5	38.45			384.15
"	12		338.79	6.91	38.45
"	13	83.92			122.97
"	14		38.07	.38	83.92

215. Accounts Receivable and Accounts Payable. If a person's account is larger on the debit side, it shows a balance owed *by* that person; it is therefore an account receivable, and the balance is an asset.

If a person's account is larger on the credit side, it shows a balance owed *to* that person; it is therefore an account payable, and the balance is a liability.

Written Work

1. Rule a cash account and enter the following transactions; find the balance, and rule the account.

August 2, you invest $5,000 in a grocery business; August 3, purchase merchandise, paying cash for the same, $1,345.75; August 6, receive cash for merchandise sold, $127.50; August 10, receive cash for goods sold, $50.25; August 12, pay for advertising, $5.60; August 17, receive cash for merchandise sold, $23.74; August 18, pay cash for an invoice of merchandise, $56.35; August 21, receive cash from Henry Belmont on account, $45.80; August 23, pay cash to Oscar Haines on account, $54.85; August 31, pay store rent, $30, and clerk hire, $35.

2. Rule a personal account similar to the illustration of Harold Booth's account, page 246, and enter the following transactions:

September 3, you purchase from R. G. Henderson, on account, 2/10; N/60, an invoice of goods amounting to $237.40; September 9, purchase an invoice of goods from Henderson, $58.35, terms 1/15; N/2 months; September 11, purchase an invoice of merchandise from Henderson, $123.60, terms 2/5; N/30; September 12, pay the invoice of the 3d, less the discount; September 14, purchase goods amounting to $25; September 16, pay the invoice of the 11th, less the discount. What is the balance of Mr. Henderson's account? Is this an account receivable or an account payable? Is the balance an asset or a liability?

3. Rule an account similar to the illustration of B. A. Newcomber's account, page 246, and enter the following transactions:

July 2, sold R. S. Clark, terms 1/5; N/30, merchandise amounting to $57.82; July 5, sold Clark goods to the value of $23.95, terms 2/10; N/60; July 7, received payment for the invoice of the 2d, less the discount; July 9, sold Clark an invoice of $75, less a trade discount of 10%, terms 1/10; N/30; July 11, sold him a bill of goods amounting to $40; July 15, received cash for the invoice of the 5th, less the discount.

Is this account an account receivable or an account payable?

Is the balance an asset or a liability?

CHAPTER XXII

TAKING INVENTORY

At certain regular intervals an inventory is taken to determine the value of the stock on hand. Two clerks usually work together in taking the inventory. One counts the number of items of each kind, and reads aloud the cost price marked on the goods. The second clerk records these facts. The inventory is then sent to the office, where the value of each item is extended and the total value of the stock on hand is determined.

Inventories are entered in various forms, depending upon the details of information desired.

216. Periodic Inventories. A simple form like the following may be used:

QUANTITY	NAME OF ITEM	COST		COST EXTENSION	
19 bbl.	Herald Flour	5	60	106	40
345 lb.	Java Coffee		26	89	70
25 lb.	Ceylon Tea		43	12	26

Written Work

Rule an inventory similar to the preceding form, enter the following items, find the cost of each item, and the total cost.

This is one of several inventory sheets used in taking stock in a grocery.

87	lb.	Mexican Java Coffee	lb.	$ .23½
136	lb.	Ceylon Tea	lb.	.42
167	pkg.	Half-pounds Ceylon Tea	lb.	.47
2	cases	Boneless Herring (4 doz.)	doz.	1.10
83	lb.	Salt Mackerel	lb.	.22
$6\frac{5}{12}$	doz.	Cans Corn	doz.	.85

$4\frac{11}{12}$	doz.	Cans Tomatoes	doz.	.95
$8\frac{3}{4}$	doz.	Cans Peas	doz.	1.25
$9\frac{1}{6}$	doz.	Cans Beans	doz.	1.35
$3\frac{5}{6}$	doz.	Bottles Olives	doz.	1.00
$4\frac{1}{3}$	doz.	Bottles Olives (18 oz.)	doz.	3.20
$\frac{7}{12}$	doz.	Bottles Stuffed Olives (10 oz.)	doz.	$.82\frac{1}{2}$
345	lb.	Evaporated Apricots	lb.	$.08\frac{3}{4}$
163	lb.	Evaporated Apples	lb.	$.07\frac{1}{4}$
109	lb.	Dried Prunes	lb.	$.05\frac{1}{2}$
230	lb.	Dried Peaches	lb.	$.07\frac{3}{4}$
67	lb.	Seedless Raisins	lb.	$.09\frac{1}{2}$
1	case	Currants (36 lb.)	lb.	$.08\frac{1}{2}$

A more elaborate form of inventory similar to the following may be used when desired.

Add	Deduct	Lot No.	Size	Name and Quantity	Cost Price	Cost Extension	% Dep.	Loss Assumed
		A 42	6	3 Pairs Mens Gun Metal Oxfords	2 80	8 40		
	1	A 42	6½	3 Pairs do	2 80	8 40		
4		A 42	7	9 Pairs do	2 80	25 20		
		A 42	7½	6 Pairs do	2 80	16 80		
		A 42	8	2 Pairs do	2 80	5 60		
		C 9	6 7 8	3 Pairs Mens Blucher Oxfords	3 10	9 30	10%	.93

217. Explanation of Form. The Lot Number is a number given an article by either the manufacturer or the merchant. Since different styles of goods are given different lot numbers, this number may be entered in the inventory in place of a description.

The " size " column is used to show the different sizes of an article in stock. As illustrated in the model inventory, each size of an article may appear on a separate line, or all sizes may be entered on the same line. When the latter method is followed, the quantity of a certain

size is indicated by writing the size below a short line, and the number of articles of that size above the line. Thus, $\frac{3}{7}$, $\frac{5}{8}$, means 3 No. 7's, 5 No. 8's.

When an error is made in the count, this error may be corrected by using the "Add" or "Deduct" columns. These columns are also used when goods are purchased or sold while the inventory is being taken. By adding goods purchased, and deducting goods sold during the taking of inventory, the value of stock at the end of the inventory may be determined.

To find the cost extension, multiply the cost by the number of items in the "Quantity" column plus the number of items in the "Add" column or minus those in the "Deduct" column.

If goods become shopworn or out of style, if the market price has decreased, or if, for any other reason, the value has become less than the cost marked on the goods and entered in the inventory, it is necessary to make some allowance for this depreciation in value.

Perhaps an article is worth only half its cost price. In that case a 50% depreciation should be entered in the "% Depreciation" column. Then 50% of the cost extension for that item is entered in the "Loss Assumed" column, and the total of this column is subtracted from the total of the "Cost Extension" column to determine the actual value of the stock on hand.

Oral Work

1. What does 25% depreciation mean? 10% depreciation? 20% depreciation?

2. If stock is damaged so that it is worth only $\frac{3}{4}$ of its original cost, what per cent depreciation should be entered?

3. Goods are worth $\frac{2}{3}$ of their original cost. What is the per cent depreciation?

4. Goods are worth $\frac{4}{5}$ of their original cost. What is the per cent depreciation?

5. Goods are worth $\frac{7}{8}$ of their original cost. What is the per cent depreciation?

Written Work

Rule an inventory form similar to the one on page 249 and enter the following items which form a portion of the inventory of a shoe stock. Enter one size to a line, find the value of each item listed, and

the total value of the stock shown on the page after making corrections for stock returned, for sales, and for depreciation.

Lot No. 219 B; $\frac{2}{6}, \frac{1}{6\frac{1}{2}}, \frac{3}{7}, \frac{4}{7\frac{1}{2}}, \frac{2}{8}, \frac{2}{8\frac{1}{2}}$. Men's Tan Oxfords, $3.15 per pair.

Lot No. 712; $\frac{2}{6}, \frac{3}{6\frac{1}{2}}, \frac{4}{7}, \frac{4}{7\frac{1}{2}}, \frac{3}{8}, \frac{4}{8\frac{1}{2}}, \frac{2}{9}$. Men's Vici Kid Bluchers, $2.90 per pair.

Lot No. 322 A; $\frac{1}{6}, \frac{1}{6\frac{1}{2}}, \frac{2}{7}$. Romeos at $1.90 per pair.

Lot No. 618 D1; $\frac{2}{7}, \frac{3}{7\frac{1}{2}}, \frac{3}{8}, \frac{2}{8\frac{1}{2}}, \frac{2}{9}, \frac{1}{9\frac{1}{2}}$. Men's Russian Calf Boots, $3.20 per pair.

One pair of Lot No. 712, Size 7, taken home by a customer before taking inventory, was returned after Lot No. 712 was listed.

During stock taking, the following sales were made:

1 pair Lot No. 712, size $7\frac{1}{2}$.
1 pair Lot No. 618 D1, size $7\frac{1}{2}$.
1 pair Lot No. 618 D1, size $9\frac{1}{2}$.

Lot No. 322 A can now be purchased for $1.75.

218. Perpetual Inventory. In some lines of business a perpetual inventory is kept. It is also called a Stock Record. When such a record is kept, a book is required for the purpose, one page being devoted to each item carried in stock. Both purchases and sales are recorded at cost price. The following model illustrates a form of perpetual inventory.

219. Value of Perpetual Inventory. Considerable labor is required to maintain a perpetual inventory, but it is valuable for several reasons.

When stock is kept in warehouses or in storerooms at some distance from the salesrooms, the perpetual inventory is a great convenience since it shows without delay the amount of stock on hand.

A " danger point " is fixed for each item. When the balance on hand has decreased to this danger point, a new supply is purchased.

An inventory may be taken at periodical intervals, and the quantities shown compared with the stock record. If the inventory and the stock record do not agree, an investigation may be made to discover the cause of the discrepancy.

No. A 603 Brace Castings

Danger Point 15

Date		Purchases			Sales		Balance	
1922		No.	Cost	Value	No.	Value	No.	Value
July	8	60	18	10 80			60	10 80
"	17				14	2 52	46	8 28
"	22				26	4 68	20	3 60
"	31	80	18	14 40				18 00
Aug.	3				25	4 50		13 50

Written Work

Rule a page of a stock record similar to the model on this page, and enter the following facts, making a perpetual inventory of the stock of Solvay Coke.

PURCHASES

October 22, 54,200#	at $4.45 per ton
November 18, 57,600#	at $4.45 per ton
December 7, 52,800#	at $4.45 per ton

SALES

October 29, 2 tons; October 30, 5 tons; October 31, 7,500#; November 2, 9,500#; November 5, 7 tons; November 17, 2 tons; November 19, 12 tons; November 20, 7,000#; November 23, 4 tons; November 24, 9,000#; November 30, 2 tons; December 9, 13,000#; December 11, 5 tons.

CHAPTER XXIII

GROSS PROFIT ON SALES

220. Definition. Gross Profit on Sales is the excess of the selling price of goods over their cost.

221. Cost of Goods Sold. The cost of goods sold is computed as follows: To the inventory at the beginning of the period add the purchases and the buying expenses such as freight and drayage; from this total deduct the inventory at the close of the period.

222. The Per Cent of Gross Profit may be computed by dividing the gross profit by the cost of the goods sold or by the selling price. Textbooks have usually stated that cost should be used as the basis, but business men almost invariably use the selling price as the basis. Therefore, when the expression " per cent of gross profit " is used in this text it will be understood to refer to the rate obtained by dividing the gross profit by the amount of the sales.

Oral Work

1. A hat which cost \$2.00 is sold for \$3.00. What is the profit? What is the per cent of profit?

2. A set of books cost a dealer \$20.00. In order to sell them the dealer lost \$2.00. What was the per cent of loss?

3. Through an error in an advertisement a merchant was obliged to sell some parasols at $16\frac{2}{3}\%$ less than their cost. He lost 30 cents on each parasol. What did they cost, and what was the selling price? At what price should they have been sold to gain 10% of the cost?

Written Work

1. Mr. Fisher bought a carriage for \$55 and sold it for \$75. What was the gross profit? What was the per cent of gross profit?

2. During a certain year a merchant purchased goods which cost him \$14,000. At the close of the year this stock inventoried \$3,000. What was the cost of the goods sold?

His sales for the year were $14,080. What was his gross profit? What was the per cent of gross profit?

3. The inventory of a dry goods merchant's stock at the beginning of the year was $3,560. The purchases during the year were $28,265. At the close of the year the inventory showed a stock of $3,940 unsold. What was the cost of the goods sold?

His sales for the year were $31,673. What was the gross profit? What was the per cent of gross profit?

4. Mr. Tracey bought the lumber business formerly conducted by Mr. Boyce. The stock on hand inventoried $5,280. Mr. Tracey's purchases during the year were $16,385. The freight was $236, the cost of labor in placing lumber in the yards and all other buying expenses $385. At the end of the year the stock inventoried $6,125. What was the cost of the goods sold during the year?

If the sales during the year were $19,245, what was the gross profit and per cent of gross profit?

5. Complete the following table:

Inventory, Jan. 1, 19—	Purchases	Buying Expenses	Total	Inventory, Dec. 31, 19—	Cost of Goods Sold	Sales	Profit or Loss	Per Cent Profit or Loss
3,125 75	16,294 90	3,70 25		4,685 60		18,495 40		
5,428 80	23,926 56	627 90		5,626 35		33,756 20		
1,290 00	6,754 00	271 25		2,140 00		8,325 75		
4,728 50	12,398 80	1,417 25		1,265 70		13,215 75		
3,680 00	15,927 00	485 00		2,726 00		14,850 00		

Enter profit and per cent of profit in black ink.
Enter loss and per cent of loss in red ink.

Written Review

1. Compute the cost of the following invoice subject to a 28% trade discount:

8 Tables	@	$27.50
12 Dressers	@	54.00
12 Beds	@	65.00
36 Chairs	@	14.80

2. If 4% is added to cost to cover buying expenses, what is the marked cost of each of the articles purchased?

3. How would the cost of each article be marked using the key word Charleston?

4. The invoice is subject to a cash discount of 2% if paid within ten days. How much is the cash discount, and what is the net amount of the invoice?

5. What is the net amount of an invoice of $745.60, subject to discounts of 20%, 10%, and 2%?

6. What single discount is equivalent to the series 40%, 20%, and 1%?

7. A's check book shows a balance of $271.60. The bank's statement at the same date shows a balance of $316.95. There are outstanding checks as follows:

No. 161	$14.75
170	23.80
172	6.95

The bank has charged 15 cents for collecting a draft and A has not deducted the charge on his check book stubs. Prepare a reconciliation of the bank account.

8. What is the fee for sending $75.60 by postal money order?

9. What is the value of £75 4*s*, 6*d* at 4.655?

10. How many francs can be obtained for $250.00 if exchange is quoted at 6.14?

11. A merchant's inventory January 1, 1922, was $6,340.65; his purchases for the year were $75,210.93; and his inventory December 31, 1922, was $7,246.80. His sales for the year were $108,215.60. What was the cost of sales? the gross profit on sales? the per cent of gross profit?

CHAPTER XXIV

INTEREST

223. Definitions. **Interest** is the income earned by loaning money or extending credit, or the expense incurred by borrowing money or obtaining credit.

Accrued interest may be either receivable or payable. Accrued interest receivable is unpaid interest due from a debtor; accrued interest payable is unpaid interest due to a creditor.

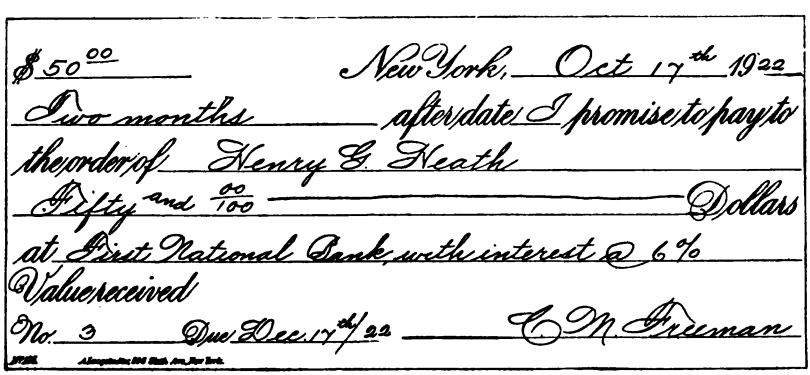
The sum borrowed is called the **principal.**

The per cent of the principal to be paid each year for the use thereof is called the **rate.**

The total of the principal and the unpaid or accrued interest is called the **amount.**

Assume that James Smith borrowed $1,000 from Henry Davis at 6% for one year. He paid a half year's interest at the middle of the year, but has not paid the interest at the end of the year. The principal is $1,000; the rate is 6%; the interest for the year is $60, and this interest is an earning for Davis who loaned the money and an expense for Smith who borrowed it. At the end of the year there is $30 unpaid or accrued interest; from Smith's standpoint this is accrued interest payable; from Davis's standpoint it is accrued interest receivable.

A **promissory** note is an unconditional promise in writing signed by the **maker** engaging to pay on demand or at a fixed or determinable future time a certain sum of money to the **payee** or his order.


50^{00} New York, Oct 17th 1922

Two months after date I promise to pay to

the order of Henry G. Heath

Fifty and $^{00}/_{100}$ Dollars

at First National Bank, with interest @ 6%

Value received

No. 3 Due Dec 17th/22 C M Freeman

PROMISSORY NOTE

If the borrower is a person of small means, or if, for any reason, it may be unsafe to loan money to him, he may be requested to have some person of recognized financial standing sign the note with him. Such notes are either **joint notes,** or **joint and several notes,** depending upon the wording.

$400.00 New York, July 15th 1922

Thirty days after date we jointly and severally promise to pay to the order of Martin Haskin

Four Hundred Dollars

at his office

Value received Fred G. Hamilton

No. Due Stephen Merrill

JOINT AND SEVERAL NOTE

$1000.00 Aurora, Ill. April 13, 1922

Ninety days after date I promise to pay to the order of First National Bank of Aurora, Ill.

One thousand Dollars

Without defalcation, value received, with interest at 6%

And further do hereby empower any Attorney of any Court of Record within the United States or elsewhere to appear for me and enter one or more confessions filed, confess judgment against me as of any term for the above sum with Costs of Suit and Attorney's commission of Ten per cent for collection and release of all errors and without stay of execution and inquisition and extension upon any levy on real estate is hereby waived, and condemnation agreed to and the exemption of personal property from levy and sale on any execution hereon, is also hereby expressly waived, and no benefit of exemption be claimed under and by virtue of any exemption law now in force or which may be hereafter passed.

Witness my hand and seal A. B. Jones (SEAL)

Due July 12, 1922 (SEAL)

JUDGMENT NOTE

224. To find interest at 6% for 6, 60, 600, or 6,000 days.

The interest on $1.00 for 1 year at 6% is	$.06
The interest on $1.00 for 2 months (60 da.) is	.01
The interest on $1.00 for 6 days ($\frac{1}{10}$ of 60 da.) is	.001

Thus, to find the interest on $1.00 for 60 days at 6%, move the decimal point two places to the left; to find the interest for 6 days, move

the point three places to the left. Since this is true if the principal is \$1.00, it will be true of any principal, and we have the following rule:

Given any principal, to find the interest at 6%.

For 6 *days, move the decimal point three places to the left; for* 60 *days, two places to the left; and for* 600 *days, one place to the left. For* 6,000 *days the interest will be the same as the principal.*

Oral Work

State the interest on each of the following principals at 6%, for 6, 60, 600, and 6,000 days:

1. \$1,000.	**2.** \$350.	**3.** \$125.60.	**4.** \$3,620.
5. \$12.20.	**6.** \$325.50.	**7.** \$6.	**8.** \$4,285

Find the interest at 6% on:

8. \$245 for 6 days. **9.** \$27 for 6 days.
10. \$37.50 for 60 days. **11.** \$12.50 for 6 days.

225. To find interest at 6% for fractions or multiples of 6, 60, 600, or 6,000 days.

The time is not always 6, 60, or 600 days, yet the above method can be easily used when the time is an easy fraction or an exact multiple of 6, 60, 600, or 6,000 days.

Problems. 1. Find the interest on \$500 for 12 days at 6%.

SOLUTION. \$.50 = the interest for 6 days.
$2 \times \$.50 = \$1.00$, the interest for 12 days.

2. Find the interest on \$700 for 40 days at 6%.

SOLUTION. \$7 = the interest for 60 days.
$\frac{2}{3}$ of \$7 = \$4.67, the interest for 40 days.

3. Find the interest on \$300 for 3 months at 6%.

SOLUTION. \$3 = the interest for 2 months (60 days).
$\$3 \div 2 = \$1.50$, the interest for 1 month.
$\$1.50 \times 3 = \$4.50$, the interest for 3 months.

Written Work

Rule a form similar to the following model; compute the interest at 6% on the various principals for the different times.

Principals	24 Da.	30 Da. (1 Mo.)	40 Da.	80 Da.	90 Da. (3 Mo.)	18 Da.	50 Da.	70 Da.	4 Mo.	6 Mo.
$ 125 60										
28 95										
378 45										
500 00										
960 00										
96 00										
9 60										
475 00										
4 75										
30 00										
146 00										
419 50										
395 00										
25,000 00										

When the time is not an easy fraction or an exact multiple of 6, 60, or 600 days, an adaptation of the method shown in the last exercise may be used.

Problems. 1. Find the interest on $500 for 99 days at 6%.

Solution.

$5.00 = the interest for 60 days.
2.50 = the interest for 30 days.
.50 = the interest for 6 days.
.25 = the interest for 3 days.
$8.25 = the interest for 99 days.

2. Find the interest on $1,000 for 82 days at 6%.

Solution.

$10.00 = the interest for 60 days.
$3.33\frac{1}{3}$ = the interest for 20 days.
$.33\frac{1}{3}$ = the interest for 2 days.
$13.67 = the interest for 82 days.

Oral Work

How would you find the interest on any principal for the following number of days at 6%:

3, 4, 5, 7, 8, 9, 36, 66, 69?

Written Work

Enter the interest at 6 % in the proper place on a form ruled like the following model:

	$845.00	$632.00	$1,296.00	$5,483.00	$74.60	$143.40	$746.95	$25.00	$130.00
10 da.									
66 da.									
63 da.									
68 da.									
75 da.									
80 da.									
85 da.									
96 da.									
98 da.									
45 da.									
82 da.									
28 da.									

226. To find interest at 6 % for any number of days.

Sometimes the method of adding the interest for different fractions or multiples of 6 and 60 days cannot conveniently be used.

In such cases

Point off three places in the principal; this will give the interest for six days. Multiply by the number of days; this will give the interest for six times the number of days. Divide by six; this will give the interest for the required number of days.

Problem. Find the interest on $500 for 19 days at 6 %.

SOLUTION. $.50 = the interest for 6 days.
$.50 × 19 = $9.50, the interest for 19 × 6 days.
$9.50 ÷ 6 = $1.58, the interest for 19 days.

Written Work

Enter the interest at 6 % on the various principals for the different times in the proper place on a form ruled like the following model:

	$435.00	$127.90	$213.65	$472.90	$123.50	$12.35
23 da.						
29 da.						
37 da.						
19 da.						
25 da.						
73 da.						
47 da.						
81 da.						
114 da.						
95 da.						
61 da.						
77 da.						
183 da.						
211 da.						

227. Interchanging Principal and Time. There is a short method which may sometimes be used when the days are not exact multiples of 6, 60, 600, or 6,000.

The interest is the same in each of the following cases:

$600.00 for 29 days at 6%, and
$ 29.00 for 600 days at 6%.

It is much easier to compute the interest in the latter case, because all that is necessary is to point off one place in the principal, giving the amount of interest immediately, $2.90.

Therefore, when the days are not exact multiples or easy fractions of 6, 60, 600, or 6,000, and when the principal *is* an exact multiple or easy fraction of one of these numbers, interchange the number of dollars and days, and proceed as before.

Problems. 1. Find the interest on $360.00 for 83 days at 6%.

SOLUTION. Interchanging, we have $ 83.00 for 360 days at 6%.
Pointing off two places, $.83 interest for 60 days.
Multiply by 6 for 360 days, and the result is $4.98.

2. Find the interest on $200.00 for 47 days at 6%.

SOLUTION. Interchanging, we have $47.00 for 200 days at 6%.
Point off one place for 600 days, $4.70.
Divide by 3 for 200 days, $1.57.

Oral Work

How would you find the interest on the following at 6%?

1. $700.00 for 71 days.
2. $150.00 for 59 days.
3. $420.00 for 113 days.
4. $30.00 for 57 days.
5. $66.00 for 171 days.
6. $1200 for 37 days.

Written Work

Compute and enter the interest at 6% on a form ruled like the following model:

	117 Da.	34 Da.	161 Da.	59 Da.	74 Da.	3 Mo. 7 Da.
$300 00						
1,200 00						
240 00						
4,200 00						
150 00						
72 00						
450 00						
54 00						
6,000 00						
5,000 00						

228. To find the interest when the rate is other than 6%.

First find the interest at 6%, then take $\frac{1}{6}$ of the result; this will give the interest at 1%. Multiply this quotient by the given rate.

The following short methods are of value in computing interest:
If the rate is 7%, add $\frac{1}{6}$ of the interest at 6%.
If the rate is 5%, subtract $\frac{1}{6}$ of the interest at 6%.
If the rate is $7\frac{1}{2}$%, add $\frac{1}{4}$ of the interest at 6%.
If the rate is $4\frac{1}{2}$%, subtract $\frac{1}{4}$ of the interest at 6%.

Oral Work

How would you find the interest at 4%? $5\frac{1}{2}$%? $6\frac{1}{2}$%? 8%? 9%?

Written Work

Rule and complete the following blank:

SIMPLE INTEREST

RATE	36 DA.	45 DA.	63 DA.	19 DA.	47 DA.	184 DA.	2 MO. 7 DA.
	$436.50	$75.80	$174.30	$66.00	$1,748.60	$73.90	$400.00
6%							
8%							
5%							
$4\frac{1}{2}$%							
$3\frac{1}{2}$%							

229. To find the interest when the time is stated in years, months, and days.

In the preceding problems the time has been stated in days and months. When the time has been stated in months, we have reduced it to days, but when the time is stated in years, months, and days, it is too laborious to reduce it to days, and the following method should be used.

Find the interest for the number of years at the given per cent. To this result add the interest for the given number of months and days.

Problem. Find the interest on $375 for 2 yr. 3 mo. 21 da. at 5%.

SOLUTION. The interest for 1 yr. is 5% of the principal.
The interest for 2 yr. is 10% of the principal.
10% of $375 = $37.50.
The interest on $375 for 3 mo. 21 da. is $5.78.
Therefore, the interest for 2 yr. 3 mo. 21 da. is $43.28.

Written Work

Rule a blank like the following model. Compute and enter the interest.

Complete the blank:

SIMPLE INTEREST

TIME			$124.00 6%	$37.62 5%	$1,245.75 3½%	$463.80 4%	$2,000.00 5½%
Years	Months	Days					
1	5	24					
1	8	12					
2	4	18					
3	8	6					
4	10	12					
10	2	7					
8	6	2					
5	7	9					
3	4	15					
2	7	28					
3	3	20					
4	7	27					
9	1	19					

230. To compute the time between two dates. In all of the problems given thus far, the time has been stated. You will often be called upon to compute the time. In such cases you will be given the date when the note was made and the date when it is due (date of maturity).

There are two methods of computing the time between dates:

First Method. Counting the actual number of days. (Not used if the interval is more than one year.)

Second Method. Compound subtraction. (May be used for any interval.)

***a.* To find the time by counting the actual number of days.**

Problem. Find the number of days between February 15, 1921, and May 24, 1921.

SOLUTION.

February has 28 days, of which 15 have expired, leaving in February	13 days
March has	31 days
April has	30 days
The note matures on May	24
Total	98 days

From this problem we derive the following rule:

Add the number of days remaining in the month after the day the note was made, the exact number of days in each complete month, and the number of days through which the note is to run in the last month.

Written Work

Find the exact number of days between the dates in the table:

Made	Due					
	Oct. 26	Nov. 13	Dec. 27	Jan. 3	Jan. 17	Feb. 26
Oct. 13						
Sept. 27						
Aug. 18						
July 16						
July 3						
May 17						
May 1						

b. **To find the interest period by compound subtraction of dates.**

Problem. Find the time between May 18, 1918, and February 16, 1921.

Solution.

	1921	2	16
	1918	5	18

Since 18 cannot be subtracted from 16, and since 5 cannot be subtracted from 2, reduce 1 month to days, and 1 year to months.

	1920	13	46
	1918	5	18
	2 yr.	8 mo.	28 da.

Written Work

Find the time by compound subtraction. The figures in the left-hand column are the dates on which the notes were given. The dates at the top are the dates of maturity.

From	To			
	Maturity 10–22–1920	Maturity 6–15–1921	Maturity 11–14–1921	Maturity 5–20–1922
6–17–1919				
8– 5–1919				
9–25–1918				
11– 6–1918				
4–19–1916				
2–26–1915				
4–30–1918				
7–14–1919				
10–11–1912				
12–31–1918				

The following table may be used in finding the exact number of days between any two dates:

For illustration, the time from Apr. 7 to Nov. 7 is found to be 214 da. From Sept. 3 to Jan. 3 is found to be 122 da. To find the time from Sept. 3 to Jan. 18, add 15 da. to 122 da.

From Any Day of	To the Same Day of the Next											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January . .	365	31	59	90	120	151	181	212	243	273	304	334
February .	334	365	28	59	89	120	150	181	212	242	273	303
March . .	306	337	365	31	61	92	122	153	184	214	245	275
April . . .	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June . . .	214	245	273	304	334	365	30	61	92	122	153	183
July . . .	184	215	243	274	304	335	365	31	62	92	123	153
August . . .	153	184	212	243	273	304	334	365	31	61	92	122
September .	122	153	181	212	242	273	303	334	365	30	61	91
October . .	92	123	151	182	212	243	273	304	335	365	31	61
November . .	61	92	120	151	181	212	242	273	304	334	365	30
December .	31	62	90	121	151	182	212	243	274	304	335	365

Review

1. In the following exercise, enter the time (in red ink [1]) on the upper of the two lines, and the interest (in black ink) on the lower line.

[1] At teacher's discretion.

If the time is less than a year, compute the actual number of days.

If the time is more than a year, find the interval by compound subtraction, and state it in years, months, and days.

From	Rate 6%			
	$216.80	$625.00	$923.40	$87.86
	To			
	9-29-1921	11-26-1921	12-24-1921	1-26-1922
9-22-1921				
6-24-1921				
4- 2-1921				
2-11-1921				
11-13-1920				
8- 6-1920				
5-22-1920				
4-25-1919				
7-13-1919				

2. Rule and complete the following, entering the time on the upper of the two lines, and the interest on the lower.

From	5%	4½%	7%	7½%
	To			
	May 26, 1921	Aug. 17, 1921	Oct. 9, 1921	Feb. 10, 1922
Principal, $219.80 April 22, 1920				
Principal, $326.00 May 17, 1920				
Principal, $80.00 September 19, 1919				
Principal, $260.00 May 14, 1919				

Written Review

Use the simplest method to find the interest on the following:

1. Interest at 6% on \$321.65 for 24 da.
2. Interest at 6% on \$523.19 for 20 da.
3. Interest at 6% on \$413.00 for 75 da.
4. Interest at 6% on \$300.00 for 57 da.
5. Interest at 5% on \$450.00 for 35 da.
6. Interest at $7\frac{1}{2}$% on \$216.80 for 42 da.
7. Interest at 4% on \$625.00 for 117 da.
8. Interest at 6% on \$80.00 for 3 yr. 8 mo. 15 da.
9. Interest at 6% on \$50.00 from March 24, 1922, to May 26, 1922.
10. Interest at 7% on \$125.00 from July 19, 1918, to January 14, 1922.

231. Accurate Interest. The methods of computing interest already explained are based on the assumption that a year contains 360 days. The slight error is ignored because of the convenience of the method. However, the United States government and some business houses prefer to use exact interest.

When we compute interest on a basis of 360 days to the year, we ignore 5 days. Five days are $\frac{5}{365}$ or $\frac{1}{73}$ of a year. When, for example, we call 180 days, $\frac{180}{360}$ or $\frac{1}{2}$ a year, we obtain a result which is $\frac{1}{73}$ too large. Since accurate interest is $\frac{1}{73}$ smaller than the interest computed by the simple methods, we can find accurate interest by the following rule:

To find accurate interest.

Find the interest by one of the common methods; diminish this result by $\frac{1}{73}$ of itself.

Problem. What is the accurate interest on \$3,500.00 for 15 days at 6%?

SOLUTION. \$35.00 is the interest for 60 days.
\$35.00 ÷ 4 = \$8.75, the interest for 15 days.
$\frac{1}{73}$ of \$8.75 = \$.12.
\$8.75 − \$.12 = \$8.63, the accurate interest.

Written Work

Find the accurate interest on the following:

1. \$584.00 for 24 days at 6%.
2. \$275.00 for 84 days at 6%.

3. $23.75 for 113 days at 5%.
4. $65.80 for 2 months 12 days at 6%.
5. $138.50 for 9 months 16 days at 7%.

232. To find the time. Situations sometimes arise in which it is necessary to determine the time, when the principal, the rate, and the interest are known.

Oral Work

The interest on $250.00 for 6 days at 6%, is $.25. $1.00 is the interest for how many days at 6%?

The interest on $1,200 for 1 day at 6% is $.20. $4.40 is the interest for what length of time at 6%?

From these problems the following rule will be understood.

To find the time when the principal, rate, and interest are known.

Find the interest on the given principal at the given rate for one day. Divide the total interest by the interest for one day to determine the number of days.

Problem. The interest on $4,500.00 at 6% for a certain time is $10.50. What is the time?

SOLUTION. The interest on $4,500.00 for 1 day at 6% is $.75.
$10.50 ÷ $.75 = 14, the number of days.

Written Work

Find the time in each of the following:

	PRINCIPAL	RATE	INTEREST	TIME
1.	$ 360.00	6%	$ 1.62	
2.	$1,500.00	6%	$ 9.75	
3.	$1,473.00	6%	$22.59	
4.	$ 279.80	6%	$ 1.91	
5.	$ 176.90	5%	$ 2.82	

233. To find the principal.

The interest on $1.00 for 72 days at 6% is $.012. $.24 is the interest on how many dollars for 72 days at 6%?

To find the principal when the rate, the time, and the interest are known.

Find the interest on $1.00 *for the given time at the given rate. Divide the given interest by the interest on* $1.00 *to determine the principal.*

Problem. The interest on a certain principal at 6% for 54 days is $2.48. What is the principal?

SOLUTION. The interest on $1.00 for 54 days at 6% is $.009.
$2.48 ÷ $.009 = 275.56, the number of dollars in the principal.

Written Work

Find the principal in each of the following:

	TIME	RATE	INTEREST	PRINCIPAL
1.	75 days	6%	$ 3.36	
2.	35 days	6%	$ 1.67	
3.	92 days	5%	$ .52	
4.	72 days	6%	$21.60	
5.	90 days	5%	$45.00	
6.	60 days	5%	$ 1.00	
7.	30 days	7%	$ 1.40	
8.	90 days	6%	$13.50	

234. To find the rate when the principal, the interest, and the time are given.

Find the interest on the given principal for the given time at 1%. Divide the total interest by the interest at 1% to find the rate.

Problem. The interest on $1,500.00 for 30 days at a certain rate is $6.25. What is the rate?

SOLUTION. The interest on $1,500.00 for 30 days at 1% is $1.25.
$6.25 ÷ $1.25 = 5, the rate per cent.

Written Work

Find the rate in each of the following:

	PRINCIPAL	TIME	INTEREST	RATE
1.	$1,375.00	42 days	$ 9.63	
2.	$ 876.25	95 days	$11.56	
3.	$ 694.75	27 days	$ 3.65	
4.	$ 940.00	2 yr. 6 mo.	$94.00	

235. Periodic Interest. Periodic interest is the simple interest on the principal, plus the simple interest on each installment of interest which was not paid when due. The interest on the principal may be due at any stated interval, usually quarterly, semi-annually, or annually.

Periodic interest is not legal in some states. Special contracts are sometimes made which provide for its payment.

Problem. What is the periodic interest on a loan of $5,000.00 at 6% for 1 yr. 6 mo., interest due quarterly?

SOLUTION. The simple interest on $5,000.00 for 1 yr. 6 mo. at 6% is $450.00.

The first interest payment of $75 is due at the end of 3 mo., and remains unpaid for	1 yr. 3 mo.
The second interest payment is due in 6 mo. and is unpaid for	1 yr.
The third interest payment is due in 9 mo. and is unpaid for	9 mo.
Fourth interest payment is unpaid for	6 mo.
Fifth interest payment is unpaid for	3 mo
The sixth interest payment is due at the maturity of the note, and is paid when due.	
The total time during which interest payments remain unpaid is	3 yr. 9 mo.

The interest on $75.00 for 3 yr. 9 mo. at 6% is $16.88

Simple interest on the principal	= $450.00
Interest on unpaid interest	= 16.88
Periodic interest	= $466.88

The amount due at the end of 1 yr. 6 mo. (principal and interest) is $5,466.88.

Written Work

Complete a form ruled like the following model. The results in the illustration are entered in the first column to illustrate how to fill in the various blanks.

	PRINCIPAL $5,000 RATE 6% TIME 1 YR. 6 MO. INT. DUE QUARTERLY	PRINCIPAL $2,000 RATE 6% TIME 2 YR. 3 MO. INT. DUE QUARTERLY	PRINCIPAL $1,200 RATE 6% TIME 4 YR. INT. DUE SEMI-ANNUALLY	PRINCIPAL $1,700 RATE 6% TIME 6 YR. 6 MO. INT. DUE SEMI-ANNUALLY	PRINCIPAL $6,500 RATE 7% TIME 1 YR. 9 MO. INT. DUE QUARTERLY	PRINCIPAL $1,500 RATE 5% TIME 2 YR. INT. DUE ANNUALLY	PRINCIPAL $3,000 RATE 8% TIME 4 YR. INT. DUE QUARTERLY
Simple interest . .	$450.00						
Amount of each interest payment .	75.00						
Sum of periods interest is unpaid .	3 yr. 9 mo.						
Interest on unpaid interest	$16.88						
Total interest . .	466.88						
Amount of note at maturity . . .	5,466.88						

CHAPTER XXV

COMPOUND INTEREST AND ITS APPLICATIONS

236. Compound Interest is based on the assumption that the simple interest earned on an investment is added to the investment so that it thereafter earns interest at the same rate as the original investment. Interest may be compounded (that is, added to the investment) annually, semi-annually, quarterly, or at the end of any other period.

Illustration. What is the compound interest on $1,200.00 at 6% for two years, compounded semi-annually?

SOLUTION. This is equivalent to a rate of 3% per period of 6 months, and there are four semi-annual periods.

Original principal	$1,200.00
Add interest on $1,200.00 @ 3%	36.00
Amount at end of first period	1,236.00
Add interest on $1,236.00 @ 3%	37.08
Amount at end of second period	1,273.08
Add interest on $1,273.08 @ 3%	38.19
Amount at end of third period	1,311.27
Add interest on $1,311.27 @ 3%	39.34
Amount at end of fourth period	1,350.61
Deduct original principal	1,200.00
Compound interest	150.61

237. Compound Interest Tables. Compound interest is usually computed with the aid of tables similar to the following:

Illustration. Using the table on page 274, compute the compound interest on $1,200.00 at 6% for two years, compounded semi-annually.

SOLUTION.

The table shows the amount of $1.00 for four periods at 3% to be	$1.125509
Deduct	1.000000
Compound interest on $1.00	.125509
Multiply by	1200.00
Compound interest on $1,200.00	$ 150.61

Compound Interest Table

Amounts of $1.00 compounded at various rates

Periods	1%	1½%	2%	2½%	3%	3½%	4%	4½%	5%	6%	Periods
1	0.010000	1.005000	1.020000	1.025000	1.030000	1.035000	1.040000	1.045000	1.050000	1.060000	1
2	1.020100	1.030225	1.040400	1.050625	1.060900	1.071225	1.081600	1.092025	1.102500	1.123600	2
3	1.030301	1.045678	1.061208	1.076891	1.092727	1.108718	1.124864	1.141166	1.157625	1.191016	3
4	1.040604	1.061364	1.082432	1.103813	1.125509	1.147523	1.169859	1.192519	1.215506	1.262477	4
5	1.051010	1.077284	1.104081	1.131408	1.159274	1.187686	1.216653	1.246182	1.276282	1.338226	5
6	1.061520	1.093443	1.126162	1.159693	1.194052	1.229255	1.265319	1.302260	1.340096	1.418519	6
7	1.072135	1.109845	1.148686	1.188686	1.229874	1.272279	1.316932	1.360862	1.407100	1.503630	7
8	1.082857	1.126493	1.171659	1.218403	1.266770	1.316809	1.368569	1.422101	1.477455	1.593848	8
9	1.093685	1.143390	1.195093	1.248863	1.304773	1.362897	1.423312	1.486095	1.551328	1.689479	9
10	1.104622	1.160541	1.218994	1.280085	1.343916	1.410599	1.480244	1.552969	1.628895	1.790848	10
11	1.115668	1.177949	1.243374	1.312087	1.384234	1.459970	1.539454	1.622853	1.710339	1.898299	11
12	1.126825	1.195618	1.268242	1.344889	1.425761	1.511069	1.601032	1.695881	1.795856	2.012197	12
13	1.138093	1.213552	1.293607	1.378511	1.468534	1.563956	1.665074	1.772196	1.885649	2.132928	13
14	1.149474	1.231756	1.319479	1.412974	1.512590	1.618695	1.731676	1.851945	1.979932	2.260904	14
15	1.160969	1.250232	1.345868	1.448298	1.557967	1.675349	1.800944	1.935282	2.078928	2.396558	15
16	1.172579	1.268986	1.372786	1.484506	1.604706	1.733986	1.872981	2.022370	2.182875	2.540352	16
17	1.184304	1.288020	1.400241	1.521618	1.652848	1.794676	1.947901	2.113377	2.292018	2.692773	17
18	1.196148	1.307341	1.428246	1.559659	1.702433	1.857489	2.025817	2.208479	2.406619	2.854339	18
19	1.208109	1.326951	1.456811	1.597650	1.753506	1.922501	2.106849	2.307860	2.526950	3.025600	19
20	1.220190	1.346855	1.485947	1.638616	1.806111	1.989789	2.191123	2.411714	2.653298	3.207136	20
21	1.232392	1.367058	1.515666	1.679582	1.860295	2.059431	2.278768	2.520241	2.785963	3.399564	21
22	1.244716	1.387564	1.545980	1.721571	1.916103	2.131512	2.369919	2.633652	2.925261	3.603537	22
23	1.257163	1.408377	1.576899	1.764611	1.973587	2.206114	2.464716	2.752166	3.071524	3.819750	23
24	1.269735	1.429503	1.608437	1.808726	2.032794	2.283328	2.563304	2.876014	3.225100	4.048935	24
25	1.282432	1.450945	1.640606	1.853944	2.093778	2.363245	2.665836	3.005434	3.386355	4.291871	25

238. Present Value of $1.00. The present value of $1.00 is a sum smaller than $1.00 which, invested at compound interest, will amount to $1.00 at the end of a certain number of periods.

The present value of $1.00 is computed by dividing $1.00 by the amount of $1.00 at the given rate for the given number of periods, as shown in the table.

Illustration. What is the present value of $1.00 due in two years, interest at 6% per annum compounded semi-annually?

SOLUTION. The amount of $1.00 for 4 periods at 3% (see table) is $1.125509.
Then $1.00 ÷ 1.125509 = $.888487 Present value of $1.00

Example. What is the present value of $1,500.00 due in two years, interest at 6% per annum, compounded semi-annually?

SOLUTION.

Present value of $1.00 (as above)	$ .888487
Multiply by	1500
Present value of $1,500.00	$1332.73
	$1332.72

The accumulation of interest by which $1,322.72 increases to $1,500.00 may be tabulated as follows:

TABLE OF ACCUMULATION

Original investment	$1,332.73
Add interest on $1,332.73 @ 3%	39.98
Total, end of first period	1,372.71
Add interest on $1,372.71 @ 3%	41.18
Total, end of second period	1,413.89
Add interest on $1,413.89 @ 3%	42.42
Total, end of third period	1,456.31
Add interest on $1,456.31 @ 3%	43.69
Total, end of fourth period	1,500.00

239. Compound Discount. The compound discount on $1.00 is the difference between $1.00 and the present value of $1.00.

Illustration. What is the compound discount on $1.00 due in two years, interest at 6% per annum, compounded semi-annually?

SOLUTION.

	1.000000
Deduct present value of $1.00 (as above)	.888487
Compound discount	.111513

Written Work

1. State the total amount accumulated at the end of five years, and the compound interest earned, on \$5,000.00 invested at

(*a*) 6% per annum, compounded annually.
(*b*) 6% per annum, compounded semi-annually.
(*c*) 6% per annum, compounded quarterly.

2. What is the present value of \$5,000.00 due at the end of five years,

(*a*) at 6% per annum, compounded annually?
(*b*) at 6% per annum, compounded semi-annually?
(*c*) at 6% per annum, compounded quarterly?

CHAPTER XXVI

CONTRACT PURCHASES AND INSTALLMENT PAYMENTS

REAL estate, furniture, musical instruments, automobiles, and other forms of property are often sold on installments due at monthly or other intervals. The contract usually specifies definite amounts which must be paid, and permits the debtor to pay larger amounts. The purchaser also pays interest on the principal.

240. Long End Interest is interest on the balance of the principal unpaid between the dates of successive installments. The contract may require:

(*a*) An equal periodical payment on the principal in addition to the long end interest. To illustrate:

On May 1, 1921, C purchased a piano from D for $1,200.00, paying $200.00 down. The balance was to be paid in monthly installments of $25.00. In addition, long end interest was to be paid monthly at the rate of 6% per annum. A record of the payments might be kept in the following manner:

DATE	MONTHLY PAYMENTS			BALANCE OF PRINCIPAL
	Principal	Interest	Total	
June 1, 1921				$ 1,000.00
July 1, 1921	$25.00	$5.00	$30.00	975.00
Aug. 1, 1921	25.00	4.88	29.88	950.00
Sept. 1, 1921	25.00	4.75	29.75	925.00

(*b*) An equal periodical payment which shall be applied first to the payment of long end interest and the balance to principal, as follows:

Date	Payment	Accrued Interest	Balance of Payment to Apply on Principal	Principal
June 1, 1921				$1,000 00
July 1, 1921	$25 00	$5 00	$20 00	980 00
Aug. 1, 1921	25 00	4 90	20 10	959 90
Sept. 1, 1921	25 00	4 80	20 20	939 70

241. Short End Interest is interest on the installment being paid, computed from the date of the purchase until the date of paying the installment. To illustrate, assume that in the preceding illustration C agrees to pay $25.00 monthly with short end interest at the rate of 6% per annum, as follows:

Date	Monthly Payments			Balance of Principal
	Principal	Interest	Total	
June 1, 1921				$1,000.00
July 1, 1921	$25.00	$0.13	$25.13	975.00
Aug. 1, 1921	25.00	.25	25.25	950.00
Sept. 1, 1921	25.00	.38	25.38	925.00

Written Work

1. Mr. Adams buys a phonograph from Z on April 1, 1922, for $300.00 to be paid for in monthly installments with interest at 6%. Prepare a table showing the monthly payments if the installments are to be $25.00 plus long end interest.

2. Prepare a table showing the payments if the installments are to be $25.00 monthly, long end interest being deducted from the installments, the balance being applied on the principal.

3. Prepare a table showing the payments if the installments are $25.00 with short end interest.

4. Compare the total payments including principal and interest in the first three problems.

5. On May 1, 1921, G. A. Sanders sold a house and lot to A. R. Crain; purchase price, $4,850.00. Mr. Crain paid $2,900.00 cash and gave the following note for the balance.

$1,950.00 BARTLETT, IOWA, MAY 1, 1921

FOR VALUE RECEIVED, I promise to pay to the order of G. A. Sanders, the sum of Nineteen Hundred and Fifty Dollars, with interest thereon at the rate of six per cent per annum, principal and interest payable in monthly installments in the sum of twenty-five dollars or more, each, the first of which is due on June 1, 1921, and one on the first day of each and every month thereafter until said principal sum of Nineteen Hundred and Fifty Dollars and interest thereon are fully paid.

Each and every of said installments is to be applied as follows:

1st. In payment of accrued interest on said sum of $1,950.00 or on the unpaid portion thereof.

2d. In monthly reduction of said principal sum of $1,950.00.

A. R. CRAIN.

Payments were made as follows:

June 1, 1921	$25.00	January 2, 1922	$40.00
July 1, 1921	25.00	February 1, 1922	25.00
August 1, 1921	30.00	March 1, 1922	32.50
September 1, 1921	25.00	April 1, 1922	25.00
October 1, 1921	27.50	May 1, 1922	45.00
November 1, 1921	35.00	June 1, 1922	25.00
December 1, 1921	25.00		

Notice that some payments are made on the second day of the month, due to the fact that the first day of the month was either Sunday or a holiday.

Prepare a statement showing a record of the payments made.

6. Assume that the contract between Mr. Sanders and Mr. Crain called for a monthly payment of $25.00 or more, and that the principal was to be decreased semi-annually by amounts computed as follows: The payments for six months shall be added and from their sum shall be deducted six months' interest on the balance of the principal unpaid at the beginning of the semi-annual period. Payments were made as indicated above. Prepare a statement showing the reduction of the debt.

7. Which of the two propositions is better for Mr. Crain? Explain why.

CHAPTER XXVII

SAVINGS BANKS

242. Checking and Savings Accounts. Most banks receive deposits under two classes of accounts, checking accounts and savings accounts.

Money deposited in checking accounts can be drawn out by checks payable to the depositor or to any other party. These checks are payable on presentation at the bank. The cash on deposit usually does not bear interest.

Money deposited in a savings account cannot be drawn out by a check payable to any person other than the depositor himself. The law usually provides that the bank may require notice of from 10 to 60 days before paying money from a savings account. Banks rarely take advantage of this privilege, however. The cash on deposit bears interest at some rate fixed by the bank. 3%, 3½%, and 4% are common rates.

243. Computing Interest on Savings Accounts. There is no uniform procedure for computing interest on deposits in savings accounts, as each bank adopts its own rules. The following are the rules of a Chicago bank:

"On the first days of January and July in each year, interest shall accrue at the rate of three per cent per annum on all savings deposits then in the Bank, and which have remained on deposit for one month or more; but no interest shall be payable for periods less than a month (except as herein otherwise provided), nor upon sums less than one dollar, nor for any part of the half year included between the first days of January and July, or July and January, as the case may be, on sums withdrawn between those periods.

"Interest shall not be paid on average balances; and all withdrawals between the interest days shall be deducted from the first deposits.

"On deposits made during the first ten days of any calendar month, interest will be allowed from the first day of that month. On all deposits made after the first ten days of any calendar month, interest will be allowed from the first day of the succeeding month."

Problem. Following is a transcript of the pass book showing the savings account of W. M. Scott.

W. M. Scott.....*Book No.* 35836

In Account with *The Snowden Trust and Savings Bank* of Chicago.

Date		Withdrawals	Deposits	Balance
Jan. 3, 1917			$100.00	$100.00
July 1, 1917	Interest		1.50	101.50
July 10, 1917			50.00	151.50
Sept. 8, 1917			40.00	191.50
Dec. 15, 1917			60.00	251.50
Jan. 1, 1918	Interest		2.66	254.16
Feb. 8, 1918		$75.00		179.16
July 1, 1918	Interest		2.68	181.84
Aug. 5, 1918			60.00	241.84
Sept. 19, 1918			70.00	311.84
Dec. 21, 1918		85.00		226.84
Jan. 1, 1919	Interest		2.71	229.55
Feb. 3, 1919			50.00	279.55
Mar. 6, 1919			100.00	379.55
May 19, 1919		125.00		254.55
June 24, 1919			167.91	422.46
July 1, 1919	Interest		3.18	425.64
July 20, 1919		75.64		350.00
July 24, 1919		100.00		250.00
July 31, 1919		100.00		150.00
Aug. 29, 1919		100.00		50.00
Sept. 6, 1919		49.00		1.00
Oct. 16, 1919			301.75	302.75
Oct. 18, 1919		50.00		252.75
Oct. 25, 1919		156.00		96.75
Dec. 14, 1919			48.00	144.75
Dec. 26, 1919			105.25	250.00
Jan. 2, 1920			100.00	350.00
Jan. 7, 1920			75.00	425.00
Jan. 1, 1920	Interest		.48	425.48

Explanation of Interest Computations

July 1, 1917. Since the $100.00 deposit was made during the first ten days of January, it bears interest from the first of January.

Interest on $100.00 for 6 months $1.50

January 1, 1918. The $50.00 deposit of July 10th bears interest from July 1st, making the balance as of that date $151.50.

Interest on $151.00 for 6 months	2.26
The September 8th deposit bears interest from September 1st.	
Interest on $40.00 for 4 months	.40
The December 15th deposit does not begin to bear interest until January 1st.	
Total	$2.66

July 1, 1918. As all withdrawals are deducted from the first deposits, the $75.00 withdrawal at February 8th is deducted from the January 1st balance, reducing the balance as of that date to $179.16.

Interest on $179.00 for 6 months	2.68

January 1, 1919. The $85.00 withdrawal on December 21st reduces the balance as of July 1st to $96.84.

Interest on $96.00 for 6 months	1.44
Interest on 60.00 for for 5 months	.75
Interest on 70.00 for 3 months	.52
Total	$2.71

July 1, 1919. The $125.00 withdrawal on May 19th reduces the balance as of January 1st to $104.55.

Interest on $104.00 for 6 months	1.56
Interest on $50.00 for 5 months	.62
Interest on $100.00 for 4 months	1.00
Total	$3.18

January 1, 1920. The withdrawals during the six months' period total $630.64. These exhaust the $425.64 balance at July 1st, and also cause a deduction of $205.00 from the deposit of October 16th. $301.75 minus $205.00 equals $96.75. Since this deposit was made after the first ten days of October, it does not begin to draw interest until November 1st.

Interest on $96.00 for 2 months	$0.48

The December 14th and December 26th deposits do not begin to bear interest until January 1.

It will be noted that the January 1st interest is entered in the pass book after January 7th. This may seem incorrect, but it is caused by the operation of the following rule of the bank:

"Interest will be credited on the books of the Bank on the first days of January and July, but will not be entered upon the depositor's pass book nor be subject to withdrawal until on or after the tenth day following such credit."

Written Work

1. Following is a statement of the deposits and withdrawals of J. O. Dorman in the First Trust and Savings Bank, which pays 4% interest according to the rules quoted above:

Date	Withdrawals	Deposits
January 6, 1920		$200.00
July 10, 1920		100.00
August 25, 1920		50.00
December 17, 1920		75.00
January 6, 1921		40.00
February 3, 1921	$125.00	
March 7, 1921		105.50
April 12, 1921		60.00
July 5, 1921	200.00	
September 7, 1921		50.00
December 10, 1921	75.00	
January 16, 1922		500.00
March 30, 1922	375.00	
May 3, 1922		105.45
May 13, 1922		155.20

Write up his pass book showing interest credits.

2. Following is a continuation of the pass book of W. M. Scott (see example). Compute interest credits and show balances.

Date	Withdrawals	Deposits
February 1, 1920		$118.75
February 7, 1920		40.00
February 28, 1920	$300.00	
March 13, 1920	84.23	
April 2, 1920		158.75
April 8, 1920		100.59
April 12, 1920	25.00	
April 24, 1920	433.00	
May 14, 1920		101.25
May 17, 1920		118.75
May 19, 1920	50.00	
May 29, 1920	25.00	
May 31, 1920	25.00	
June 10, 1920	75.00	
July 11, 1920		95.50
July 17, 1920		100.00
August 2, 1920		118.75

September 2, 1920		499.85
September 6, 1920	25.00	
September 11, 1920	25.00	
September 13, 1920	25.00	
October 4, 1920		114.45
October 14, 1920	800.00	
March 16, 1921	75.00	
June 2, 1921		150.00
July 10, 1921		498.87
August 30, 1921	200.00	
September 10, 1921	225.00	
September 16, 1921	75.00	

No further deposits or withdrawals were made until July 10, 1922, when the entire balance was withdrawn.

244. Postal Savings Banks. The United States Government has provided a savings bank system operated in conjunction with the postal service, whereby savings may be deposited at interest with the security of the United States Government.

Deposits. Any person ten years of age or over may become a depositor. Sums less than $1.00 cannot be deposited. No person can deposit more than $100.00 in any one calendar month, or have a balance at any time of more than $500.00, exclusive of interest. Depositors receive a postal savings certificate for the amount of each deposit. Interest is allowed on these certificates at the rate of two per cent for each *full year* that the money remains on deposit, *beginning with the first day of the month following the one in which it was deposited.*

Withdrawals. Money may be withdrawn by surrendering to the officer where the deposit was made the savings certificates covering the desired amount.

Bonds. Under certain conditions a depositor may surrender certificates in amounts of $20.00 or any multiple of $20.00, up to and including $500.00 and receive in return government bonds bearing interest at 2½% per year. These postal savings bonds may be held in addition to the $500.00 deposit allowed to one depositor.

Written Work

1. How much interest would a man receive if he deposited $15.00 in a postal savings bank on March 1, 1921, and withdrew it three months later?

2. How much interest would he receive if he withdrew the money on March 11, 1922?

3. A man made the following deposits in a postal savings bank:

May 20, 1922	$15.00
June 17, 1922	6.00
August 11, 1922	26.00
October 30, 1922	17.00

When would he be entitled to the full year's interest on each of his deposits, and how much interest would he receive?

CHAPTER XXVIII

PARTIAL PAYMENTS

PART payment of an interest-bearing debt is known as a "partial payment."

One of the following methods is usually employed in computing the balance remaining after partial payments have been made.

245. The United States Rule. The United States Rule has been approved by the Supreme Court of the United States, and is usually employed when the time of the interest-bearing debt exceeds one year. This method provides that:

a. Payments must be applied to pay accrued interest before any deduction can be made from the principal.

b. Payments which do not pay the accrued interest leave the principal undiminished until other payments are made which are sufficient to cover *all* accrued interest.

c. Any surplus remaining after accrued interest is paid in full is applied toward the reduction of the principal.

Partial payments may be made only by agreement between the maker and the payee. This agreement may appear in the note.

$1800.00 New York, January 16 1919

Three years — after date I promise to pay to the order of Allan J Kilmer

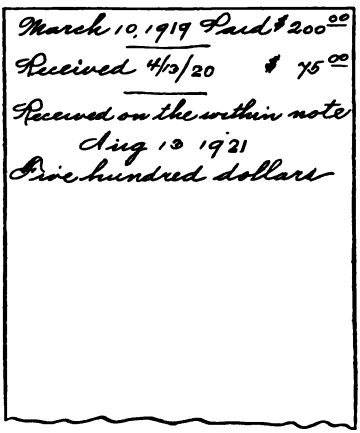
Eighteen Hundred no/100 — Dollars

at his office, with interest @ 6%

Value received

No 8 Due Jan 16, 1922 — D O White

The above note had the following indorsements, showing payments made with the consent of the payee.



Problem. What amount was necessary to pay the note and interest at maturity?

SOLUTION. Time from January 16, 1919, to March 10, 1919, 53 days.

Interest on $1,800.00 for 53 days is $15.90.

$200.00 (first payment) – $15.90 (accrued interest) = $184.10, surplus to apply on principal.

$1,800.00 – $184.10 = $1,615.90, unpaid principal.

Time from March 10, 1919, to April 13, 1920, is 1 yr. 1 mo. 3 da.

Interest on $1,615.90 for 1 yr. 1 mo. 3 da. is $105.84.

The payment of $75.00 does not cover the accrued interest, and the principal therefore remains undiminished.

Time from April 13, 1920, to August 13, 1921, is 1 yr. 4 mo.

Interest on $1,615.90 for 1 yr. 4 mo. is $129.27.

$105.84 + $129.27 = $235.11, total accrued interest.

$75.00 + $500.00 = $575.00, total payments.

$575.00 – $235.11 = $339.89, surplus remaining to apply on principal.

$1,615.90 – $339.89 = $1,276.01, unpaid principal.

Time from August 13, 1921, to January 16, 1922, is 5 mo. 3 da.

Interest on $1,276.01 for 5 mo. 3 da. is $32.54.

$1,276.01 + $32.54 = $1,308.55, amount required to pay principal and accrued interest at maturity.

The work may be arranged in tabular form, as shown by the following illustration:

PARTIAL PAYMENTS ON A NOTE COMPUTED BY THE UNITED STATES RULE

DATE	ACCRUED INTEREST					PAYMENT		SURPLUS TO APPLY ON PRINCIPAL		PRINCIPAL	
	Time			Amount							
	Yr.	Mo.	Da.								
Of Note, 1919–1–16										1,800	00
1st Payment, 1919–3–10		1	23	15	90	200	00	184	10	1,615	90
2d Payment, 1920–4–13	1	1	3	105	84	75	00				
3d Payment, 1921–8–13	1	4	0	129	27	500	00				
				235	11	575	00	339	89	1,276	01
Maturity, 1922–1–16		5	3	32	54					1,308	55
										Amount due at maturity	

246. The Merchants' Rule. The Merchants' Rule is generally used by banks and by business men to find the balance due when partial payments are made on interest-bearing notes in which the time is one year or less. It provides that:

a. The principal shall draw interest from the date the loan is made until the date of the final settlement.

b. Each payment shall draw interest from the date the payment is made until the final settlement.

Problem. On a note of $500.00, dated July 8, 1920, payable in one year with interest at 6%, the following payments were made: Sept. 15, 1920, $200.00; Jan. 8, 1921, $150.00; May 8, 1921, $60.00. What amount was necessary to make final settlement on July 8, 1921?

SOLUTION. Principal		$500.00	
Int. on $500.00 or 1 yr. at 6%		30.00	
Total			$530.00
First payment	$200.00		
Int. on first payment from 9–15–20 to 7–8–21	9.78		
Total		$209.78	
Second payment	$150.00		
Int. on second payment from 1–8–21 to 7–8–21	4.50		
Total		154.50	

Third payment	$60.00		
Int. on third payment from 5–8–21 to 7–8–21	.60		
Total		$60.60	
Sum of payments and interest			424.88
Balance due July 8, 1921			$105.12

This work may be arranged in tabular form as shown in the following illustration:

TABULAR FORM FOR INDICATING RESULTS OF PARTIAL PAYMENTS COMPUTED BY THE MERCHANTS' RULE

DATES		LOAN						PARTIAL PAYMENTS							
		Principal		Interest		Amount		Payment		Interest				Amount of Payment and Int.	
										Time mo.	Time da.	Dollars			
Of note	7–8–1920	500	00												
Maturity	7–8–1921			30	00	530	00								
First payment	9–15–1920							200	00	9	25	9	78	209	78
Second payment	1–8–1921							150	00	6	0	4	50	154	50
Third payment	5–8–1921							60	00	2	0		60	60	60
														424	88
								Bal.	due	at final settl		m't		105	12
						530	00							530	00

Written Work

The first two problems, being for long-time loans, should be computed by the United States Rule. Prepare a ruled form similar to the model and show your results thereon. In each problem, find the balance due at maturity.

1. Principal, $4,000.00.
Date of paper, November 16, 1918.
Time, 4 yr.
Rate, 6%.

Payments:

Jan. 20, 1919	$ 75.00
Sept. 26, 1919	125.00
April 5, 1920	500.00
Oct. 21, 1920	100.00
July 10, 1921	250.00
Oct. 21, 1921	1,500.00

2. Principal, $1,200.00.
Date of paper, October 29, 1919.
Time, 2 yr. 6 mo.
Rate, 6%.

Payments:

Jan. 16, 1920	$ 60.00
Nov. 6, 1920	25.00
March 12, 1921	165.00
July 10, 1921	15.00
Sept. 28, 1921	150.00
Dec. 3, 1921	275.00

The following problems, being for short-time loans, should be computed by the Merchants' Rule. Rule a form similar to the model.

3. Principal, $600.00.
Date of paper, September 15, 1922.
Time, 1 yr.
Rate, 6%.

Payments:

Oct. 20, 1921	$ 75.00
Nov. 11, 1921	60.00
Feb. 16, 1922	115.00

4. Principal, $1,000.00.
Date of paper, March 19, 1921.
Time, 6 mo.
Rate, 5%.

Payments

April 21	$ 65.00
May 27	125.00
June 24	180.00
Aug. 19	200.00

CHAPTER XXIX

DISCOUNTING NOTES AND OTHER COMMERCIAL PAPER

247 Borrowing from a Bank. Banks prefer to have the borrower pay interest in advance. The interest may be paid in cash, or the bank may subtract it from the face of the note, giving the borrower the difference, which is called the **proceeds.**

If you gave a bank the following note:

$100.00 CHICAGO, ILL., March, 2, 1922.

Ninety days after date, I promise to pay to the order of the Bowen National Bank One Hundred Dollars.

.....

Your Signature

the bank would deduct ninety days' interest, $1.50, and you would receive the proceeds, $98.50.

NOTE. There is no promise to pay interest, because the interest is prepaid. Banks may charge any rate they choose, within the limits set by the state law. In this text, when no rate is mentioned use 6 %.

The above illustration shows the method of **discounting a note.**

248. Terms Used in Bank Discount. The following terms are used frequently, and you should become familiar with them:

Maturity means the date on which the note is due.

The **value** of the note is the amount due at maturity.

If the note draws interest, the *value* is the sum of the principal and the interest. Otherwise, the *value* is the face of the note.

The **discount period** is the exact number of days between the date of discounting the paper and its maturity.

The **bank discount** is the simple interest on the value for the discount period.

The difference between the value and the bank discount is called the *proceeds.*

If the proceeds of a discounted note exceed the face, the excess is interest; if the proceeds are less than the face, the difference is discount.

Oral Work

In the illustration on the previous page what are each of the following:

The value? The maturity? The discount period?
The bank discount? The proceeds?

249. To find the bank discount and the proceeds.

To compute the bank discount, compute the simple interest on the value of the note for the discount period.

To find the proceeds, subtract the bank discount from the value of the note.

Written Work

How much would you receive from the bank if you discounted the following notes?

1. A note for \$500, without interest, due in 30 days, discounted at 6%.

2. A note for \$725, without interest, due in two months, discounted at 6%.

3. A note for \$85.50, without interest, due in 90 days, discounted at 7%.

4. The following blank shows the value of several notes and the periods for which they were discounted. Compute the bank discount at 6%, and find the proceeds.

Note Due in		Value						
		\$87.00	\$265.00	\$962.50	\$1,000.00	\$275.00	\$765.00	\$35.00
30 days	Bank discount Proceeds	.44 86.56						
60 days	Bank discount Proceeds							
80 days	Bank discount Proceeds							
90 days	Bank discount Proceeds							
120 days	Bank discount Proceeds							

250. Discounting the Paper of Other Persons. If you have a note signed by some person of good financial standing, you can borrow money on it by discounting it in the same way that you would discount a note made by yourself.

Suppose John Doe had given you the following note:

$500.00 CHICAGO, ILL., January 18, 1922.

Ninety days after date I promise to pay to the order of (Your Name) Five Hundred Dollars.

JOHN DOE.

By the following indorsement,

Pay to the order of
The First National Bank
(Your Name)

you transfer the note to the bank. Moreover you promise to pay the bank $500 on April 18, in case Doe fails to do so. The bank will loan you $500 minus the discount.

When you discounted your own note (in the exercise just given), you did so on the same day that the note was made. The discount period and the time of the note were therefore the same. But you may keep the notes of other people some time before discounting them. The bank will charge you interest from the day you discount a note until it is due.

Thus, if you had not discounted the above note until March 6, the bank would have charged you interest for the number of days from March 6 to April 18, the date of maturity. How many days in this discount period?

What would be the discount period if you discounted the note January 24? February 16? March 21?

251. To find the discount period.

Find the maturity of the note.

Find the exact number of days between the date of discount and the date of maturity.

252. To find the maturity. Notes due a certain number of *months* after date fall due on the same day of the month as the day on which they were made, with the following exceptions:

Notes made on the 31st, maturing after a specified number of months, falling due in a month having only 30 days, mature on the 30th.

Notes made on the 28th, 29th, 30th, or 31st, maturing after a specified number of months, falling due in February, mature on the last day of February.

Illustrations.

Note Made	Due in	Maturity
Jan. 17, 1922	2 months	March 17, 1922
August 31, 1922	3 months	November 30, 1922
Dec. 31, 1921	2 months	February 28, 1922

Notes due a certain number of *days* after date, mature after the exact number of days has elapsed.

Illustrations.

	Note Made	Due in	Maturity
	1921		
a.	Feb. 16	30 days	March 18

Since February has only 28 days, the 30 days of the note will be divided as follows:

12 February days (after the 16th)
18 March days

b. March 18 — 30 days — April 17

March has 31 days; 13 left after the 18th; the remaining 17 of the 30 days must be in April.

c. April 17 — 30 days — May 17

April has 30 days; 13 left after the 17th; the remaining 17 of the 30 days will be in May.

From these illustrations, the following should be clear:

To find the maturity:

Change the days to months; (30 *days* = 1 *month*).

Assume that the note will mature on the same day of some following month.

Correct this result by subtracting 1 *day for each month of* 31 *days through which the note runs; and by adding* 2 *days if the note runs through February.* (*In case of leap year, add* 1 *day.*)

The following table provides a convenient method for indicating the months for which corrections are to be made.

Month	Number of Month		Days	Correction
January	1	13	31	− 1
February	2	14	28	+ 2 (or + 1)
March	3	15	31	− 1
April	4	16	30	
May	5	17	31	− 1
June	6	18	30	
July	7	19	31	− 1
August	8	20	31	− 1
September	9	21	30	
October	10	22	31	− 1
November	11	23	30	
December	12	24	31	− 1

Problems. 1. Find the maturity of a note dated May 16, 1922, due in 4 months.

Solution. May is the fifth month. Add 4 months. The note will be due in the ninth month, which by the table is September.

Maturity, September 16.

2. Find the maturity of a note of the same date, due in 120 days.

Solution. Call 120 days 4 months.

May is the fifth month. The note is due in the ninth month, shown by the table to be September.

Call the maturity September 16. Correct as follows:

Note runs through May	Subtract 1 day
Note runs through July	Subtract 1 day
Note runs through August	Subtract 1 day
Total	3 days

Therefore the maturity is September 13.

3. Find the maturity of a note dated December 19, 1921, due in 150 days.

Solution. Call 150 days 5 months.

December is the twelfth month. The note is due in the seventeenth month, which, by the table, is May.

Call the maturity May 19, 1922.

Correct as follows:

December	− 1
January	− 1
February	+ 2
March	− 1
April (has 30 da.)	
Note does not run through May.	
Total	− 1

Maturity, May 18, 1922.

4. Find the maturity of a note dated June 16, 1922, due in 70 days.

SOLUTION. Call 70 days 2 months, 10 days.
June 16 plus two months is August 16; plus 10 days is August 26.
To correct:

For June	
For July	− 1
Total	− 1

Maturity, August 25, 1922.

Written Work

Enter the maturity of each note in the proper column:

NOTE DATED	DUE IN 1 MONTH	DUE IN 30 DAYS	DUE IN 2 MONTHS	DUE IN 60 DAYS	DUE IN 150 DAYS	DUE IN 45 DAYS
Sept. 23, 1922						
Oct. 10, 1922						
May 2, 1922						
July 20, 1922						
Dec. 15, 1922						
Feb. 9, 1922						

253. To find the discount period. *Compute the actual number of days between the date of discount and the date of maturity.*

Illustration. A note, dated July 17, due in 4 months, was discounted on August 5. What was the discount period?

SOLUTION. Four months from July 17 is November 17, the date of maturity. The discount period, therefore, extends from August 5, the date of discount, to November 17, the date of maturity.

To find the number of days between August 5 and November 17:

31	total number of days in August
5	number of August days expired before discounting
26	number of August days in the discount period
30	number of September days in the discount period
31	number of October days in the discount period
17	number of November days in the discount period
104	total number of days in the discount period

Oral Work

After studying the illustration, state a method for finding the number of days in the discount period.

Written Work

1. On a form ruled like the following, enter the date of maturity on the upper line, and the discount period on the lower line.

The notes dated February 14 were discounted April 8.
The notes dated February 17 were discounted March 2.
The notes dated March 23 were discounted April 29.
The notes dated March 18 were discounted April 15.
The notes dated March 30 were discounted May 20.

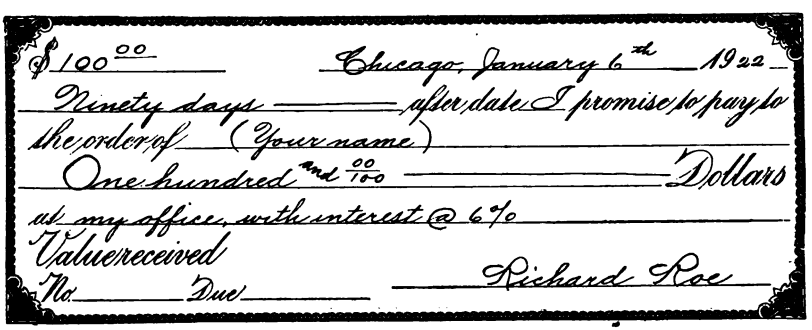
Dates Notes were Made	Time				
	60 Days	75 Days	100 Days	120 Days	6 Months
Feb. 14, 1922					
Feb. 17, 1922					
Mar. 23, 1922					
Mar. 18, 1922					
Mar. 30, 1922					

2. Complete a table ruled like the following model. Enter the date of maturity on the first line, the discount period on the second line, the

discount on the third line, the proceeds on the fourth line, as illustrated by the first problem in the table.

Date of Paper and Face	Time, 90 Days	Time, 3 Months	Time, 120 Days	Time, 6 Months
	Discounted April 13	Discounted April 11	Discounted April 24	Discounted May 20
Jan. 17, 1922 $300	April 16			
	3 days			
	$.15			
	$299.85			
Jan. 31, 1922 $285				
Feb. 20, 1922 $126				

254. Discounting Interest-bearing Notes. If you had in your possession the following note and wished to discount it, you would first add 90 days' interest ($1.50) to the face of the note to determine its *value* at maturity. This note is really Roe's promise to pay you $101.50 at maturity, and the bank will give you as much for it as for a note for $101.50 without interest.


$100.00 Chicago, January 6th 1922

Ninety days after date I promise to pay to the order of (Your name)

One hundred and 00/100 Dollars

at my office, with interest @ 6%

Value received

No. Due Richard Roe

When discounting interest-bearing paper, compute the interest for the *full time* the note is to run, and add this interest to the face, to de-

termine the *value* at maturity. Find the discount on this value. Subtract the discount from the *value* to determine the proceeds.

Written Work

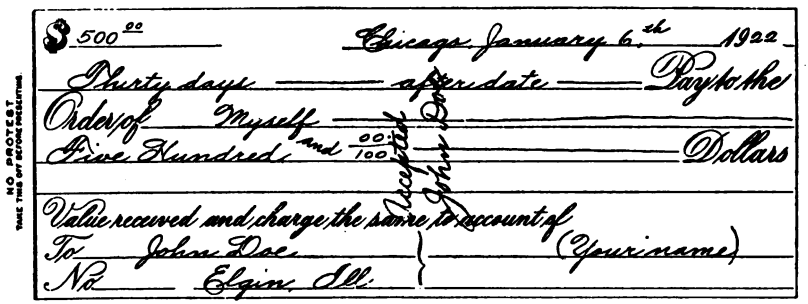
Complete a table ruled like the following model, entering the facts as illustrated in the first problem.

All notes discounted at 6%.

		Time, 90 Days		Time, 4 Months		Time, 6 Months	
		Without Int.	With Int. 6%	Without Int.	With Int. 6%	Without Int.	With Int. 5%
Principal, $500.00	Maturity	Sept. 1	Sept. 1				
	Total Interest		$ 7.50				
Date of Paper, June 3, 1921	Value	$500.00	$507.50				
	Dis. Period	11 days	11 days				
Date of Discount Aug. 21, 1921	Bank Dis.	$ .92	$ .93				
	Proceeds	$499.08	$506.57				
	Net Dis.* or Int.	.92*	6.57				
Principal, $1,250.00	Maturity						
	Total Interest						
Date of Paper, Aug. 6, 1921	Value						
	Dis. Period						
Date of Discount Sept. 20, 1921	Bank Dis.						
	Proceeds						
	Net Dis.* or Int.						
Principal, $47.96	Maturity						
	Total Interest						
Date of Paper, July 16, 1921	Value						
	Dis. Period						
Date of Discount Aug. 27, 1921	Bank Dis.						
	Proceeds						
	Net Dis.* or Int.						
Principal, $55.00	Maturity						
	Total Interest						
Date of Paper, Aug. 5, 1921	Value						
	Dis. Period						
Date of Discount Oct. 5, 1921	Bank Dis.						
	Proceeds						
	Net Dis.* or Int.						

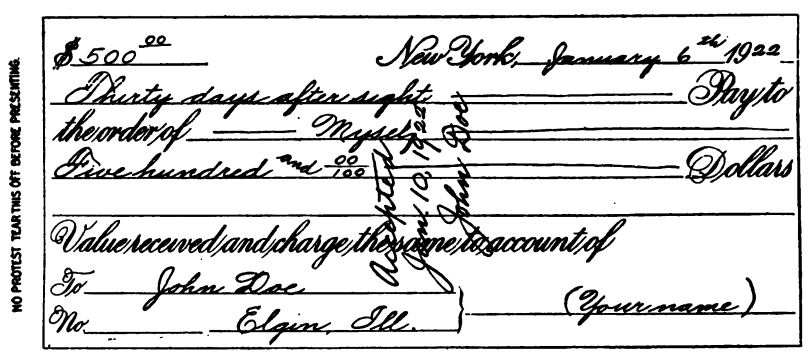
255. Discounting Acceptances. Time drafts were discussed in Chapter XIX. An accepted draft is a written promise of the acceptor to pay money at a definite date, and accepted drafts may be discounted, just as notes may be discounted. In case the person who has accepted the draft does not pay it when due, the person who discounted it will be required to pay it.

The following draft drawn by you on John Doe and accepted by him is just as truly his promise to pay $500.00 at a given date, as a promissory note signed by him would be.


$500.00
Chicago, January 6th 1922
Thirty days after date Pay to the
Order of Myself
Five Hundred and 00/100 Dollars
Value received and charge the same to account of
To John Doe
No. Elgin, Ill.
(Your name)
Accepted Jan 10 1922 John Doe
NO PROTEST. TEAR THIS OFF BEFORE PRESENTING.

This draft could be discounted in exactly the same manner as a note dated January 6, payable 30 days after date.

If the draft were drawn payable 30 days after sight, instead of after date, the bank discount would be different because the date of maturity would be different.


$500.00
New York, January 6th 1922
Thirty days after sight Pay to
the order of Myself
Five hundred and 00/100 Dollars
Value received and charge the same to account of
To John Doe
No. Elgin, Ill.
(Your name)
Accepted Jan 10 1922 John Doe
NO PROTEST. TEAR THIS OFF BEFORE PRESENTING.

Written Work

1. What is the date of maturity of the draft on this page payable 30 days after date?

2. What is the date of maturity of the draft on page 300 payable 30 days after sight?

3. What is the discount period of each draft if discounted on Jan. 14?

4. Find the bank discount and the proceeds of each draft.

Review

Write the notes and drafts called for in the following. Show all acceptances and indorsements.

On October 3, you received from B. A. Anderson of Batavia, New York, a note dated October 1, due in two months, for $135.00, without interest.

On August 28, you sold to N. M. Davis, of Toledo, Ohio, an invoice of goods amounting to $84.00; terms 1/10; N/30. This invoice was due September 27, but Mr. Davis was unable to pay it. He agreed, however, to accept a thirty-day draft for the amount. You drew the draft on September 30, payable thirty days after date, and sent it to him for acceptance. He accepted it and returned it to you.

On October 5, you sold a bill of goods amounting to $68.00 to J. D. Robinson, of Burlington, Iowa. Terms, 30 day draft less 1%. On October 5, you drew a draft on him, payable 30 days after sight; he accepted the draft on October 7.

On July 10, you received from J. F. Cook of Lime Springs, Iowa, a note made by him on July 8, in your favor, for $500.00, due six months after date, with interest at 6%.

On June 4, you received a note for $600.00 from A. B. Hicks of Chicago. The note was drawn on April 16, by D. F. Fairchild of Omaha, Nebraska, in favor of Hicks, payable nine months after date, without interest, and was transferred to you by full indorsement.

On October 15, you took these notes and drafts to the Dairy State Bank to discount. The bank's discount rate was 6%. Finding that the proceeds would not be sufficient to meet your needs, you gave the bank your own note, dated October 15, due in 20 days, for $250.00, without interest.

1. Find the proceeds of each of the papers discounted.

Your balance at the bank before discounting this paper was $167.25. You deposited the proceeds of the discounted paper. What was your balance after discounting the paper?

2. On October 25, the bank wished to increase its deposit with its correspondent bank at Chicago. It therefore rediscounted the notes and drafts received from you. How much did the bank receive for the paper, when discounted on October 25 at 6%?

3. Marshall owed Daniels $485.90; on September 24 he gave Daniels the following note, properly indorsed.

$450.00 BRAINERD, IOWA, September 4, 1922

Sixty days after date, I promise to pay to the order of J. F. Marshall, Four Hundred and Fifty Dollars.
Value received
Without interest. F. S. CROSBY.

How much did Marshall owe Daniels after transferring the note?

Written Review

1. Compute the interest on the following:

$ 375.60	for	6	days at 6%
2,145.30	for	90	days at 6%
1,216.80	for	27	days at 6%
723.65	for	56	days at 6%
600.00	for	111	days at 6%
800.00	for	71	days at 6%
425.30	for	83	days at 5%
215.65	for	29	days at 4½%
90.00	for	7 months and 8 days	at 6%
242.80	for	2 years, 4 months, and 9 days,	at 5%

2. Compute the interest on $560.00 from May 1, 1919, to February 3, 1922, at 7%, finding the time by compound subtraction.

3. Compute accurate interest (on the basis of 365 days to the year) on $2,360.45 from January 3, 1921 to May 19, 1922, at 5%.

4. Find the balance due on April 7, 1922, on the following loan:

$2,500.00	borrowed January 6, 1921, at 6% interest
265.00	paid June 2, 1921
15.00	paid December 4, 1921
745.00	paid February 3, 1922

(*a*) Compute by United States Rule.
(*b*) Compute by Merchants' Rule.

5. A buys a piano for $850.00, paying $200.00 down and the balance in monthly installments of $25.00 plus long end interest at 6% per annum. How much is each of the first five payments? What is the balance of the debt after the fifth payment, ignoring accrued interest? How much interest is accrued and unpaid?

6. If the monthly payments are $25.00 each, long end interest being deducted and the balance applied on the principal, what is the balance of the debt after the fifth monthly payment?

7. If A pays $25.00 monthly plus short end interest, how much is each of the first five payments, and what is the balance of the debt after the fifth payment, ignoring accrued interest? How much interest is accrued and unpaid?

8. What is the discount at 6% on a $500.00 non-interest-bearing note dated May 3, due in 60 days, and discounted May 24? What are the proceeds?

9. What would be the proceeds if the note bore 5% interest?

BUSINESS EXPENSES

CHAPTER XXX

WAGES AND PAY ROLLS

THERE are several wage systems in use, the most common of which will be discussed in this chapter.

256. The Day or Hour Rate. This system is the one generally used because it is the simplest. When the day or hour rate is used. the employee's wage depends upon the time he has worked. Work done overtime or on holidays is usually paid for at one and one-half times the regular rate. In many factories time clocks are installed. As each employee begins work, he registers the time on his card; when he stops work, he again registers the time. At the end of the week, each employee's card shows his actual hours of labor. A pay roll is prepared showing the total number of hours each employee has worked, his hourly rate, and his total wage.

Written Work

1. Supply the missing facts in the following pay roll.

No.	Name	Hours worked per day						Time			Overtime			Total Wages Earned		Money Advanced		Wages Due	
		M	T	W	T	F	S	Hrs.	Rate	Amount	Hrs.	Amount							
1	Andrew White	9	8	9	9	9	8	52	30	15	60			15	60			15	60
2	August Piper	9	9	11	9	12	9	54	28	15	12	5	2	10	17	22			17	22
3	Edward Hazlett	10	9	11	9	10	9	54	32	?		4	1	92	?		4	—	?	
4	James Davis	8	7	9	8	9	9	?	29	?					?				?	
5	Oscar Bennett	9	9	10	10	11	9	?	34	?		?	?		?		1	20	?	
6	James Calkins	8	8	9	9	9	8	?	32	?					?				?	
7	Fred Calkins	8	9	9	10	11	9	?	30	?		?	?				6	—	?	
8	Charles Davis	9	6	10	11	9	10	?	28	?		?	?		?				?	
9	Harry McGee	10	9	10½	9	10	11½	?	25	?		?	?		?		1	75	?	

In this factory a full day is 9 hours. Any over hours are paid for at 1½ times the regular rate. Thus, Calkins gets 1½ times regular pay

for the overtime on Thursday and Friday, even though he did not work full time on Monday.

2. Rule a pay roll similar to the model, and enter the following data, finding the amount of wages due each employee.

The superintendent supplies the cashier with the following list of the employees, together with their numbers, and the hourly wage rate paid each. A full day in this factory is 8 hours, and overtime is paid for at $1\frac{1}{2}$ times the regular rate.

No.	Employee's Name	Hourly Rate
1.	Fred Smith	$.35
2.	Frank Bailey	.32
3.	Charles Hanchett	.30
4.	Harry Davis	.36
5.	Carl Bell	.38
6.	Lewis Clark	.40
7.	Arthur Helms	.29
8.	Tony Martin	.31
9.	Henry Dickinson	.26
10.	Nathan Wright	.37

During the week ending March 15, time tickets were turned in by the foreman, showing the hours worked by each employee.

Monday: 1, 9 (Workman No. 1, 9 hours); 2, 11; 3, 8; 4, 11; 5, 8; 6, 10; 7, 9; 8, 12; 9, 9; 10, 8.

Tuesday: 1, 9; 2, 11; 3, 9; 4, 9; 5, 10; 6, 9; 7, 9; 8, 9; 10, 9.

Wednesday: 1, 10; 2, 12; 3, 11; 4, 12; 5, 9; 6, 11; 7, 8; 8, 11; 9, 10; 10, 9.

Thursday: 1, 9; 2, 9; 3, 11; 4, 10; 5, 8; 6, 10; 7, 11; 8, 11; 9, 12; 10, 10.

Friday: 1, 5; 2, 9; 3, 11; 4, 10; 5, 9; 6, 9; 7, 9; 8, 9; 9, 10; 10, 8.

Saturday: 1, 9; 2, 11; 3, 11; 4, 10; 5, 11; 6, 9; 7, 12; 8, 9; 9, 4; 10, 9.

The cashier has already advanced Charles Hanchett $3.00; Carl Bell, $2.25; Tony Martin, $3.75; and Nathan Wright, $5.00.

257. Piecework Wage System. The piecework wage system is based on the theory that if the employee is paid for the actual amount of work he accomplishes, he will produce more in a day than if he is paid a straight hour or day wage.

The pay roll prepared under a piecework system is similar to that prepared under the hourly rate system, with the exception that overtime is usually paid for at the same rate that regular time receives, and the basis of the wage is the number of pieces produced and not the number of hours employed.

Written Work

Find the missing facts in the following pay roll.

PIECEWORK PAY ROLL FOR WEEK ENDING JUNE 8, 192–

No.	Name	Operation No.	Number Produced						Total	Rate	Earned		Advanced		Due	
			M.	T.	W.	T.	F.	S.								
1	John Sanford	264	16	14	17	13	15	18	93	.14	13	02	2		11	02
2	Irving Danner	178	23	26	22	27	24	25	?	.12	?				?	
3	Byron Shepley	361	37	33	34	39	36	30	?	.08	?				?	
4	Edward Magee	422	28	27	29	28	26	27	?	.05	?		3	25	?	
5	Percival Conrad	267	42	45	41	39	43	20	?	.07	?				?	
6	Ernest Anderson	207	24	26	20	23	30	27	?	.09	?		2	60	?	

258. The Differential Rate. The differential rate is a modification of the piecework system. A standard day's work is determined, and each employee is expected to produce this standard amount of work. If he produces the standard, he receives a certain rate per piece. If he produces less than the standard, he receives a lower rate per piece. If he produces more than the standard, he receives a higher rate per piece.

This system is based on the theory that the expenses for heat, light, rent, power, etc., remain about the same, whether the employees produce a small amount of product, or a large amount. By increasing the amount of output, these expenses are distributed over a larger quantity of manufactured goods, and the cost of making each article is thereby decreased. The saving effected by increasing the output is divided between the owner of the factory and the workmen who, by their skill or industry, increase the output. On the other hand, the employees who, by producing a small quantity, tend to increase the cost per article are paid at a lower rate.

Problem. A manufacturer learned by experience that the average workman in his factory could produce ten articles per day, and that

25 cents per article could be paid for the work. He then adopted a differential rate, as follows:

Number of Pieces Produced	Rate per Piece
7	$.20
8	.21
9	.23
10, Standard	.25
11	.26
12	.27

Jones produced the standard quantity on Wednesday, and received $10 \times \$.25 = \$2.50$.

Smith produced 8 pieces, and received $8 \times \$.21 = \$1.68$.

Brown produced 12 pieces, and received $12 \times \$.27 = \$3.24$.

Written Work

The following differential rate applies in a certain factory:

Number of Pieces	Rate
10	$.12
11	.13
12	.14
13	.15
14	.17
15	.19
16	.21
17	.22
18, Standard	.24
19	.25
20	.27
21	.29
22	.31
23	.33
24	.33½
25	.34

The production sheet for the week ending September 17, 19— was as follows:

No.	Name	Daily Production					
		M.	T.	W.	T.	F.	S.
1	Edward Dye	17	19	18	16	19	18
2	Frank Bartlett	14	15	14	16	15	14
3	Arthur Bassett	19	20	19	21	22	19
4	James Hamilton	22	21	21	23	20	21
5	Den Slocum	12	14	13	15	12	14
6	John Gaylord	13	16	15	16	15	17
7	Sam Bayliss	19	20	19	21	18	17
8	George Bates	19	22	23	20	19	18
9	Wm. Osgood	16	14	12	14	11	16

1. Devise a pay roll blank which will show all necessary facts, and enter each laborer's production and wages. See how good a pay roll form you can devise without asking for suggestions. It should be simple and convenient, and, while not being too elaborate, it should give all necessary information.

2. Find the average daily production of each workman. Prepare a graph, showing the standard production by a red line across the sheet, and the workmen's averages by a black curve.

Commission

259. Definitions. An **agent** is a person who transacts business for another.

The **principal** is the person whose business is transacted by the agent.

Principals often pay their agents a commission for their services. The **commission** is usually a per cent of the money value involved in the transaction.

Problem. A wholesaler engaged a salesman, agreeing to pay him 8 % of his gross sales as commission. How much did the salesman earn if he sold $3,425 worth of goods during a month?

Solution.

$3425.00 sales
.08 rate of commission
$ 274.00 commission

Written Work

1. A real estate agent sold a farm belonging to Mr. Robinson for $20,450. The agent received 3% of the selling price as payment for his services. How much did the agent receive? How much did Mr. Robinson receive after paying his agent?

2. A manufacturer engaged Mr. Seeley as a traveling salesman. Mr. Seeley's contract stipulated that he should receive 7% of his gross sales as commission. During four weeks his sales were as follows:

$576.25; $498.49; $376.45; $723.50.

What was his commission each week?

3. Mr. Brooks, a merchant, engaged an attorney to collect a number of accounts. The attorney charged for his services 15% of the amounts collected. What did the attorney receive for collecting $354.75? What was the net amount received by Mr. Brooks?

4. James Hunter bought eggs for a Chicago produce merchant. For his labor in purchasing and hauling them to the station, he received 1½% of the cost of the eggs. On Tuesday he bought 27 cases, of 30 dozen each, paying 29 cents per dozen for them. How much did Hunter pay for the eggs? How much was his commission? How much did the eggs cost the Chicago merchant, not including the transportation?

5. The salesmen of the Wickman Specialty Company receive the following commissions:

On goods sold in Department A, 9%.
On goods sold in Department B, 13%.
On goods sold in Department C, 12½%.
On goods sold in Department D, 15%.

The following table shows the sales made by five salesmen.

SALESMAN	DEPT. A	DEPT. B	DEPT. C	DEPT. D
C.G. PINES	356.27	473.28	293.75	459.23
F.L. SNOW	523.74	625.90	483.37	824.84
E.D. VICKS	283.57	285.37	295.64	472.84
L.S. BASSETT	923.74	825.36	736.46	462.47
FRANK HOLTZMAN	735.84	375.47	375.47	462.85

Devise a form which will show the data and enter the following:

Each salesman's commission on goods sold in each department.
The total commissions paid to each salesman.
The total commissions paid for each department.
The total commissions paid to the five salesmen.
The total sales in each department.
The total sales of each salesman.
The total sales in all departments.

260. Pay Roll Slips. Laborers are sometimes paid by check and sometimes by the envelope system. When the latter method is used, each employee receives an envelope containing the exact amount of his wages. Each employee receives an envelope containing the exact amount of his wages.

THE FIRST NATIONAL BANK

CURRENCY FOR PAY ROLL

Depositor Hunter & Colby

	DOLLARS	CENTS	
			$ 50 Bills.
			20
12	120	00	10
4	20	00	5
5	10	00	2
4	4	00	1
3	1	50	Halves.
3		75	Quarters.
7		70	Dimes.
3		15	Nickels.
10		10	Pennies.
	157	20	TOTAL

PAY ROLL SLIP

The bills and coins required for the payroll are ascertained by preparing a coin sheet, showing the money needed for each envelope. The following coin sheet was drawn up to determine the money required for the payroll.

No.	Name	Wages		Bills				$.50	$.25	$.10	$.05	$.01
				$10.00	$5.00	$2.00	$1.00					
1	James Downs	22	67	2		1		1		1	1	2
2	Francis Canfield	31	69	3			1	1		1	1	4
3	Arthur Hubbard	26	38	2	1		1		1	1		3
4	Frank Munger	29	35	2	1	2			1	1		
5	George Bailey	28	96	2	1	1	1	1	1	2		1
6	Ezra Colfax	18	15	1	1	1	1			1	1	
		157	20	12	4	5	4	3	3	7	3	10

COIN SHEET

When the currency is delivered a check is made out to the bank for the amount of the pay roll.

Written Work

Rule a coin sheet and enter the number of coins of each denomination necessary to pay each of the employees in the pay roll prepared in the differential wage rate problem on page 308.

Make out a pay roll slip on the First National Bank of your city, to obtain the money to pay these employees.

CHAPTER XXXI

POSTAGE, FREIGHT, AND EXPRESS RATES

Postage

261. The Rates for Domestic Postage. Domestic mail matter includes mail addressed for local delivery, or for transmission from one place to another within the United States, or to or from or between the possessions of the United States, and to or from the United States postal agency at Shanghai, China. Domestic rates and conditions, with certain exceptions, apply to mail addressed to Canada, Cuba, Mexico, and the Republic of Panama.

Classes of Domestic Mail

First Class: Written matter, letters and all mail which is sealed.

Rate: Letters and other first-class matter, 2 cents for each ounce or fraction thereof. Post cards and postal cards, 1 cent each.

Aeroplane mail, consisting of first-class matter, including sealed packages not exceeding 30 inches in length and girth combined, is chargeable with postage at the rate of 2 cents for each ounce or fraction thereof.

Second Class: Newspapers and periodicals bearing notice of entry as second-class matter.

Rate: When sent unsealed by others than the publisher or a news agent, 1 cent for each 4 ounces or fraction thereof.

Third Class: Circulars, newspapers and periodicals not admitted to the second class, nor embraced in the term "book." Books are included in fourth-class mail, as are also parcels of miscellaneous printed matter weighing more than 4 pounds.

Rate: 1 cent for each 2 ounces or fraction thereof. The limit of weight is 4 pounds. Parcels of printed matter

weighing more than 4 pounds which do not exceed the limit of weight and size for fourth-class matter come within that class and are mailable at the parcel-post rates.

Fourth Class: Merchandise, farm and factory products, books, miscellaneous printed matter weighing more than 4 pounds, and all other mailable matter not embraced in the first, second, and third classes.

Rates: Parcels weighing 4 ounces or less, except books, seeds, plants, etc., 1 cent for each ounce or fraction thereof, any distance.

Parcels weighing 8 ounces or less containing books, seeds, cuttings and plants, 1 cent for each 2 ounces or fraction thereof, regardless of distance.

Parcels weighing more than 8 ounces containing books, seeds, plants, etc., parcels of miscellaneous printed matter weighing more than 4 pounds, and all other parcels of fourth-class matter weighing more than 4 ounces are chargeable, according to distance or zone, at the pound rates shown in the following table, a fraction of a pound being computed as a full pound.

FOURTH-CLASS (DOMESTIC PARCEL) ZONE RATES

WEIGHT IN POUNDS	LOCAL	ZONES							
		1st, up to 50 mi.	2d, 50 to 150 mi.	3d, 150 to 300 mi.	4th, 300 to 600 mi.	5th, 600 to 1,000 mi.	6th, 1,000 to 1,400 mi.	7th, 1,400 to 1,800 mi.	8th, over 1,800 mi.
1	$0.05	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09	$0.11	$0.12
2	.06	.06	.06	.08	.11	.14	.17	.21	.24
3	.06	.07	.07	.10	.15	.20	.25	.31	.36
4	.07	.08	.08	.12	.19	.26	.33	.41	.48
5	.07	.09	.09	.14	.23	.32	.41	.51	.60
6	.08	.10	.10	.16	.27	.38	.49	.61	.72
7	.08	.11	.11	.18	.31	.44	.57	.71	.84
8	.09	.12	.12	.20	.35	.50	.65	.81	.96
9	.09	.13	.13	.22	.39	.56	.73	.91	1.08
10	.10	.14	.14	.24	.43	.62	.81	1.01	1.20
11	.10	.15	.15	.26	.47	.68	.89	1.11	1.32
12	.11	.16	.16	.28	.51	.74	.97	1.21	1.44
13	.11	.17	.17	.30	.55	.80	1.05	1.31	1.56

FOURTH-CLASS (DOMESTIC PARCEL) ZONE RATES

WEIGHT IN POUNDS	LOCAL	ZONES							
		1st, up to 50 mi.	2d, 50 to 150 mi.	3d, 150 to 300 mi.	4th, 300 to 600 mi.	5th, 600 to 1,000 mi.	6th, 1,000 to 1,400 mi.	7th, 1,400 to 1,800 mi.	8th, over 1,800 mi.
14	.12	.18	.18	.32	.59	.86	1.13	1.41	1.68
15	.12	.19	.19	.34	.63	.92	1.21	1.51	1.80
16	.13	.20	.20	.36	.67	.98	1.29	1.61	1.92
17	.13	.21	.21	.38	.71	1.04	1.37	1.71	2.04
18	.14	.22	.22	.40	.75	1.10	1.45	1.81	2.16
19	.14	.23	.23	.42	.79	1.16	1.53	1.91	2.28
20	.15	.24	.24	.44	.83	1.22	1.61	2.01	2.40
21	.15	.25	.25	.46	.87	1.28	1.69	2.11	2.52
22	.16	.26	.26	.48	.91	1.34	1.77	2.21	2.64
23	.16	.27	.27	.50	.95	1.40	1.85	2.31	2.76
24	.17	.28	.28	.52	.99	1.46	1.93	2.41	2.88
25	.17	.29	.29	.54	1.03	1.52	2.01	2.51	3.00
26	.18	.30	.30	.56	1.07	1.58	2.09	2.61	3.12
27	.18	.31	.31	.58	1.11	1.64	2.17	2.71	3.24
28	.19	.32	.32	.60	1.15	1.70	2.25	2.81	3.36
29	.19	.33	.33	.62	1.19	1.76	2.33	2.91	3.48
30	.20	.34	.34	.64	1.23	1.82	2.41	3.01	3.60
31	.20	.35	.35	.66	1.27	1.88	2.49	3.11	3.72
32	.21	.36	.36	.68	1.31	1.94	2.57	3.21	3.84
33	.21	.37	.37	.70	1.35	2.00	2.65	3.31	3.96
34	.22	.38	.38	.72	1.39	2.06	2.73	3.41	4.08
35	.22	.39	.39	.74	1.43	2.12	2.81	3.51	4.20
36	.23	.40	.40	.76	1.47	2.18	2.89	3.61	4.32
37	.23	.41	.41	.78	1.51	2.24	2.97	3.71	4.44
38	.24	.42	.42	.80	1.55	2.30	3.05	3.81	4.56
39	.24	.43	.43	.82	1.59	2.36	3.13	3.91	4.68
40	.25	.44	.44	.84	1.63	2.42	3.21	4.01	4.80
41	.25	.45	.45	.86	1.67	2.48	3.29	4.11	4.92
42	.26	.46	.46	.88	1.71	2.54	3.37	4.21	5.04
43	.26	.47	.47	.90	1.75	2.60	3.45	4.31	5.16
44	.27	.48	.48	.92	1.79	2.66	3.53	4.41	5.28
45	.27	.49	.49	.94	1.83	2.72	3.61	4.51	5.40
46	.28	.50	.50	.96	1.87	2.78	3.69	4.61	5.52
47	.28	.51	.51	.98	1.91	2.84	3.77	4.71	5.64
48	.29	.52	.52	1.00	1.95	2.90	3.85	4.81	5.76
49	.29	.53	.53	1.02	1.99	2.96	3.93	4.91	5.88
50	.30	.54	.54	1.04	2.03	3.02	4.01	5.01	6.00
51	.30	.55	.55	1.06	.....	.....	.....	.....	.....
52	.31	.56	.56	1.08	.....	.....	.....	.....	.....

The limit of weight of fourth-class matter is 70 pounds for parcels mailed for delivery within the first, second, and third zones, and 50 pounds for all other zones. Parcels may not exceed 84 inches in length and girth combined.

Special Delivery. By affixing a "special delivery" stamp (cost 10 cents), or ten cents' worth of ordinary stamps in addition to the regular postage and writing the words "Special Delivery," delivery of letters and parcels by special messenger is assured.

Registry. The registry system provides greater security for valuable mail. The registry fee is ten cents. If registered mail is lost, the sender is indemnified for the actual value up to $50 for first-class mail, and up to $25 for third-class domestic mail.

Insurance of Parcels. Mail sent by parcel post may be insured by the payment of fees as follows:

3 cents	for value not exceeding	$ 5.00
5 cents	for value not exceeding	25.00
10 cents	for value not exceeding	50.00
25 cents	for value not exceeding	100.00

C. O. D. Service. Parcel post matter may be sent C. O. D. by affixing the following fees in stamps in addition to the regular postage:

10 cents when amount to be remitted to sender does not exceed $ 50.00
25 cents when amount to be remitted to sender does not exceed 100.00

The remittance is made by postal money order, the fee being collected from the addressee. The C. O. D. fees automatically insure parcels for their actual value up to $50.00 for a 10-cent fee, and up to $100.00 for a 25-cent fee.

262. Rates for Foreign Postage. The rates of postage applicable to articles for foreign countries are as follows:

	Cents
Letters for Canada, Cuba, Mexico, Republic of Panama, city of Shanghai (China), England, Ireland, Newfoundland, Scotland, Wales, Bahamas, Barbados, British Guiana, British Honduras, Dominican Republic, Dutch West Indies, Leeward Islands, and New Zealand, Trinidad (including Tobago) and the Windward Islands (including Grenada, St. Vincent, The Grenadines and St. Lucia), per ounce	2
Letters for Germany by direct steamers, per ounce	2
Letters for all other foreign countries:	
For the first ounce or fraction of an ounce	5
For each additional ounce or fraction of an ounce	3
Single post cards (including souvenir cards), each	2
Reply postal cards, each	4

Printed matter of all kinds, for each 2 ounces or fraction of 2 ounces 1
Commercial papers, for the first 10 ounces or less 5
For each additional 2 ounces or fraction of 2 ounces 1
Samples of merchandise, for the first 4 ounces or less 2
For each additional 2 ounces or fraction of 2 ounces 1
Registration fee, in addition to postage 10
(Newfoundland is not included in the Dominion of Canada.)

Written Work

Compute the cost of postage on the following:

Domestic mail.

1. Letter weighing 1½ oz.
2. Book weighing 5 oz.
3. Book weighing 19 oz., to a point in the fourth zone.
4. Books weighing 5 pounds to a point in the second zone.
5. Three separately wrapped newspapers mailed by a person not a publisher; weight, 3 oz., 5 oz., and 8 oz.
6. Ten separately addressed printed circulars, each weighing 1½ oz.
7. Package of merchandise weighing 3 oz.
8. Package of seeds weighing 12 oz. to a point in the seventh zone.
9. Merchandise weighing 16 lb. 7 oz., to a point in the sixth zone.
10. Merchandise weighing 35 pounds, to a point 130 miles distant.
11. Registered letter weighing 7 oz.
12. Books weighing 11 lb. 9 oz., to a point in the fifth zone; package insured. (Ins. fee 5¢.)

Foreign mail.

13. Letter to London, weighing 1½ oz.
14. Letter to Paris, weighing 1½ oz.
15. Package of merchandise samples weighing 11 oz., to London.

Freight Rates

A freight rate is the charge made by a railroad company for conveying a specified quantity of property between two stations.

The rate is usually stated in cents per hundred pounds; but it may be stated in dollars and cents per ton or other unit of weight.

263. Factors Determining Rates. Different rates are charged on different kinds of freight, the rate depending in general upon two things:

a. **The expense of handling.**

Suppose two boxes, each weighing 100 pounds, are sent by freight. One is a large box occupying 40 cubic feet of car space; the other occu-

pies only 16 cubic feet. The rate per 100 pounds on the first box will probably be more than on the second because fewer such boxes can be placed in one car.

b. **The risk of loss or damage.**

Railroads are required by law to pay for goods lost or damaged. The rate will therefore depend upon:

1. The value. For example, the rate on a prize-winning cow, worth $900, will be higher than on a cow worth $60.
2. The risk of damage. The rate on glassware, for example, is higher than on wooden ware. Careful packing usually lowers the rate.
3. The risk of loss. The rate on small packages which may easily be misplaced or lost is higher than on those of larger size having the same value.
4. The risk of an article damaging other articles in the same car.

264. Classifications. For convenience in determining rates, commodities are grouped into ten different classifications. These classifications are made by the railroads after considering the elements of cost of transportation and risk. When goods are to be shipped from one state to another, the classification must be approved by the Interstate Commerce Commission.

A book called a " Classification " is published, containing an alphabetical list of commodities which can be shipped by freight, together with the classification in which each commodity is placed.

The following list is selected from a page of a Classification:

Notice that two classifications are given for some articles. The first applies when goods are shipped in less than car loads (L. C. L.); the second applies to car load shipments.

	C. L.		C. L.
Cutlery, blades or handles plated or sterling silver, boxed	$1\frac{1}{2}$	Display Cases, window shade, boxed or crated	1
Cutters, Sage Brush (knife attachments for grading machines):	5 Min. wt. 30,000 lbs.	Dog Benches (for exhibition purposes only), K. D. flat, min. C. L. wt. 30,000 lbs.	4 B
In bundles	2	Door Hangers and Rail:	
In boxes or barrels .	3	Barn Door Hangers (iron), boxed or crated . . .	3 5 Min. wt. 36,000 lbs.
Designs, Floral, Artificial or Natural, dried, in boxes	D 1	Barn Door Rails (iron), in bundles	3
		Barn Door Hanger Rails (wooden), in bundles .	3

Dextrine, in bags, bbls. or boxes, min. C. L. wt. 30,000 lbs.	2	4	
Dish-Washing Machines, N. O. S., boxed or crated.	1		
Formaldehyde, in barrels or kegs or in glass bottles, boxed, min. C. L. wt. 30,000 lbs.	2	4	
Parlor Door Hangers, boxed	2		
Parlor Door Hanger Rail, wooden, in bundles .	3		
Tubing, for Barn Door Hanger Rail, nested in bundles	4		

TERMS EXPLAINED: N. O. S. means Not Otherwise Specified.

1½ means one and one half times first-class.
D 1 means double the first-class rate.
C. L. means car loads.
K. D. means knocked down, taken apart.

265. Freight Tariff. The tariff is a list of the rates applying to different classifications of freight when shipped between the points named.

The following illustration is selected from a freight tariff.

BETWEEN	AND	CLASS RATES IN CENTS PER 100 POUNDS									
		1st Class	2d Class	3d Class	4th Class	5th Class	Class A	Class B	Class C	Class D	Class E
Stoddard, Wis. Calvert, Wis. La Crosse, Wis. Grand Crossing, Wis. Onalaska, Wis. Lytle, Wis. Trempealeau, Wis. East Winona, Wis. Winona, Minn.	Chicago, Ill. Group No. 1	50	42	33	23	18	23	18	15	12	11
	Springfield, Ill. Group No. 2	53	43	33	25	20	23	18	15	12[9]	11
	St. Louis, Mo. Group No. 3	50	42	33	23	18	23	18	15	12	11
	Danville, Ill. Group No. 4	63	52[5]	42	26	21	26	21	18	15	13[5]
	Cairo, Ill. Group No. 4	80	65	52	33	26	33	26	22	19	17

NOTE. 52[5] means 52½ cents.

266. To find the freight charge. The rates differ for each of the ten classifications. Therefore, when a shipment of goods is made, the following steps are necessary to determine the freight charge:

a. Weigh the commodity.

b. Find its classification.

c. Determine from the tariff the rate applying to that classification between the points of shipment.

d. Multiply the number of hundreds of pounds by the rate.

Problem. A shipment of barn door hangers (crated) is sent from Chicago to La Crosse, Wis. The weight crated is 185 pounds. Find the freight charges.

SOLUTION. The classification for less than car load lots is 3.

The rate on third-class commodities between Chicago and La Crosse, as shown by the tariff, is 33 cents per hundred.

$0.33 freight rate per hundred pounds.
1.85 number of hundreds of pounds.
$0.61 freight charge.

267. Minimum Charges. A minimum charge is fixed for shipment of any commodity between two points. When the charge is computed and is less than this minimum charge, the minimum charge is made. The minimum charge is sometimes the third-class rate per hundred, or it may be an arbitrarily fixed number of cents.

268. Car Loads. When goods are shipped in car load lots, they frequently go under a different classification and at a lower rate. Minimum weights for car loads are specified. For example, 36,000 pounds is the minimum weight for a car of door hangers. This does not mean that the car must contain 36,000 pounds, but the shipper will be charged freight on at least that weight, and more if the car contains a greater weight.

Problem. Shipped from Chicago to La Crosse a car load 39,000# of door hangers. Find the freight charges.

SOLUTION. Classification C. L., 5 (see classification, page 317).
Rate, $0.18.
390.00 × $0.18 = $70.20, the freight charge.

If the door hangers weighed only 32,380#, a charge would be made for the minimum weight of 36,000#.

SOLUTION. Classification, 5.
Rate, $0.18 per 100 pounds.
360 × $0.18 = $64.80, freight charge.

269. Commodity Rates. There are some articles which are not listed in the classification, but which have a rate of their own, called a "commodity rate." The most important of these commodities are lumber, grain, live stock, coal, coke, and hay.

Written Work

Find the freight on each of the following:

1. 360# of formaldehyde in glass bottles, boxed, shipped from St. Louis, Mo., to Winona, Minn.
2. 500# dextrine in bags, shipped from Winona, Minn., to Danville, Ill.
3. 260# display cases (wood and wire, crated), shipped from Springfield, Ill., to Onalaska, Wis.
4. Car load of dextrine 28,000#, shipped from Winona, Minn., to Cairo, Ill.
5. Box of silver-plated cutlery, 60#, shipped from Chicago, to La Crosse, Wis.
6. Box of dish washing machines, shipped from Chicago to Stoddard, Wis. Weight 175#.
7. Box of parlor door hangers, 80#, shipped from Springfield, Ill., to Grand Crossing, Wis.
8. Flower design, 35#, from Chicago to East Winona, Wis.
9. Car load of iron door hangers, 41,350#, from St. Louis, Mo., to La Crosse, Wis.

Express Rates

270. The Classification. The express classification is simpler than the freight classification, most articles being unspecified and taking the first-class rate.

Rate Scale Numbers. The agent at each express station is supplied with a directory showing the scale number to be used in computing the cost of shipment to every other express station in the United States. The scale numbers run from 1 to 294, depending on the distance between point of shipment and point of delivery.

The following table shows the rate scale numbers from the Chicago directory for shipment to certain Iowa points.

Station	Scale No.	Station	Scale No.
Burdette	27	Campbell	26
Burlington	15	Carlisle	26
Burt	31	Carnes	39
Buxton	21	Carney	26
		Carpenter	27
C		Carroll	29
Calmar	17	Casey	30
Calumet	33	Cedar Falls	23
Cambridge	26	Cedar Rapids	15

271. Schedule of Rates. The table on the following page gives the rates per pound on shipments between points where scale numbers from 26 to 33 apply. The complete table states the rates on shipments weighing from 1 to 100 pounds.

272. To find the express charge for shipments of 100 pounds or under.

a. Find the weight of the commodity.

b. Determine the classification.

c. Find the rate scale number applicable between the points of shipment and delivery.

d. The table of rates will show the charge for first and second-class commodities weighing 100 *pounds or less.*

Problem. What is the express charge on a case of medicine shipped from Chicago to Burdette, Iowa? Weight, 40 pounds.

Solution. Classification, 1.
Rate scale number (see table) 27.

The schedule shows the charge under rate scale number 27, first class, 40 pounds, to be $1.23.

273. To find the express charge for shipments over 100 pounds.

(*a*) First-class Rates:

Find the charge for a shipment of 100 *pounds, first class. Multiply this charge by the number of pounds in the shipment, and divide by* 100.

Problem. Find the charge on 216 pounds of first-class matter when the scale rate number is 31.

Schedule of First and Second Class Express Rates in Cents

Scale Numbers

Lbs.	26		27		28		29		30		31		32		33	
	1st Class	2d Class	1st Class	2d Class	1st Class	2d Class	1st Class	2d Class	1st Class	2d Class	1st Class	2d Class	1st Class	2d Class	1st Class	2d Class
25	88	66	89	67	92	69	93	70	96	72	97	73	98	74	100	75
26	89	67	93	70	95	71	96	72	97	73	100	75	101	76	102	77
27	93	70	95	71	97	73	98	74	100	75	102	77	103	77	106	80
28	95	71	97	73	98	74	101	76	102	77	103	77	107	80	108	81
29	97	73	98	74	101	76	102	77	106	80	107	80	110	83	111	83
30	98	74	101	76	102	77	106	80	107	80	110	83	111	83	113	85
31	101	76	103	77	106	80	108	81	110	83	112	84	113	85	116	87
32	103	77	106	80	108	81	110	83	112	84	115	86	116	87	120	90
33	106	80	108	81	110	83	112	84	115	86	116	87	120	90	122	92
34	108	81	110	83	112	84	115	86	116	87	120	90	122	92	125	94
35	110	83	112	84	115	86	116	87	120	90	122	92	125	94	126	95
36	112	84	115	86	116	87	120	90	122	92	125	94	127	95	129	97
37	113	85	116	87	120	90	122	92	125	94	127	95	129	97	131	98
38	116	87	120	90	122	92	125	94	126	95	129	97	131	98	135	101
39	117	88	121	91	123	92	126	95	129	97	131	98	135	101	137	103
40	121	91	123	92	126	95	129	97	131	98	135	101	137	103	140	105

SOLUTION. Rate for 100 pounds, first-class matter, scale number 31, is found by the complete table to be $2.84.

$$\frac{216 \times \$2.84}{100} = \$6.1344$$

Under the express company's rule that all fractions of a cent must be equalized as one cent, the charge would be $6.14.

(*b*) Second-class Rates:

Multiply the first-class rate on 100 *pounds by the number of pounds in the shipment, and divide by* 100.

Deduct 25% *of this result to find the second-class charge.*

Problem. What is the charge on 216 pounds of second-class matter, when the rate scale number is 31?

SOLUTION. Charge for 216 pounds, scale number 31, first class, is $6.14.
75% of $6.14 = $4.605, or $4.61.

Written Work

Find the charge for the following shipments by express from Chicago:

1. 26-pound box of cameras to Burdette, Iowa. Classification 1 (first class).
2. 25 pounds of candy to Calumet, Iowa. Classification 1.
3. 30 pounds of seeds to Carpenter, Iowa. Classification 2.
4. 28 pounds of telephone instruments, unboxed, to Casey, Iowa. Classification $1\frac{1}{2}$ times first class.
5. 163 pounds of statuary to Carroll, Iowa. Classification 1. First-class rate for 100 pounds, scale number 29, $2.70.
6. 191 pounds of shrubs to Burt, Iowa. Classification 2. First-class rate for 100 pounds, scale number 31, $2.84.

CHAPTER XXXII

DEPRECIATION

274. Definitions. Depreciation is the inevitable decrease in the value of fixed assets due to use and the lapse of time.

Fixed assets are the permanent investments in property used in the conduct of business, such as buildings and machinery.

Carrying value is the cost of an asset less the depreciation provided.

275. Factors of Depreciation. While repairs may be made to prolong the usefulness of a building or other fixed asset, there is sure to come a time when the property is either entirely useless, or when it is good business economy to replace it. Suppose a machine which cost $2,400 can be used for ten years, at the end of which time it is worth $400. During the ten years a loss of $2,000 will take place and this loss should be spread over the ten year period. The amount of depreciation to charge as an expense each year will depend upon:

(*a*) The cost of the asset ($2,400 in the illustration).
(*b*) The estimated life of the asset (10 years in the illustration).
(*c*) The estimated scrap or residual value ($400 in the illustration).
(*d*) The method of dividing the total estimated depreciation among the years of life.

Methods of Depreciation. There are a number of methods of computing the annual depreciation, the following being those used most frequently:

First, a fixed rate computed each year on the cost of the property. This is known as the straight line method because it results in an equal annual charge.

Second, a fixed rate on the diminishing value, which results in a diminishing annual charge.

Third, the sum of the years' digits method, which also results in a diminishing annual charge. This method is explained on page 326.

276. The Straight Line Method. This is the simplest and the customary method. Applied to the illustration, the annual depreciation would be computed as follows:

$2,400.00	Cost
400.00	Estimated scrap value
2,000.00	Total depreciation in 10 years

$2,000.00 ÷ 10 = $200.00 the annual depreciation charge.

If it is desired to know the rate of depreciation, it can be computed as follows:

$$\$200.00 \div \$2,400.00 = 8\tfrac{1}{3}\%$$

The depreciation may be tabulated as follows:

Year	Depreciation	Carrying Value
		$2,400.00
1	$ 200.00	2,200.00
2	200.00	2,000.00
3	200.00	1,800.00
4	200.00	1,600.00
5	200.00	1,400.00
6	200.00	1,200.00
7	200.00	1,000.00
8	200.00	800.00
9	200.00	600.00
10	200.00	400.00
Total	$2,000.00	

277. Diminishing Value Percentage Method. This method has a theoretical advantage. The depreciation and repairs are both expenses; since the repairs will tend to increase from year to year it is desirable to have the depreciation charge decrease so that the sum of the repairs and the depreciation will be fairly uniform. It has a practical disadvantage, however, because it is necessary to use tables of logarithms to compute the rate. The formula for computing the rate is:

$$r = 1 - \sqrt[N]{\frac{s}{c}}$$

In this formula, r is the depreciation rate; s is the scrap value; c is the cost; and n is the number of years of estimated life. Applied to the illustration

$$r = 1 - \sqrt[10]{\frac{400}{2400}}$$

The value of r can be computed with the aid of logarithms. With such a table the rate is found to be 16.404%. The annual depreciation may be tabulated as follows:

Year	Depreciation	Carrying Value
		$2,400.00
1	$ 393.70	2,006.30
2	329.11	1,677.19
3	275.13	1,402.06
4	229.99	1,172.07
5	192.27	979.80
6	160.73	819.07
7	134.36	684.71
8	112.32	572.39
9	93.89	478.50
10	78.50	400.00
Total	$2,000.00	

The depreciation the first year is 16.404% of $2,400.00; the second year, 16.404% of $2,006.30; et cetera.

278. Diminishing Value, Sum of Years' Digits Method. This method has the theoretical advantage of a diminishing annual charge, while avoiding the practical disadvantage of requiring logarithms to compute the rate. Each year's depreciation is computed by multiplying the total depreciation by a fraction, the numerator of which is a year's digit (taken in inverse order) and the denominator of which is the sum of the digits of all the years.

Applied to the illustration the asset has an expected life of ten years; hence the sum of the years' digits is:

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$$

The total depreciation is $2,000.00, and the depreciation the first year is $\frac{10}{55}$ of $2,000.00; the depreciation the second year, $\frac{9}{55}$ of $2,000.00; et cetera. The depreciation may be tabulated as follows:

Year	Fraction	Depreciation	Carrying Value
			$2,400.00
1	$\frac{10}{55}$	363.64	2,036.36
2	$\frac{9}{55}$	327.27	1,709.09
3	$\frac{8}{55}$	290.91	1,418.18
4	$\frac{7}{55}$	254.55	1,163.63
5	$\frac{6}{55}$	218.18	945.45
6	$\frac{5}{55}$	181.82	763.63
7	$\frac{4}{55}$	145.45	618.18
8	$\frac{3}{55}$	109.09	509.09
9	$\frac{2}{55}$	72.73	436.36
10	$\frac{1}{55}$	36.36	400.00
Total	$\frac{55}{55}$	$2,000.00	

Written Work

1. Find the annual depreciation on a building worth $15,560, if 4% is charged off each year.

2. How much is charged off annually for depreciation by a manufacturer who owns property which depreciates at the following rates?

Property	Value	Depreciation Rate
Factory building	$40,000	5%
Machinery	4,800	$7\frac{1}{2}$%
Tools	1,250	$12\frac{1}{2}$%
Patents	5,000	$\frac{1}{17}$

3. The owner of a building estimates the annual depreciation as 3% of its cost. The building cost $4,000. What is the amount of the annual depreciation?

The building is rented at $40 per month. If the taxes, insurance, and other expenses amount to $80 per year, what net income does the owner of this property receive on his investment after allowing for depreciation?

4. It is estimated that a machine costing $2,220 can be sold at the end of eight years for $500. What per cent should be charged annually for depreciation by the straight line method, and what will be the amount of the annual depreciation?

5. A machine cost $5,000.00; it has an estimated life of five years and an estimated residual value of $100.00. Prepare a table showing

the annual depreciation charges and carrying values by the straight line method.

6. Prepare a table using the same facts as in problem 5 but writing off depreciation on the diminishing value, using 54.27 % as the rate.

7. Prepare a table using the same facts as in problem 5 but writing off depreciation by the sum of the years' digits method.

CHAPTER XXXIII

ADVERTISING

DURING the last fifty years the annual cost of advertising in this country has increased to more than seven hundred million dollars. The advertising methods employed are numerous. The principal mediums are newspapers, magazines, street car signs, posters, electric signs, circulars, booklets, and novelties.

279. Newspapers. Newspaper space is sold by the page, inch, and agate line. The word "agate" refers to the size of the type.

This is a line of agate type.

An inch contains nine lines of agate type when a lead is placed between each line; or 12 lines when set solid. The price varies with the circulation of the newspaper; 3 cents per inch per thousand subscribers is considered a fair basis on which a newspaper may determine its rates. On this basis, a newspaper with a circulation of 5,000 would charge 15 cents per inch for its space. The larger newspapers generally compute their rates on the basis of 7 cents per line per thousand subscribers, but there are many exceptions to this.

RATE CARD OF A DAILY NEWSPAPER

DISPLAY RATES PER AGATE LINE		
	Daily	Sunday
Run of paper, 5 lines or more	$.40	$.50
Next to reading matter	.50	.60
Specified page	.45	.55

DISCOUNTS FOR YEARLY CONTRACTS		
Lines	Per Cent	Insertions
2,500 or more lines	10	26
5,000 or more lines	25	52
10,000 or more lines	28	312

280. Magazines. The rate charged by magazines for advertising space also depends upon the circulation, a fair basis being considered ½ cent per line, or $1.00 per page, per thousand subscribers. Higher rates are charged for the more desirable positions, such as cover page, and for printing in colors.

SPECIMEN RATE CARD OF THE —— MAGAZINE

RATES PER ISSUE	
One page	$100.00
Half page	50.00
Quarter page	27.00
Eighth page	15.00
Less than ⅛ page, per agate line	.75
Inside cover page	150.00
Back cover, 3 colors	350.00

Three per cent discount for cash.
Forms close on the 10th day of the month preceding date of issue.

One well-known magazine with a very large circulation charges $4,000 per page, and there are several magazines which charge more than $250 per page.

281. Street Car Signs. Space is provided above the windows of most street cars for printed signs. These signs differ in size, but the standard size is 11 in. × 21 in. Some cards of double length are used; these are 11 in. × 42 in. The ordinary rate for street car advertising space is 50 cents per month per car for a half run, which means six months in all the cars, or one year in one half the cars. It is 45 cents a car per month when all cars are used for a full year. Exceptions to this rate apply in New York, Chicago, and some other large cities. The cards are furnished at the expense of the advertiser.

282. Posters. Posters are printed in various sizes, but the standard unit is a sheet 28 in. × 42 in. The billboards are usually about 10 feet high, in order to accommodate 4-sheet posters. The charge made for rent of boards and posting signs depends upon the size of the city and the desirability of the location. The rates range from 4 cents to 30 cents a sheet per month. A discount of 5% is usually given on a contract for three months' service, and 10% on a contract for six months' service. The cost of printing the posters ranges from 1½ cents to 4 cents per sheet depending on the quality and quantity.

283. The Advertising Agency. In many of the larger cities there are advertising agencies which assist advertisers in preparing copy, in directing advertising campaigns, and in selecting the best medium for advertising a particular article. On the theory that the agency creates new business, publishers usually allow agencies a commission of from 10% to 15% of the gross cost of all advertising placed with them for publication.

284. Checking Results. Large advertisers make special efforts to determine the mediums which bring the greatest number of inquiries, and which result in the most sales and the largest amount of profit. For this purpose, the advertisements in the various newspapers and magazines are "keyed"; for example, the address may be differently stated in each medium, or different booklets may be offered, so that when the replies are received in the office, it will be possible to determine the medium which attracted the reader. Each periodical is credited with the replies received from its readers. Further records are kept to determine the value of the goods sold to the readers of the periodical, and the profit resulting therefrom.

Written Work

Refer to the newspaper rate card on page 329 to obtain facts for Problems 1, 2, and 3.

1. On the basis of nine agate lines to the inch, what is the cost of a four-inch, single-column advertisement? The advertisement appears next to reading matter.

2. What is the cost of a double-column, 40-line display advertisement, printed by request on the financial page? How much would the same advertisement cost if printed on a specified page of the Sunday paper? Can you account for the increased cost?

3. A clothing store contracted for 10,000 lines to be printed in the course of a year. 8,000 lines were printed in the daily editions, "run of paper." 1,200 lines were printed in the daily editions on specified pages, and the balance appeared in the Sunday editions next to the reading matter. What was the total cost?

4. A breakfast food manufacturer ran the following amount of advertising in the magazine whose rate card is printed on page 330.

January	$\frac{1}{8}$ page	March	$\frac{1}{2}$ page
February	1 page	April	12 agate lines

May	1 page	September	Back cover
June	Inside cover page	October	1 page
July	Inside cover page	November	1 page
August	Inside cover page	December	½ page

What was the cost for the year if he paid cash?

5. The advertiser in Problem 4 placed his copy through an agency which received from the publishers 12½ % of the gross cost as its commission. How much commission did the agency receive? How much did the publishers receive?

6. An insurance company advertised in the street cars of a city in which one of its offices was located. There were 850 cars, and the insurance company contracted for a " full run " at the rate of 45 cents per month per car. What was the cost (not including the cost of printing)?

If the cards were changed every two months, and if 400 extras were printed to allow for " torns " and " dirties," how many were required for the year? What was the cost of the cards at $25 per thousand?

What was the total cost of this company's street car advertising?

7. A manufacturer advertised his product with posters. Each poster was 112″ × 126″. How many sheets 28″ × 42″ to each poster?

He signed a six months' contract for 250 posters at 16 cents per sheet per month, posters to be changed only when worn out, and received the customary 10 % discount. What was the cost of the service?

He purchased 15 % more posters than were actually required for the boards, estimating that this number would be necessary to allow for damaged sheets. What was his printing bill at 2¼ cents per sheet?

What was the total cost of the six months' advertising?

8. Complete the table on the following page.

Which columns do you think give the best evidence as to the value of each magazine as an advertising medium for this product?

Which is the better evidence of the value of an advertising medium, the cost per reply or the cost per sale?

List these magazines in the order of their value as advertising mediums for this particular advertiser (on the basis of per cent of net profit).

RECORD OF ADVERTISING FOR JUNE, 1922

PUBLICATION	SPACE	COST OF ADVERTISING	NUMBER OF REPLIES	COST PER REPLY	NUMBER OF SALES	PER CENT OF REPLIES SOLD	COST PER SALE	VALUE OF GOODS SOLD	ADVERTISING COST WHAT PER CENT OF SALES	PROFIT FROM SALES	NET PROFIT AFTER DEDUCTING ADVERTISING COST	PER CENT OF NET PROFIT
Literary Guide . .	page	$400	500	80	400	80	$1	$2,000	20	$600	$200	50
Readers' Monthly .	½ page	250	385		156			863		315		
Inland Monitor . .	page	180	600		88			1,275		420		
Firelight	⅛ page	60	93		68			1,640		395		
Romance and Story	¼ page	75	370		148			1,290		396		
Youth and Age . .	80 ag. lines	30	147		95			670		115		
Farming Age . . .	¼ page	60	320		90			280		38		

NOTE. Cost per reply is found by dividing the total cost by the number of replies.
Per cent of replies sold is found by dividing the number of sales by the number of replies.
Cost per sale is found by dividing the total cost of advertising by the number of sales.
To find what per cent the advertising cost is of the sales, divide the advertising cost by the value of the goods sold.
Net profit is found by subtracting the advertising cost from the gross profit.
To find what per cent the net profit is of the cost of advertising, divide the net profit by the cost of the advertising.
If a net loss should occur, show the net loss and the per cent of net loss in red ink.

CHAPTER XXXIV

PROPERTY INSURANCE

PROPERTY is in danger of destruction from many causes; insurance companies have been organized to assume the risks of destruction, thus relieving the owners from anxiety and loss. Any person having a financial interest in a property may insure himself against its loss or destruction by paying an insurance company a certain per cent of the value of the property insured.

The person who purchases insurance is called the **Insured.**

The company which assumes the risk is called the **Underwriter.**

The contract between the insured and the underwriter is called a **Policy.**

285. The Value of Insurance.

a. In case of loss the insured receives a certain sum, called an indemnity, which wholly or partially repays his loss.

b. Even though the property is not destroyed, the insured has the security which the insurance gives.

c. Some wholesale houses are unwilling to sell goods on credit to persons whose property is inadequately insured.

286. Risks for which Insurance is Written. Insurance companies are organized to furnish protection against numerous kinds of risks. Some of the most common classes of insurance are:

Fire.
Lightning and Tornado.
Marine, on vessels and cargoes at sea.
Live stock.
Transportation.
Burglary.
Plate Glass.
Boiler Explosion.
Automobile.
Crop.

The principles underlying all types of property insurance are the same, and for the purposes of this text fire insurance only will be explained. It will be understood that other forms of insurance may be written, settled, or canceled in a similar manner.

287. Process of Insuring. When a person wishes to secure a policy insuring his property, he should first ascertain the value of the property. To do so, it may be necessary to take an inventory, applying to the goods a price at which they would sell in the market at the time of taking the inventory.

After the valuation has been determined, the next thing to decide is the amount of insurance it is desirable to carry. If the property is insured at its total valuation, the owner will bear no loss in case of fire. But since, in most fires, a portion of the property is saved, it is customary to insure only a fraction of the total valuation.

It is a matter of choice how much risk the insured wishes to take. If he wishes to be free from all risk, he should insure his property at its full valuation. Insurance companies, however, are rarely willing to insure property at its full value. Business men usually insure from $\frac{1}{2}$ to $\frac{4}{5}$ of the valuation of their property.

In some communities where the fire protection is poor, insurance companies sometimes refuse to insure more than $\frac{3}{4}$ of the valuation of property. On the other hand, in cities where the fire protection is good and the probability of total loss is comparatively slight, people are not inclined to insure a large fraction of the valuation. To counteract this tendency, the insurance companies frequently offer a lower rate, or price per hundred dollars of insurance carried, to those who insure 80% or more of the valuation than they offer to persons insuring less than 80%.

288. The Cost of Insurance. Insurance companies charge a certain number of cents or dollars for insuring each \$100.00 worth of property. Thus, if the insurance rate were 60 cents per hundred dollars of insurance carried, and you insured a building worth \$5,000.00 at its full value, the insurance cost would be computed as follows:

\$.60 the rate per \$100 insurance.
50 the number of hundred dollars of insurance purchased.
\$30.00 the premium, or cost of the insurance per year.

289. To find the premium:

Multiply the valuation of the property by the fraction of the value insured.

Point off two places in this product, to determine the number of hundred dollars of insurance purchased.

Multiply by the rate per $100.

Problem. What is the premium for insuring, at 80 % of its value, a building worth $10,000.00, if the rate is 90 cents per year?

SOLUTION.
$10,000.00 value of the property
.80 fraction of property to be insured
$ 8,000.00 insured value
$ .90 rate per hundred
80 number of hundred dollars of insurance carried
$72.00 premium

290. Insurance Rates. The insurance rate per hundred dollars per year varies from about $.30 on brick dwellings in cities, where the fire protection is good, to $7 or more on very dangerous risks, such as garages storing gasoline, and buildings located in districts where the fire protection is poor.

Thus it is seen that the rate depends upon the hazard, or the danger of the property burning. This hazard is determined by four things, which are always taken into account in fixing the rate on different pieces of property. They are:

a. The **Construction** of the building. If it is built of wood, a higher rate is charged than if it is built of brick, or other slow-burning material. If it has many stories, so that a fire starting near the roof will be difficult to extinguish, a higher rate will be charged.

b. Its **Occupancy.** If the building is occupied by some hazardous business, such as a woodworking mill in which wood shavings accumulate, or a clothes-cleaning establishment containing dangerous chemicals, a higher rate will be charged than if the business caused no particular hazard.

c. Its **Exposure.** By "exposure" is meant: first, the distance from other buildings from which it might take fire; and second, any hazard which might arise from its being in the neighborhood of a dangerous building.

d. Its **Protection.** The rate will be lower if the building is close to a fire hydrant, in a city with an efficient fire department, or if the building is equipped with automatic sprinklers which are opened by

the heat from a fire. If a night watchman is engaged, this also may reduce the rate.

The rate depends also upon the length of time for which the insurance is purchased.

291. Typical Rates. The following table will illustrate the rates which apply on different classes of property. On business houses and factories the rate is fixed after an inspection of the property made by a representative of the insurance company. The insurance companies of a state frequently join in organizing an "inspection bureau" which fixes the rates.

The rates on city property will be lower than those which apply generally throughout the country, because of the more adequate fire protection of a large city.

A CITY INSURANCE TARIFF

DWELLINGS

	ONE YEAR, PER $100
Brick, stone, tile, cement-block, or concrete dwellings	$0.30
Single frame dwellings and contents, detached not less than 50 feet in all directions	.50
Single frame dwellings and contents, detached not less than 25 feet in all directions	.60
Single frame dwellings and contents, detached 50 feet on one side	.60
All other frame dwellings not less than	.75
Brick-veneered or tile-veneered dwellings	.40
Dwellings plastered outside, with tile or other noncombustible roof	.40
Dwellings plastered outside, with shingle or other combustible roof, rate same as frame dwellings.	
Dwellings in part brick and frame rate as frame, subject to survey and rating by the board.	
Streets, without reference to their width, to be considered as cutting off charge for exposure.	

BRICK BUILDINGS OCCUPIED FOR APARTMENT HOUSES OR FLATS

	ONE YEAR, PER $100
Three stories in height	$0.40
Four stories in height and 50 feet or less wide	.50
Five stories in height and 50 feet or less wide	.60
Six stories or more in height and 50 feet or less wide	.75

BRICK BUILDINGS (OTHER THAN APARTMENT HOUSES, CHURCHES, SCHOOLS, FLATS AND WOODWORKING RISKS) IN PROCESS OF CONSTRUCTION

	One Year, per $100
Two stories or less in height	$0.40
Three stories in height	.50
Four stories in height	.60
Five stories in height	.75

Over five stories in height, add 25 cents for each story.

SCHOOLHOUSES

	One Year, per 100
Brick or stone, with metal, slate, or composition roof	$0.60
Brick or stone, with shingle roof	.75
Frame	1.00

CHURCHES

	One Year, per $100
Brick or stone	$0.75
Frame	1.00

Contents of buildings take a rate 10 % higher than the buildings themselves.

TERM RATES

When policies are written for several years, the following will apply:

2 years, $1\frac{3}{4}$ annual rates.
3 years, $2\frac{1}{2}$ annual rates.
4 years, $3\frac{1}{4}$ annual rates.
5 years, 4 annual rates.

FARM PROPERTY

	One Year, per $100	Three Years, per $100	Five Years, per $100
Dwellings, barns, outbuildings, when written under same policy	$0.50	$1.00	$1.50
When farm barns are written without the dwellings	.75	1.50	2.25

Written Work

Compute the premiums on the following fire insurance policies. The proper rates will be found by a study of the preceding tariffs.

One-year Policies — City Property

1. Single frame building, value $4,000. No buildings nearer than 60 feet. Insured at 80 % of its valuation.

2. Single frame dwelling, value $5,400. House on north, 70 feet away; store on south, 25 feet away. Insured at full value.

3. Brick dwelling, value $6,800. Insured at 75% of its value. Nearest building, 50 feet.

4. Dwelling, plastered outside; shingle roof. Value, $4,200. Insured for $3,600. No building nearer than 35 feet.

5. Five-story apartment house, 48 feet wide. Value, $36,000. Insured at $\frac{3}{4}$ of value.

6. Four-story brick store in process of construction. Value, $3,900. Insured at 80% of its valuation.

TERM POLICIES — CITY PROPERTY

7. Three-year policy on brick schoolhouse, with tile roof. Value $40,000. Insured at full value.

8. Five-year policy on frame church. Value, $7500. Insured at full value.

POLICIES ON HOUSEHOLD GOODS — CITY

9. Three-year policy of $1,000 on household goods in frame dwelling. Building on north, 10 feet; building on south, 60 feet.

10. One-year policy of $500 on household goods in brick-veneered building.

FARM POLICIES

11. One-year policy of $8,500, covering dwelling and barns.

12. Five-year policy of $3,000 on barn and silo.

292. Insurance Agent's Commission. Local agents of the fire insurance companies are located in almost every city and town. They act as the representatives of the companies, soliciting the business, and collecting the premiums. For this service they receive a certain per cent of the premiums, usually varying from 15% on mercantile risks to 25% on dwellings.

To find the agent's commission.

Problem. Suppose that a store building valued at $6,000 is insured for $\frac{3}{4}$ of its value, the rate being 70 cents per hundred. What is the agent's commission if he receives 15% of the premium?

SOLUTION. $\frac{3}{4}$ of $6,000 = $4,500, the insured value.
45 × $.70 = $31.50, the premium.
15% of $31.50 = $4.73, the agent's commission.

Written Work

In each of the following problems find

The premium;
The agent's commission;
The amount of the premium received by the insurance company.

1. A one-year policy of $3,000 on a store building. Rate, 90 cents. Agent's commission, 20 %.

2. A three-year policy on a dwelling valued at $6,500, insured at 80 % of its valuation. Rate, 65 cents. Three-year policy for 2½ annual rates. Agent's commission, 18 %.

3. A one-year policy of $800 on household goods. Rate, 50 cents. Agent's commission, 15 %.

293. Settlement of Losses. When a fire occurs, causing a small loss, settlement may be made by the local agent. When the loss is large, a special **adjuster** is sent by the company to make the settlement.

The adjuster first determines with the owner the actual cash value of the property immediately preceding the fire. This may be done by finding the cost of the property when new, and subtracting a certain amount for **depreciation**; that is, the decrease in value due to use, age, change of style, or any other cause. This cash value is called the **sound value.**

The adjuster next determines the value of goods which were saved from the fire. In determining this value he deducts any loss due to damage by the fire, or by water used in extinguishing the fire. The property saved is called **salvage.**

It is evident that:

Sound Value − Salvage = Loss.

The companies pay this actual loss up to the value of the insurance carried.

Problems.

1.

Valuation of property by insured when taking the policy	$10,000.00
Insured for 80 % of its value; Insured value	8,000.00
Sound value agreed upon by adjuster and insured after allowing $500 for depreciation	9,500.00
Salvage	3,000.00
Actual loss	$ 6,500.00

Since the insured carried $8,000 insurance, the loss would be paid in full.

2. Valuation of property by insured when taking policy .	$ 8,000.00
Insured for ¾ of its value; Insured value.	6,000.00
Sound value, after deducting 10 % for depreciation . .	7,200.00
Salvage	500.00
Actual loss.	$6,700.00

Since the insured carried only $6,000 insurance, the company is liable for only $6,000 of the loss; the insured bearing the remaining $700 loss.

In adjusting losses, the insurance companies reserve the right to:

a. Settle the loss as shown above; or

b. Pay the sound value of the property, taking possession of all the salvage; or

c. Replace the property by other property of like kind and quality.

294. The Co-Insurance Clause. Policies often contain what is called a co-insurance clause, worded somewhat as follows: "In consideration of the premium for which this policy is issued, it is expressly stipulated that in event of loss this company shall be liable for no greater proportion thereof than the sum hereby insured bears to eighty per cent of the cash value of the property described herein at the time when such loss shall happen."

This is an "80 % co-insurance clause." The clause may contain any per cent, but 80 % is customary. The theory of this clause is as follows: the insured agrees that if he carries a policy less than 80 % of the sound value of the property he becomes a co-insurer with the company. For example, if the face of the policy is only 70 % of the sound value, the insured is a co-insurer for 10 %. Any losses will be divided ⅞ to the company and ⅛ to the insured, except that the company will not be liable for more than the face of the policy. If the policy is 80 % of the sound value of the property, the company will be liable for all losses up to the face of the policy.

Example A.

Sound value of property	$10,000.00
Policy	8,000.00 (or 80 %)

The company is liable for all losses up to $8,000.00.

Example B.

Sound value of property	$10,000.00
Policy	8,500.00 (Or more than 80 %)

The company is liable for all losses up to $8,500.00

Example C.

Sound value of property	$10,000.00
Policy	7,000.00 (Less than 80 %)
Loss	4,000.00

Company is liable for $\frac{7}{8}$ of $4,000.00, or $3,500.00.

Example D.

Sound value of property	$10,000.00
Policy	6,500.00 (Less than 80 %)
Loss	4,000.00

Company is liable for $\frac{65}{80}$ of $4,000.00, or $3,250.00.

Example E.

Sound value	$10,000.00
Policy	6,500.00
Total loss	

Company is liable for $6,500.00. $\frac{65}{80}$ of $10,000.00 is $8,125.00, but as this is more than the face of the policy, the company is liable for only $6,500.00.

From these examples, the following rule should be clear:

To determine the company's liability under a policy with the 80 % co-insurance clause, when the policy is less than 80 % of the sound value, multiply the total loss by a fraction, the numerator of which is the face of the policy, and the denominator of which is 80 % of the sound value of the property. The product is the liability of the company, except that the liability cannot be greater than the face of the policy.

Example F.

Sound value	$12,500.00
Policy	7,000.00
Loss	4,000.00

Numerator of fraction = 7,000

Denominator of fraction = 80 % of 12,500 = 10,000

Liability $= \frac{7,000}{10,000}$ of $4,000 = $2,800.00

Written Work

(No co-insurance clause in Problems 1 to 8)

1. An apartment building worth $45,000 was insured at 80% of its valuation. A fire occurred resulting in a total loss. If the sound value was considered to be the same as the original valuation, how much insurance did the owner of the property receive, and what was his loss?

2. A dwelling valued at $3,500.00 was insured at its full valuation. A fire resulted in a partial loss, the salvage being estimated at $1,250.00. If the sound value of the property was $3,325.00, how much insurance did the owner of the property receive?

3. If the property in Problem 2 had been insured for $1,500.00, how much insurance would have been paid?

4. A policy of $1,500.00 was written on a man's household goods. When adjustment of the loss was made, the following facts were determined: Value of household goods immediately preceding the fire, $1,855.00; salvage, $575.00. How much money did the insured receive as a settlement?

5. If the salvage in Problem 4 had been $215.00, how much would the insured have received?

6. A building worth $5,000.00 was insured under a five-year policy at its full value. Four years after taking the policy, the house was completely destroyed by fire. In making adjustment of the loss, it was agreed that the sound value was 5% less than the insured value. How much did the owner of the house receive in settlement?

7. If the salvage in Problem 6 had been $1,350.00, how much insurance would the insured have received?

8. Cost of building, $6,000; insured for 80% of its value under five-year policy secured by the payment of four annual rates of 75 cents. Burned at end of third year. Sound value determined by deducting 4% annual depreciation. Salvage, $1,200. Find premium and loss, if any, to owner.

9. Property is insured for $7,500.00 under a policy with an 80% co-insurance clause. The property was totally destroyed. Its sound value was $12,000.00. What was the liability of the insurance company?

10. What would the liability of the company have been if only a partial loss of $7,000.00 had occurred?

11. Sound value of property $25,000.00. Insurance carried

$20,000.00. Co-insurance clause, 80%. Total loss. What is the liability of the company?

12. Same facts except that the loss was only $15,000.00. What is the liability of the company?

13. Sound value of property $25,000.00. Insurance carried, $15,000.00. Co-insurance clause, 80%. Total loss. What is the liability of the company?

14. Same facts except that the loss was only $10,000.00. What is the liability of the company?

295. Canceling Policies. Both the insurance company and the insured have the right to cancel an insurance policy at any time.

When the policy is canceled by the *insurance company*, the portion of the premium to be repaid to the insured is determined *pro rata*.

The exact fraction of the time of the policy which has not expired is determined and this fraction of the premium is returned to the insured.

Problem. On July 7, 1922, the owner of a building insured his property for one year. The premium was $24.00. On September 17, the policy was canceled by the insurance company. What rebate did the insured receive for the unexpired term?

SOLUTION. Time from September 17, 1922, to July 7, 1923, 293 days, unexpired term of policy.

$\frac{293}{365}$ of $24.00 = $19.27, amount of premium returned.

The table on page 267 will be of assistance in finding the unexpired term of a policy.

Problem. An insurance policy for one year was purchased on July 9, and was canceled by the company on the 17th of the following December. How many days in the unexpired term of this policy? What fraction of the premium will be refunded?

SOLUTION. The number of days between December 17 and July 17 is shown by the table to be 212. Subtracting 8, we have 204, the number of days from December 17 to July 9.

$\frac{204}{365}$ of the premium will be refunded.

When the policy is canceled by the *insured*, the amount of premium returned is determined by the "short rate." The short rate is an arbitrary per cent fixed by the insurance companies, and is shown by a table, a portion of which is shown on page 345.

PER CENTS OF THE ANNUAL PREMIUMS TO BE CHARGED OR RETAINED FOR PERIODS LESS THAN ONE YEAR. ARRANGED BY DAYS FROM ONE TO THREE HUNDRED SIXTY DAYS.

DAYS	PER CENT	DAYS	PER CENT	DAYS	PER CENT	DAYS	PER CENT
66	34.32	71	36.30	76	37.50	81	38.85
67	34.97	72	36.58	77	37.90	82	39.13
68	35.46	73	36.79	78	38.20	83	39.34
69	35.79	74	36.93	79	38.40	84	39.48
70	35.95	75	37.00	80	38.50	85	39.55

Problem. 1. On October 28, 1922, a one-year policy for $2500.00 was written on a dwelling. Premium, $12.50. On January 15, 1923, the policy was canceled at the request of the insured. What rebate did the insured receive?

SOLUTION. Time from October 28, 1922, to January 15, 1923, 79 days.

The table shows that 38.40 % of the premium is to be *retained* when the policy has been in force 79 days.

38.40 % of $12.50 = $4.80.

$12.50 − $4.80 = $7.70, amount of premium returned.

Written Work

Find the premium rebates on the following canceled policies:

1. One-year policy of $2,400.00. Premium, $16.00. Policy written May 19, 1922; canceled by the company on July 29, 1922.

2. Same facts as Problem 1, except that the policy is canceled at the request of the policyholder.

CHAPTER XXXV

TAXATION

296. Purpose of Taxation. The national government requires money to support the army and navy, to pay the salaries of government employees, to pay pensions, and to finance other activities carried on by the nation.

The state governments require money for the expense of their officers, and to support their various institutions, schools, universities, asylums, and penitentiaries.

The counties require money for the building of bridges, the trial of criminal cases, the salaries of officers, the relief of the poor, etc.

Cities must pay for police and fire protection, care of streets, etc.

School districts contribute to the support of the public schools.

The money required for these expenses is raised by taxes, licenses, fees, assessments, and fines.

297. A **tax** is a sum of money levied by the proper officers of a government to defray government expenses.

The funds of the national government are raised largely in three ways:

Customs Duties on Imports;
Internal Revenue;
Income Tax.

The funds of the state and local governments are raised largely by direct taxes on real estate and personal property.

State and Local Taxes

298. Taxable Property. The amount of property tax paid by any individual to state and local governments depends on the assessed value of the property which he owns and on the tax rate.

Real estate consists of land and buildings.

Personal property consists of movable property, such as merchandise, furniture, machinery, live stock, cash, notes, stocks, bonds, and mortgages.

299. Determining the Assessed Value of Property. The value of each person's taxable property is determined by the assessor. The **assessed value** is usually a fractional part of the real value.

For example, in a certain state, property is assessed at $\frac{1}{3}$ of its real value. Jones owns a farm which the assessor considers to have a real value of \$30,000. He therefore assesses it at \$10,000.

Although the law gives the assessor the power to determine the taxable value of property, property owners usually have the privilege of appearing before a Board of Equalization to prove a claim that their property has been assessed at too high a valuation. The Board of Equalization judges between the values fixed on property by the assessor and by the owner.

300. The Tax Rate. The tax rate may be expressed in several ways, the most common of which are:

A per cent;

A certain number of mills on each dollar of assessed valuation;

A certain number of dollars or cents on each hundred dollars of assessed valuation.

The following tax rates are equivalent:

1.6%;
16 mills (on the dollar);
\$1.60 (on each hundred dollars).

301. To find the amount of tax. To find the amount of tax to be paid by any property owner:

Multiply the assessed value of the property by the tax rate.

(*a*) When the rate is stated as a per cent.

Problem. Bennet's property is assessed at \$3,000; the tax rate is 1.6%.

SOLUTION.

\$3,000 assessed valuation
.016 tax rate
\$48.00 tax

(*b*) When the rate is stated as a certain number of mills on the dollar.

Problem. Taylor's property is assessed at \$3,800. The rate is 24 mills.

SOLUTION.
$3,800 assessed valuation
.024 tax rate
$91.20 tax

(*c*) When the rate is stated as a certain number of dollars on each hundred dollars of assessed value.

Problem. Finch's property is assessed at $5,470. The tax rate is $1.95.

SOLUTION.
$1.95 the rate per hundred dollars
54.70 the number of hundreds of dollars assessed value
$106.67 the tax

Oral Work

1. State each of the following tax rates in two other ways:

3.6 % 27 mills $2.83

2. A certain state assesses property at $\frac{2}{5}$ of its real value. What is the assessed value of a house worth $5,000? Of a factory worth $15,000? Of a building worth $4,000?

3. Dickinson has a house assessed at $2,000. The rate is 2.5 %. What is his tax?

4. Barnes has a store building assessed at $3,000 and a house assessed at $2,000. The tax rate is 30 mills. What is his tax?

5. If property is assessed at $\frac{1}{3}$ of its real value, what is the assessed value of a city lot worth $1,200? How much tax will the owner pay if the tax rate is 22 mills?

6. Property valued at $6,000 is assessed at $\frac{1}{2}$ of its value and taxed at a rate of $2.50. What tax does the owner pay?

7. Three men living in different cities were comparing their tax rates. A's rate was $3.56; B's rate was 28 mills; C's rate was 3.14 %. Which had the largest rate? How much tax should each pay on an assessed value of $2,000?

8. Bailey owns property worth $15,000 and is taxed $4.50 per hundred dollars on $\frac{1}{2}$ of the real value. Osborne owns property worth $15,000 and is taxed 32 mills on $\frac{1}{3}$ the real value. Which pays the larger tax? How much does each pay?

9. Powell pays 3 % on $\frac{1}{3}$ of the real value of his property. What per cent of the value of his property does he pay annually as taxes?

10. A loaned money at 6 % and took a mortgage. Since a mortgage is personal property, it is taxable. The mortgage was taxed at a rate

of \$4 on $\frac{1}{5}$ the real value. What per cent of the value of the mortgage did A pay annually as taxes? What was his per cent of net income on the loan?

Written Work

1. Complete the following table:

Real Value of Property	Fraction of Value Assessed	Assessed Value	Rate	Tax
\$16,250	$\frac{1}{5}$		\$5.42	
7,920	$\frac{1}{3}$		3.18%	
22,000	$\frac{2}{5}$		\$2.89	
960	$\frac{1}{5}$		64 mills	
4,000	$\frac{1}{3}$		2.92%	

2. Hill rents his house for \$30.00 per month. The house cost him \$3850.00. What is the per cent of net profit on his investment, after paying the following annual expense?

Insurance, \$15.40.

Repairs, \$60.00.

Taxes: Property assessed at $\frac{1}{5}$ its value and taxed at \$6.23 per hundred dollars.

302. To determine the tax rate. Each of the taxing governments (state, county, township, city, school district, etc.) prepares a budget or estimate of the money needed for the following year.

The assessed value of the property available for taxes is found from the assessors' lists.

The amount of funds needed, divided by the assessed value of the property, gives the rate which must be levied in order to obtain the required amount.

The rates for the state, county, city, school district, etc., are added to form a single rate.

The following illustration shows the method of determining a tax rate.

State Tax

State budget \$3,600,000

Assessed value of the property in the state . \$1,000,000,000

$$\frac{\$36,000,000}{3,600,000} \div \$1,000,000,000 = .36\% \text{ state rate}$$

COUNTY TAX

County budget $30,000
Assessed value of the property in the county . $4,000,000
$30,000 ÷ $4,000,000 = .75 % county rate

CITY TAX

City budget $12,000
Assessed value of property in the city $1,000,000
$12,000 ÷ $1,000,000 = 1.20 % city rate

SCHOOL TAX

School district budget $24,000
Assessed value of the property in school district $1,000,000
$24,000 ÷ $1,000,000 = 2.40 % school tax rate
4.71 % total tax rate

Problem. Mr. Owen has a building worth $15,000.00, which is assessed at $\frac{1}{3}$ of its real value. What is the tax on the building at the above rate?

SOLUTION. Assessed value, $5,000.00.
Rate, 4.71 %.
.0471 × $5,000.00 = $235.50, tax.

Oral Work

Explain why the tax rate in your city may be different from the rate in a neighboring city.

Written Work

1. Find the tax rate (approximate).
State budget, $2,850,000.
Assessed value of property in the state, $1,450,000,000.
State rate?
County budget, $33,000.
Assessed value of property in county, $5,600,000.
County rate?
City budget, $14,800.
Assessed value of property in city, $1,345,000.
City rate?
School district budget, $32,500.
Assessed value of property in school district, $1,345,000.
School tax rate?
Total rate?

2. The following is the tax rate in the town of M.

State tax	$ .60
County tax	1.25
Township tax	.26
City tax	2.18
School district tax	2.86
	$7.15

In this town, property is assessed at $\frac{1}{3}$ of its real value. Snyder owns the following property:

	Real Values
Residence	$ 6,500.00
Store	4,950.00
Office building	13,000.00

What is his total tax?

3. How much of this tax goes to the state, and how much to each of the other taxing units?

4. What per cent of his tax goes to each of the taxing units?

5. The assessed value of Miller's real estate is $13,395.00. What is his total tax?

6. How much school tax does Miller pay?

CHAPTER XXXVI

CUSTOMS DUTIES

303. Tax on Imports. The national government collects a tax from persons who import certain kinds of merchandise into the United States from foreign countries. The taxes thus imposed and collected are called **Customs Duties.**

The process of importing goods is, very briefly, as follows:

The foreign shipper prepares an invoice of the goods, showing their value in the coinage of the exporting country.

Invoices of goods with a value of more than $100.00 are certified by the United States consul in the foreign city. The consul retains one copy of the invoice and sends another copy to the collector of the United States port to which the goods are shipped.

When the goods are delivered to the steamship company, the shipper receives a bill of lading, which is a receipt for the merchandise. This bill of lading and an invoice are sent to the American importer.

When the goods arrive at port in this country, the importer presents the invoice and the bill of lading at the custom house, pays the duty, and receives his merchandise.

Customs duties are of two kinds: **ad valorem** and **specific.**

304. Ad Valorem Duties. "Ad valorem" means: *according to value.* When an ad valorem duty is placed on goods, the tax is a certain per cent of the value of the merchandise. Ad valorem duties are not computed on fractions of a dollar. If the cents are less than fifty, they are dropped; if fifty or more, they are considered an additional dollar.

To compute ad Valorem duty:

Reduce the value of the goods stated in foreign money to United States money. Multiply the value by the tariff rate.

The Secretary of the Treasury proclaims the value of foreign coinage in terms of United States money, and the most recent proclamation is used as the basis of money reduction. A copy of such a proclamation will be found on page 235.

TREASURY DEPARTMENT.
Customs Cat. No. 7551.

CONSUMPTION ENTRY.

No. 10650
Date of Entry 7/19/22
Port of Chicago

DISTRICT OF Chicago

MERCHANDISE IMPORTED BY John Jones & Co

in the Br S/s Laconia, ——, Master, from Palermo

on Apr 2, Invoice No. 2108, Dated at Palermo Italy, on Mar 21 1922

Forwarded from N Y. ; under I. T. No. 57926, Dated Apr 4 1922
(Strike out this line for direct importations.)

MARKS.	NOS.	PACKAGES AND CONTENTS.	QUANTITY	INVOICE VALUE	DUTIABLE VALUE.	RATE OF DUTY.	DUTIES
J J & C.	100/108	9 Cases		Lire			
		Cheese		286 48	56	20%	11 00
		Olives	2 gals	6 32	1	15¢ gall	30
		Figs	108 lbs	26 60	5	2¢ lb	2 16
		Unshelled Walnuts	76 lbs	17 47	3	2¢ "	52
		Olive Oil in 5 gal tins	48 gals	347 56	67	20¢ gal	9 60
		Tomato sauce		99 00	19	25%	4 75
		Raw Wool	5 lbs	13 17	3	free	—
				791 50	153		$ 28 33
		Gross Lire invoiced 871.50					
		Less non dutiable charges 30.00					
		Inland freight 17.00					
		Consular fees 13.00		791 50 dutiable value			
		1 lira = .193¢					

Estimated duty paid, $ 28 33

Bond, Cat. No. ....., taken.

c2—7117

John Jones & Co
Importer.

Problem. What is the duty on an importation of woolen dress goods valued at £30, the tariff rate being 35 % ad valorem?

SOLUTION. Value of shipment in foreign coinage, £30.
Value in United States coinage, $146.00.
35 % of $146.00 = $51.10, amount of duty.

305. Specific Duties. Specific duties are levied according to the quantity of the goods imported, without respect to their value. The rate is usually stated as a certain number of cents per pound, per hundred pounds, per ton (2,240 lb.), per yard, or other standard of measure. Allowance is sometimes made for tare and breakage.

To compute specific duty. When the metric system of measurement is used in the foreign invoice, the quantity must be reduced to the American standards. Duties are not computed on fractions of a unit of measure. Fractions less than $\frac{1}{2}$ are dropped; $\frac{1}{2}$ or larger fractions are considered as an additional unit.

Problem. The tariff rate on macaroni is 1 cent per pound. What is the amount of the duty on a shipment of 200 kilos of macaroni?

SOLUTION. 1 kilo = 2.2046 lb.
200 kilos = 440.92 lb., or 441 lb.
Duty on 441 lb. at 1 cent per pound is $4.41.

306. Ad Valorem and Specific Duties. In some cases both an ad valorem and a specific duty are levied on the same importation of merchandise.

Problem. The duty on perfumery containing alcohol is 40 cents a pound and 60 % ad valorem. What is the amount of the tariff tax on an importation of 10 kilos of perfumery valued at 970 francs?

SOLUTION. 10 kilos = 22.046 lb., or 22 lb.

$ .40 duty per pound
22 number of pounds
$8.80 specific duty

1 franc = $.193
$.193 × 970 = $187.21 value of perfumery
60 % of $187 = $112.20 ad valorem duty

$ 8.80 specific duty
112.20 ad valorem duty
$121.00 total duty

307. The Tariff. Congress revises the tariff law at various intervals. The law prescribes the tariffs, or rates, to be charged on various imports. The *free list* comprises those articles on which no duty is placed.

The following are some of the customs duties from a recent tariff.

Artificial flowers, 60% ad valorem
Cabbage seed, 6 cents per pound
Castile soap, 10% ad valorem
Cheese, 20% ad valorem
Cigars, $4.50 per pound, and 25% ad valorem
Cocoa, unsweetened, 8% ad valorem
Cotton batting, 25% ad valorem
Lithographed booklets, 7 cents per pound
Oriental rugs, 50% ad valorem
Poppyseed, 15 cents per bushel
Silk ribbons, 45% ad valorem
Toys, 35% ad valorem
Tulip bulbs, $1.00 per thousand
Woolen dress goods, 35% ad valorem

Written Work

Refer to the tariff rates above, and compute the duty on the following importations of merchandise.

1. 50,000 tulip bulbs from Holland.
2. Woolen dress goods from England, valued at 125 pounds sterling.
3. Castile soap from Italy, valued at 40 lires.
4. An invoice of cotton batting valued at £20 15*s.* 6*d.*
5. Lithographed booklets, weight 247 lb. 9 oz.
6. A shipment of cheese valued at 820 marks.
7. Silk ribbons from Paris, invoiced at 5,400 francs.
8. Toys from Germany, valued at 1,000 marks.
9. An invoice of 3,000 cigars from Germany. The cigars weigh $12\frac{3}{8}$ ounces per hundred, and are invoiced at 52 marks per thousand.
10. The tariff rate on Cuban cigars is 20% less than the rate applying to cigars shipped from other countries. What is the duty on a shipment of 7 cases, each containing 3000 cigars, weight $14\frac{3}{8}$ oz. per hundred; value, $50.00 per thousand?

Review Problems

Written Work

1. The following differential piece work rates apply in a factory:

19 pieces or less	$1.60 each
20	1.65
21	1.69
22	1.73
23	1.75
24	1.77
25	1.79
26	1.83½
27	1.89½
28	1.95
29 or more	2.01

What is the pay roll for the week, if the production records show the following?

Employee	Pieces Produced
A	26
B	29
C	21
D	16
E	30
F	25
G	28

2. An attorney takes $3,500.00 of accounts to collect for a merchant, under an agreement for commissions as follows:

10% on accounts less than 3 months old.
15% on accounts more than 3 months and less than 6 months old.
20% on accounts more than 6 months and less than 12 months old.
25% on accounts more than 12 months old.

He makes collections as follows:

$ 675.90	two months old.
139.80	four months old.
1,465.30	ten months old.
617.19	fifteen months old.

What is his total commission and how much does the merchant realize from the collections?

3. Show the annual depreciation for 5 years on an asset which cost $15,000.00 with an estimated life of 12 years and an estimated residual value of $2,800.00.

(*a*) Straight line method.
(*b*) Sum of years' digits method, diminishing charge.
(*c*) Fixed rate on the diminishing value, using 13.05%.

4. What is the cost of insurance on a building valued at $12,500.00, insured for 5 years at 80% of its value if the rate is 53.4 cents?

5. Assume that this policy contains an 80% co-insurance clause and that, at the date of a fire, market values have increased to such an extent that the property had a sound value of $15,000.00. If there was a loss amounting to $8,000.00, how much should the insured receive?

6. A one-year policy was written on May 7 for $12,000.00 at a rate of 42 cents. The policy was canceled July 22. What portion of the premium was refunded if the policy was canceled by

(*a*) the insurance company?
(*b*) the insured?

7. Jones owns property taxed at $5.92 on $\frac{2}{5}$ of an assessed valuation of $19,300.00. What is the tax?

8. What is the duty on an importation of 10 cases of cigars, each case containing 3,000 cigars, weight $15\frac{5}{8}$ ounces per hundred, value $65.00 per thousand? Rate: $4.50 per pound and 25% ad valorem.

BUSINESS ORGANIZATION

CHAPTER XXXVII

INDIVIDUAL PROPRIETORSHIP

308. Types of Organization. Business enterprises are usually organized in one of the following forms:

Individual Proprietorship;
Partnership;
Corporation.

Definitions. The things of value owned by an individual or a business are called **resources** or **assets.** These include amounts due from debtors because these claims are valuable.

The debts of a business or an individual are called **liabilities.**

The excess of the assets over the liabilities is called the **Net Worth, Net Assets, Net Resources,** or **Capital.**

If the liabilities exceed the assets, the difference is called the **Net Insolvency.**

309. Statement of Assets and Liabilities. The net worth or net insolvency is shown by a statement called Statement of Assets and Liabilities, or Balance Sheet. The following is such a statement showing the financial condition of the business owned by J. W. Waterbury.

J. W. WATERBURY
STATEMENT OF ASSETS AND LIABILITIES
June 30, 1922

	Assets			
Real Estate	Store and lot	7,500 00		
Cash	Balance on hand	3,500 00		
Merchandise	Inventory	6,325 00		
James Fitch	Owes the business	110 00		
F. G. Beach	Owes the business	65 35		
Total Assets			17,500	35
	Liabilities			
T. E. Madison	Business owes him	123 60		
Henry Marsden	Business owes him	962 85		
Total Liabilities			1,086	45
J. W. Waterbury's net worth			16,413	90

The statement might be prepared in the following form:

J. W. WATERBURY
STATEMENT OF ASSETS AND LIABILITIES
June 30, 1922

Assets		*Liabilities*	
Cash	$3,500.00	T. E. Madison	$123.60
Merchandise	6,325.00	Henry Marsden	962.85
James Fitch	110.00	J. W. Waterbury, Capital	16,413.90
F. G. Beach	65.35		
Real Estate	7,500.00		
	17,500.35		17,500.35

310. Notes Receivable and Notes Payable. Notes receivable include such written obligations as promissory notes and acceptances, payable to the business. **Notes payable** include such written obligations as promissory notes and acceptances, payable by the business.

Interest-bearing notes are worth their face plus accrued interest. The accrued interest on notes receivable should therefore be included among the assets in the statement of assets and liabilities, and the accrued interest on notes payable should be included among the liabilities.

Problem. J. C. Danner is making a statement of assets and liabilities on December 31, 1922. Among his assets is a note for $600.00, made by I. M. Henderson on December 1, 1922, due 90 days after date, with interest at 6%. Thirty days have expired since the note was made, and thirty days' interest has therefore accrued. The amount of this interest, being an obligation payable to the business, should be entered among the assets.

Among his liabilities is a note for $500.00, made by him on December 16, payable in two months, with interest at 6%. Fifteen days' interest has accrued on this note. The amount of the accrued interest, $1.25, being an obligation payable by the business, should be entered among the liabilities.

Written Work

1. Complete the following statement:

HENRY PETERMAN

STATEMENT OF ASSETS AND LIABILITIES

June 30, 1922

	Assets		
Cash		$2,365.90	
Merchandise		8,926.95	
Notes Receivable	H. E. Benton's note	975.00	
Interest		?	
Smith and Edwards		825.19	
Total Assets			xx,xxx.xx
	Liabilities		
Notes Payable	Favor of E. A. Clark	360.00	
Interest		?	
William Davis		38.26	
Total Liabilities			xxx.xx
Henry Peterman, Net Worth			xx,xxx.xx

H. E. Benton's note was dated June 6, payable in 3 months, with interest at 6%. Peterman's note in favor of Clark was made on June 18, with interest at 6%.

2. Prepare a statement of assets and liabilities from the following facts relative to the financial condition of O. E. Benton, December 31, 1922:

Cash, $4,624.27; merchandise inventoried at $12,417.95; value of store and lot, $5,000.00; furniture and fixtures valued at $420.00; delivery equipment cost $300.00 one year ago, depreciation being estimated at 10% per year.

Personal accounts:

F. E. Fairchild owes the business $215.20.
Geo. Finch owes the business $28.45.
Edgar Knight owes the business $103.25.
The business owes Justin & Co. $856.75.
The business owes F. R. Rickert $49.00.

Notes and Acceptances:

On October 27, 1922, Benton gave a note to Eldridge and Peabody for $300.00. The note is due 4 months from date and bears interest at 6%. Compute interest for exact number of days.

On December 26, 1922, Benton accepted a draft payable 30 days after sight for $85.00, without interest, in favor of S. B. Gridley.

311. Profit and Loss Statement. The Profit and Loss Statement shows the earnings, expenses and net profit earned by a business during a stated period of time. The following is an illustration of such a statement.

D. L. BAILEY

PROFIT AND LOSS STATEMENT

For the Month of December, 1922

Net Sales		$6,210.72
Deduct Cost of Goods Sold:		
Inventory, December 1, 1922	$1,233.60	
Purchases, December	4,394.80	
Total	5,628.40	
Less Inventory, December 31, 1922	975.62	4,652.78
Gross Profit on Sales		1,557.94
Deduct Expenses:		
Rent	175.00	
Salaries	460.00	
Advertising	56.20	
Heat and Light	32.00	
Delivery Expense	56.35	
Insurance and Taxes	73.85	
Depreciation	70.00	
General Expense	92.74	
Bad Debts	15.75	1,031.89
Net Profit on Operations		526.05
Add Net Interest Earned:		
Interest Earned on Money Loaned	35.60	
Interest Paid on Money Borrowed	6.92	28.68
Net Profit for the Month		$554.73

Written Work

1. Copy the foregoing statement, and in a column at the right of the money values, show what per cent each item is of the net sales.

2. Prepare a Profit and Loss Statement with a percentage analysis from the following facts: The business is owned by Oscar Clark, and the statement covers the year 1922.

Net Sales, 1922	$27,546.27.
Purchases, 1922	22,860.39.
Inventory, Jan. 1, 1922	4,219.26.
Inventory, December 31, 1922	5,126.99.

Interest:

6 months' accrued interest at 6% on Clark's note of $345.00 in favor of Jenkins.

30 days' accrued interest at 5% on Clark's note of $280.00 in favor of Michelson.

2 months' accrued interest on Bailey's note in favor of Clark. Face of note, $375.00; rate, 6%.

28 days' interest has accrued on a note given by Snyder to Clark. Face of note, $240.00; rate, 6%.

An extra salesroom was rented for the year at a monthly rental of $20.00.

Salaries:

Clerk,	12 months at	$75.00 per month.
Delivery boy,	3 months at	$35.00 per month;
	9 months at	$40.00 per month.

Heat and light, $119.25.

Feed and care of delivery horse, $64.00.

Value of store building, $3,000.00; value of lot, $800.00.

Store and lot assessed at $\frac{1}{3}$ of its value and taxed at a rate of $2.16.

Stock valued at $4,000.00; assessed at $\frac{1}{3}$ of its value and taxed at a rate of $2.16.

An insurance policy was carried on the store and one on the stock.

Store insured at 80% of its value; rate, $1.15.

Stock insured under a policy of $3,000.00; rate, $1.15.

Annual depreciation on store building charged at 3%.

Loss from bad debts estimated at $\frac{1}{4}$% of net sales.

General expense, $114.17.

CHAPTER XXXVIII

PARTNERSHIP

"A **partnership** or **firm** is an association composed of two or more persons, formed for the purpose of carrying on, as co-owners, a business with a view to profit." * Partners are liable individually for the debts of the partnership which cannot be paid from the resources of the partnership.

312. Profit and Loss Ratio. Since the profits are to be shared by the partners they should agree as to the ratio for dividing them. The following methods of dividing profits are frequently found:

(*a*) Equally.
(*b*) In an arbitrary ratio.
(*c*) In the ratio of the capitals at the beginning of the period.
(*d*) Interest on capital at the beginning of the period, and the balance in an arbitrary ratio.
(*e*) In the average capital ratio.

Each of these methods will be discussed in this chapter.

(*a*) *Equally*

This ratio is usually adopted when the partners contribute equal values of skill, services, and capital. It is the simplest of all ratios. Where there has been no agreement, the law assumes that the profits will be divided equally.

Written Work

1. Oldham, Barnes, and Snowden each invests $4,000.00 in a partnership, the gains and losses of which are to be shared equally. The following statements show the method of ascertaining and dividing the profits.

* Uniform Partnership Law.

OLDHAM, BARNES & SNOWDEN
PROFIT AND LOSS STATEMENT
For the Year Ending December 31, 1922

Net Sales		$18,226.55	
Less Cost of Goods Sold:			
Purchases	$17,847.90		
Less Inventory, Dec. 31, 1922	3,472.00	xx,xxx.xx	
Gross Profit on Sales			x,xxx.xx
Deduct expenses:			1,245.25
Net Profit on Operations			x,xxx.xx
Add Net Interest and Discount:			
Discount on Purchases	135.85		
Interest Earned	3.50	139.35	
Discount on Sales	57.80		
Interest Paid	28.35	86.15	xx,xx
Net Profit for the Year			x.xxx.xx
Divided as follows:			
G. F. Oldham $\frac{1}{3}$			xxx.xx
D. M. Barnes $\frac{1}{3}$			xxx.xx
Wm. A. Snowden $\frac{1}{3}$			xxx.xx
Total (as above)			x,xxx.xx

OLDHAM, BARNES & SNOWDEN
STATEMENT OF PARTNERS' CAPITALS
Year Ending December 31, 1922

	Oldham	*Barnes*	*Snowden*	*Together*
Investments, Jan. 1, 1922	$4,000.00	$4,000.00	$4,000.00	$12,000.00
Add Profits for 1922	xxx.xx	xxx.xx	xxx.xx	x,xxx.xx
Total	x,xxx.xx	x,xxx.xx	x,xxx.xx	xx,xxx.xx
Less Cash Withdrawn during Year	150.00	125.00	160.00	xxx.xx
Net Worth, December 31, 1922	x,xxx.xx	x,xxx.xx	x,xxx.xx	xx,xxx.xx

OLDHAM, BARNES & SNOWDEN
BALANCE SHEET
December 31, 1922

Assets		*Liabilities*		
Land and Buildings	$6,400.00	Accounts Payable		$1,800.50
Cash	3,240.90	Notes Payable		500.00
Merchandise	3,472.00	Total Liabilities		2,300.50
Accounts Receivable	3,132.70	Partners' Net Worth (as above)		
Notes Receivable	275.00	G. F. Oldham	x,xxx.xx	
Accrued Interest	3.50	D. M. Barnes	x,xxx.xx	
		Wm. A. Snowden	x,xxx.xx	xx,xxx.xx
	xx,xxx.xx			xx,xxx.xx

2. Prepare completed copies of the following statements:

WADHAMS & LEE
PROFIT AND LOSS STATEMENT
Year Ending December 31, 1922

Net Sales		$11,147.80
Less Cost of Goods Sold:		
Inventory, January 1, 1922	$ 3,000.00	
Purchases	12,416.20	
Total	xx,xxx.xx	
Less Inventory, December 31, 1922	x,xxx.xx	x,xxx.xx
Gross Profit on Sales		x,xxx.xx
Deduct Expenses:		
Taxes	56.00	
Insurance	80.00	
General Expense	1,378.90	x,xxx.xx
Net Loss on Operations		xx.xx
Add Net Financial Expense:		
Sales Discount	86.40	
Less Purchase Discount	69.25	xx.xx
Net Loss for the Year		xx.xx

WADHAMS & LEE
STATEMENT OF PARTNERS' CAPITALS
Year Ending December 31, 1922

	Wadhams	*Lee*	*Together*
Investments, January 1, 1922	$ 00.00	$6,000.00	$12,000.00
Less Drawings during the Year	6.93	340.70	767.63
Capitals before Dividing Loss	x,xxx.xx	x,xxx.xx	xx,xxx.xx
Net Loss for the Year	xx,xx	xx,xx	xx.xx
Balances, December 31, 1922	x,xxx.xx	x,xxx.xx	xx,xxx.xx

WADHAMS & LEE
BALANCE SHEET
December 31, 1922

Assets		*Liabilities*		
Real Estate	$4,195.00	Notes Payable		$2,000.00
Cash	246.90	Accounts Payable		1,250.33
Merchandise Inventory	5,724.60	Total Liabilities		x,xxx.xx
Notes Receivable	1,600.00	Partners' Capitals		
Accounts Receivable	2,640.35	James Wadhams	x,xxx.xx	
		Edwin C. Lee	x,xxx.xx	xx,xxx.xx
	xx,xxx.xx			xx,xxx.xx

(b) *In an Arbitrary Ratio*

When there is a variation in the capital contributions of the partners, their business ability, or the amount of time devoted to the business, the division of the profits may be in an arbitrary ratio intended to give recognition to such a variation.

Written Work

1. Davis, an attorney, admits two younger attorneys, Fritz and Healy, into partnership. Since Davis has an established practice, it is agreed that he shall receive $\frac{2}{5}$ of the net earnings, and that Fritz and Healy shall divide the remainder equally. The net earnings the first year were \$8,000.00; the second year they were \$9,260.00. Compute the division of the earnings for each year.

2. Two firms of public accountants decide to consolidate. The old firm of Jones and Robinson shared profits in the ratio of $\frac{3}{5}$ to Jones and $\frac{2}{5}$ to Robinson. The firm of Sikes and Wilson divided their profits in the ratio of $\frac{4}{5}$ to Sikes and $\frac{1}{5}$ to Wilson. It is agreed that in the new partnership of Jones, Robinson, Sikes and Wilson, the profits shall be divided as follows:

The books shall be kept in such a manner as to show the earnings from clients of each of the old firms, and the earnings due to entirely new business. 60 % of the earnings from clients of each old firm shall go to the old firm in its former profit and loss ratio. The remaining 40 % shall go into a common fund for equal division among the four partners. Any earnings due to new business shall be divided equally among the four partners.

The profits for three years after the consolidation were:

	First Year	Second Year	Third Year
From J. & R. Clients	\$6,200.00	\$7,500.00	\$7,800.00
From S. & W. Clients	5,480.00	5,960.00	6,200.00
From new business	7,000.00	7,800.00	8,100.00

Show the annual division of profits.

(c) *In the Ratio of the Capitals at the Beginning of the Period*

This method makes the capital investments the deciding factor in the division of the profits. The profits are divided in the ratio of the capitals at the beginning of the year, six months' period, or other period for which the profits are being shared. Additional contributions during the period, or drawings, have no effect on the ratio.

Problem. Nelson invested \$2,000.00, Baker \$3,000.00, and Anderson \$5,000.00, agreeing to divide profits in the ratio of their investments. The profits the first year were \$1,256.70. How should the profits be divided?

SOLUTION.

PARTNER	INVESTMENT	RATIO	PROFITS
Nelson	\$2,000.00	$\frac{2}{10}$	\$ 251.34
Baker	3,000.00	$\frac{3}{10}$	377.01
Anderson	5,000.00	$\frac{5}{10}$	628.35
Total			\$1,256.70

Written Work

1. A invests \$4,500. B invests \$6,000. C invests \$7,500. Profits and losses are to be shared in proportion to investment. The first year there was a gain of \$2,461.25. What was the amount of profit received by each partner?

2. On July 1, 1921, D. M. Paine, F. R. Adamson, and B. O. Cone entered into partnership. Gains and losses were to be shared in proportion to investments.

Paine, who had been conducting a grocery business, turned in to the partnership his entire assets and liabilities, as shown by the following statement:

D. M. PAINE

STATEMENT OF ASSETS AND LIABILITIES

July 1, 1921

	Assets		
Real Estate	Store and Lot	\$3,000.00	
Furniture and Fixtures		350.00	
Cash	On hand	316.75	
Merchandise	Per inventory	4,135.60	
Accounts Receivable	Per ledger	1,237.80	
Total			x,xxx.xx
	Liabilities		
Accounts Payable	Per ledger	215.60	
Notes Payable	Per bill book	1,245.00	
Interest	Accrued on notes payable	3.11	
Total			x,xxx.xx
Net Capital	D. M. Paine		x,xxx.xx

He gave his note with interest at 6 %, dated July 1, 1921, payable to Paine, Adamson & Cone, for an amount sufficient to make his total investment $8,000.

Adamson invested the following:

Merchandise, $1,200.

A note in his favor, signed by O. G. Dunlap, for $2,500, dated April 1, 1921, due in 1 year, bearing 6 % interest. (Is this note worth its face, its face plus accrued interest, or its face minus the discount until maturity?)

Cash to make his total investment $4,000.

Cone invested a draft for $740, drawn by himself on Graf & Knight, on June 18, 1921, payable 30 days after sight, without interest. Graf & Knight accepted the draft on June 20, 1921. This draft was received by the partnership at its discounted value. (Used 6 %.) Cone also invested enough cash to make a total investment of $3,000.

Prepare a statement of assets and liabilities, showing the net capital of the firm of Paine, Adamson & Cone, at the beginning of business on July 1, 1921. The following illustration will show the form of the statement:

PAINE, ADAMSON & CONE

STATEMENT OF ASSETS AND LIABILITIES

July 1, 1921

	Assets			
Cash	Invested by Paine	xxx.xx		
	Invested by Adamson	xxx.xx		
	Invested by Cone	x,xxx.xx	x,xxx.xx	
Merchandise	Invested by Paine	x,xxx.xx		
	Invested by Adamson	x,xxx.xx	x,xxx.xx	
Real Estate			x,xxx.xx	
Furniture and Fixtures			xxx.xx	
Notes Receivable	Paine's note favor P. A. & C.	xxx.xx		
	Dunlap's note favor Adamson	x,xxx.xx		
	Cone's draft on G. & K.	xxx.xx	x,xxx.xx	
Interest	Accrued on Dunlap's note		xx.xx	
Accounts Receivable	from Paine		x,xxx.xx	
Total				xx,xxx.xx
	Liabilities			
Notes Payable	Received from Paine		x,xxx.xx	
Interest	Accrued on note rec'd from Paine	x.xx		
	Discount on Cone's draft	x.xx	x.xx	
Accounts Payable	Received from Paine		xxx.xx	
Total				x,xxx.xx
Net Capital	Paine, Adamson & Cone			xx,xxx.xx

What fraction of the net gains should each partner receive if gains are shared in proportion to capital invested?

On December 31, 1921, a statement of assets and liabilities of the firm showed the following facts:

STATEMENT OF ASSETS AND LIABILITIES

PAINE, ADAMSON & CONE

December 31, 1921

	Assets		
Real Estate		$ 2,910.00	
Furniture and Fixtures		339.50	
Cash	On hand	2,215.60	
Merchandise	Inventory	15,465.70	
Accounts Receivable		4,285.37	
Total			xx,xxx.xx
	Liabilities		
Accounts Payable		1,941.20	
Total			x,xxx.xx
Net Capital Paine, Adamson & Cone			xx,xxx.xx

What was the increase in the capital of the business during the 6 months' period?

This increase represents the profits remaining in the business. During the 6 months' period, the partners withdrew profits as follows:

PARTNER	AMOUNT
Paine	$ 1,413.32
Adamson	1,206.66
Cone	1,654.99
	$ 4,274.97

What was the total gain for the 6 months' period?

Prepare a statement showing how much of the total profit each partner was entitled to receive.

Also prepare a Statement of Partners' Capital Accounts, showing the original investments, the total profits of each partner, the amounts drawn out, and the capitals at December 31, 1921.

The profits for the next six months are to be divided on the basis of the capitals at December 31, 1921. At June 30, 1922, a statement of assets and liabilities showed the following facts.

PAINE, ADAMSON & CONE

STATEMENT OF ASSETS AND LIABILITIES

June 30, 1922

Assets		*Liabilities*	
Cash	$1,846.03	Accounts Payable	$1,240.65
Merchandise	8,420.50	Notes Payable	1,000.00
Accounts Receivable	5,627.38	Interest Accrued on Notes Payable	13.33
Real Estate	2,820.00	Net Capital	xx,xxx.xx
Furniture and Fixtures	329.00		
	xx,xxx.xx		xx,xxx.xx

The partners drew nothing during the period.

Prepare a statement showing the computation of the net profit or loss for the period and the distribution thereof. Also prepare a Statement of the Capital Accounts covering the period from January 1, 1922, to June 30, 1922.

(d) Interest on Capital at the Beginning of the Period, and the Balance in an Arbitrary Ratio

The purpose of this method is to allow each partner profit equal to a fair return on his capital before dividing the remainder of the profits in an arbitrary ratio. It is used when the partners' capitals are in a different ratio from that in which they divide profits. Three different conditions may be met.

(1) When the profits are greater than the agreed interest.

Problem. At the beginning of 1922 Briggs invests $8,000.00 and Duncan invests $6,000.00. It is agreed that in the division of profits each partner shall be allowed interest at the rate of 6% per annum, and that the remaining profits shall be divided equally. The profits for the year were $2,800.00. How should they be divided?

SOLUTION.	*Briggs*	*Duncan*	*Together*
Total Profits			$2,800.00
Interest on Capital	$ 480.00	$ 360.00	840.00
Balance Equally	980.00	980.00	$1,960.00
Total	$1,460.00	$1,340.00	

(2) When the profits are less than the agreed interest.

Problem. Assume that the profits were only $600.00. How should they be divided?

SOLUTION.	*Briggs*	*Duncan*	*Together*
Total Profits			$600.00
Credits for Interest	$480.00	$360.00	840.00
Debits for excess of Interest over Profits	120.00	120.00	$240.00
Net profits distributed	$360.00	$240.00	

(3) When a loss occurs.

Problem. Assume a loss of $500.00. How should it be divided?

SOLUTION. In spite of the fact that there is a loss, the interest must be allowed in accordance with the agreement. Then the sum of the loss and the interest is divided in the agreed ratio.

	Briggs	*Duncan*	*Together*
Net Loss			$ 500.00
Interest on Capital — Credited	$480.00	$360.00	840.00
Total Loss and Interest — Debited	670.00	670.00	$1,340.00
Net Debit for Loss	$190.00	$310.00	

Written Work

1. A, B, and C entered into partnership at January 1, with the following investments: A, $6,000.00; B, $10,000.00; C, $20,000.00. It is agreed that interest shall be allowed on investments at the rate of 6 % per annum and that the remaining profits shall be divided equally. Show the division of profits and losses under the following cases:

(*a*) Net gain for the year, $5,460.00.
(*b*) Net gain for the year, $1,860.00.
(*c*) Net loss for the year, $900.00.

2. Jones and Morgan began a partnership July 1, 1922; Jones invested $9,500.00 and Morgan invested $11,500.00. Each partner was to receive interest on his investment at the rate of 5 % per year, the remaining net profit or loss to be divided equally. On December 31, 1922, a statement of the losses and gains of the firm showed a net gain of $815.00. What amount of interest and remaining profit should each partner have received?

3. Wallace and Montgomery entered into partnership on January 1, 1922. Each of these men had formerly conducted a business of his

own, and each turned into the partnership his entire resources and liabilities, as shown by the following statements:

L. F. MONTGOMERY

STATEMENT OF ASSETS AND LIABILITIES

December 31, 1921

Assets		
Real Estate	$3,000.00	
Fixtures	419.75	
Cash	1,215.75	
Merchandise	3,565.80	
Accounts Receivable	625.90	
Interest	4.18	
Total		x,xxx.xx
Liabilities		
Accounts Payable	1,275.90	
Notes Payable	1,418.00	
Total		x,xxx.xx
Net Investment, L. F. Montgomery		x,xxx.xx

IRVING WALLACE

STATEMENT OF ASSETS AND LIABILITIES

December 31, 1921

Assets		
Fixtures	$ 319.50	
Cash	865.00	
Merchandise	3,462.90	
Accounts Receivable	2,419.17	
Total		x,xxx.xx
Liabilities		
Accounts Payable	1,425.00	
Total		x,xxx.xx
Net Investment, Irving Wallace		x,xxx.xx

It is agreed that each of the partners shall receive interest on his investment at the rate of 6 % per annum, remaining profit to be divided equally.

On December 31, 1922, the following statement showed the condition of the firm:

WALLACE AND MONTGOMERY
STATEMENT OF ASSETS AND LIABILITIES
December 31, 1922

Assets		
Real Estate	$3,000.00	
Fixtures	632.86	
Cash	1,462.80	
Merchandise	7,292.90	
Accounts Receivable	1,853.27	
Interest Accrued	2.09	
Total		xx,xxx.xx
Liabilities		
Accounts Payable	615.20	
Notes Payable	1,418.00	
Total		x,xxx.xx
Net Capital, Wallace and Montgomery		xx,xxx.xx

There were no drawings.

Prepare a statement showing the computation and division of the profits, and a statement of the capitals.

(e) *In the Average Capital Ratio*

This method is adopted when there are investments and withdrawals during the period which affect the capitals to such an extent that it is not considered fair to divide the profits in the ratio of the capitals at the beginning of the period.

The method of computing the average capital ratio is shown in the following problem.

Problem. Compute the average capitals of Smith and Jones, whose capital accounts appear below.

JAMES SMITH, CAPITAL

1922			1922		
March 1	Withdrawal	$500.00	January 1	Investment	$5,000.00
August 1	"	$400.00	June 1	"	2,000.00
			October 1	"	3,000.00

WALTER JONES, CAPITAL

1922			1922		
February 1	Withdrawal	$400.00	January 1	Investment	$8,000.00
July 1	"	600.00	May 1	"	2,000.00
			December 1	"	1,000.00

SOLUTION:

JAMES SMITH

	DATE	MONTHS TO END OF YEAR	AMOUNT	PRODUCT	
Investments:					
	Jan. 1	12	$5,000.00	$60,000.00	
	June 1	7	2,000.00	14,000.00	
	Oct. 1	3	3,000.00	9,000.00	$83,000.00
Withdrawals:					
	Mar. 1	10	500.00	5,000.00	
	Aug. 1	5	400.00	2,000.00	7,000.00
Net:					$76,000.00

$76,000.00 ÷ 12 = $6,333.33, Average Capital

WALTER JONES

	DATE	MONTHS TO END OF YEAR	AMOUNT	PRODUCT	
Investments:					
	Jan. 1	12	$8,000.00	$96,000.00	
	May 1	8	2,000.00	16,000.00	
	Dec. 1	1	1,000.00	1,000.00	$113,000.00
Withdrawals:					
	Feb. 1	11	400.00	4,400.00	
	July 1	6	600.00	3,600.00	8,000.00
Net:					$105,000.00

$105,000.00 ÷ 12 = $8,750.00 Average Capital

The profits are divided in the ratio of the average capitals; that is, in the ratio of $6,333.33 and $8,750.00. But these numbers were obtained by dividing $76,000.00 and $105,000.00 by 12. Therefore the ratio of 76 and 105 is the same as the ratio of the average capitals, and these numbers are used as the basis of the division of profits. In other words, if for any reason the average capitals must be known, it is necessary to divide by 12; but for purposes of dividing profits all that is required is the average capital **ratio** and for this purpose it is not necessary to divide by 12.

Problem. The profits of the firm of Smith and Jones for the year 1922 were $10,860.00. How should they be divided?

SOLUTION. 76 + 105 = 181

Smith's share =	76/181 of $10,860 =	$ 4,560.00
Jones's " =	105/181 " 10,860 =	6,300.00
Total		$10,860.00

Written Work

1. Divide in the average capital ratio the profits of the firm of Bates, Wharton and Dutton for the year 1922.

F. S. BATES, CAPITAL

1922			1922		
March 1	Withdrawal	$2,500.00	Jan. 1	Investment	$7,500.00
			May 1	"	3,000.00
			Nov. 1	"	2,000.00

D. E. WHARTON, CAPITAL

1922			1922		
July 1	Withdrawal	$650.00	Jan. 1	Investment	$6,000.00
Oct. 1	"	600.00	Apr. 1	"	3,000.00
			Oct. 1	"	4,000.00

S. N. DUTTON, CAPITAL

1922			1922		
April 1	Withdrawal	$200.00	Jan. 1	Investment	$2,000.00
			July 1	"	4,000.00

The net profit for the year was $9,650.75.

CHAPTER XXXIX

INSOLVENCY AND BANKRUPTCY

INDIVIDUALS, firms, or corporations which are not able to pay their debts when due, or whose liabilities exceed their assets, are said to be insolvent. Under the Federal law, any person in a condition of insolvency may be declared a bankrupt if he so desires. His assets, with the exception of certain exempt property, are turned over to a trustee representing the creditors, and divided proportionately among them, all creditors except those whose claims are preferred or secured receiving the same fractional part or per cent of the amount due them.

313. Bankruptcy Proceedings. The creditors of any insolvent person except a "wage-earner" earning less than $1,500 per year, or a "person engaged chiefly in farming or tillage of the soil," may institute bankruptcy proceedings against him, if he has done one of the following acts:

a. Transferred or concealed any of his property with intent to delay payment or to defraud his creditors.

b. Transferred property to one creditor for the purpose of giving him an advantage over the other creditors.

c. Allowed one creditor to obtain a preference through legal proceedings.

d. Made an assignment of all his property to a trustee for creditors.

e. Admitted in writing his inability to pay his debts, and his willingness to be adjudged a bankrupt.

After a debtor has "gone into bankruptcy" and made a settlement by turning over his assets, he is legally discharged from his indebtedness. Any resources which he may possess at a later time cannot be taken in further payment of debts settled in bankruptcy.

Illustration of a Settlement in Bankruptcy. On October 20, 1922, J. D. Murphy was declared a bankrupt, and a trustee was elected by the creditors to take over his assets, convert them into cash, and pay the creditors. Mr. Murphy filed the following schedule of assets and liabilities at book values:

SCHEDULE OF ASSETS AND LIABILITIES

Assets

Cash	$ 465.00	
Merchandise	3,225.00	
Real estate	8,500.00	
Accounts receivable	3,688.18	$15,878.18

Liabilities

Taxes	$ 165.00	
Wages	233.00	
Mortgage on real estate	2,000.00	
Hartman, Oelberg & Co.	2,146.80	
D. G. Barnhard	364.00	
Franklin & Gertz	7,685.25	
D. O. Mitchell	8,146.30	$20,740.35

The trustee sold the real estate to the holder of the mortgage for $5,800 in excess of the mortgage.

Cash receipts were as follows:

From Murphy	$ 465.00	
From the sale of the real estate after paying the mortgage	5,800.00	
From the sale of merchandise	2,900.00	
From accounts receivable collected	3,500.00	$12,665.00

He made payments as follows:

Wages (which are a preferred claim and must be paid before the other liabilities)	$ 233.00	
Taxes (which are also a preferred claim)	165.00	
His fee as receiver, allowed by court	250.00	$ 648.00
Balance of cash on hand		$12,017.00

The unpaid liabilities are as follows:

Hartman, Oelberg & Co.	$2,146.80	
D. G. Barnhard	364.00	
Franklin & Gertz	7,685.25	
D. O. Mitchell	8,146.30	$18,342.35

The receiver will be able to pay $\frac{12,017.00}{18,342.35}$ or 65.5 % of the liabilities. Each creditor will receive 65.5 % of his claim. The common expression of this settlement is, " Mr. Murphy paid 65 cents on the dollar."

Written Work

1. Fred Helms made an assignment to his creditors, turning over the following assets to a receiver:

Cash on hand and in bank	$ 85.00
Merchandise inventoried at	2,300.00
Store property, book value	3,500.00
Accounts receivable	1,632.50

His liabilities were as follows:

Wages, preferred claim	$ 95.00
Accounts payable:	
Weston and Burton	1,036.49
George Seeley & Co.	378.57
J. G. O'Shea	1,275.85
Holden & Plehn	5,300.00
Wm. Meyers	3,000.00

The receiver is able to realize $2,000 on the merchandise. The store property is old and no depreciation has been deducted on Mr. Helms' books. The receiver is able to sell it for only $2,400. He collects all of the accounts receivable with the exception of $212.50.

The receiver pays the wages, $95; the expenses of the receivership, $75, and his own fee of $100, which is allowed by the court. The remaining cash is distributed among the creditors.

How much does each creditor receive?

CHAPTER XL

CORPORATIONS, STOCKS, AND BONDS

THE method and the advantages of organizing a business as a corporation may be shown by an illustration.

William Austin has $50,000.00 which he wishes to invest in a factory for manufacturing carriages.

George Fox has $30,000.00, and he is willing to join Austin in the organization of a carriage manufacturing business.

Austin and Fox believe that a capital of $100,000.00 is necessary for the successful operation of a business such as they wish to organize, but together they are able to invest only $80,000.00. In order to obtain the remaining $20,000.00, they may take one or more persons into partnership, or they may organize a corporation.

314. Partnership and Corporation Contrasted. The disadvantages of a partnership are:

a. Each partner is personally liable for the debts of the partnership.

b. Partners frequently have difficulty in withdrawing from a partnership.

Unless at the time of organizing a partnership a definite date is set when the partnership may be dissolved, no partner can withdraw from the business without the consent of the other partners. The law does not permit a partner to sell his interest in the partnership without the consent of the other partners.

c. The death of a partner dissolves the partnership.

The advantages of a corporation are:

a. Each member (stockholder) of the corporation is liable for only the amount he agrees to contribute to the capital of the corporation. His private resources cannot be taken for the debts of the corporation.

b. Any stockholder may sell his interest in the corporation without the consent of the other stockholders, and without dissolving the corporation.

c. The death or bankruptcy of a stockholder does not dissolve a corporation.

d. By the method of selling shares of stock (to be explained later), an individual may invest a small amount of money in a large corporation. The capital of the railroads and other large corporations is usually furnished by thousands of stockholders. Partnerships of such a size would be unwieldy and impracticable.

Considering these advantages, Austin and Fox decide to incorporate, and after complying with the legal requirements they prepare a subscription list on which to receive subscriptions to the capital stock.

Subscription List

The Eclipse Carriage Company

We, the undersigned, hereby subscribe for the number of shares and the amount of the Capital Stock of the Eclipse Carriage Company set opposite our names, agreeing to pay for the same on June 1, 1921.

Date		Number of Shares Face Value $100.00	Amount Par Value	Signature	Residence
1921 April	27	Five Hundred	$ 50,000	William Austin	Chicago, Ill.
	27	Three Hundred	30,000	George Fox	Chicago, Ill.
	27	One Hundred	10,000	Edward Lyons	Gary, Ind.
	29	Fifty	5,000	F. L. Robbins	Wheaton, Ill.
May	6	Twenty	2,000	P. F. Aldrich	Aurora, Ill.
	8	Twenty	2,000	C. B. Mains	Chicago, Ill.
	11	Ten	1,000	W. F. Hildebrandt	Fayette, Iowa

315. Terms Used. The **Capital Stock** of a corporation is the amount of capital authorized by the charter.

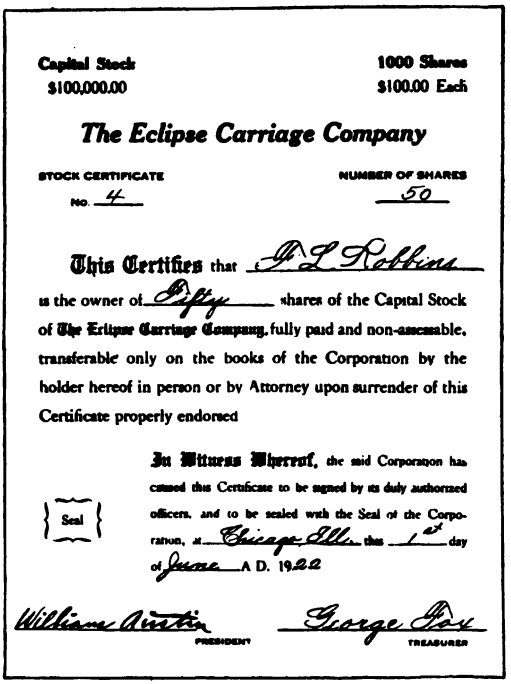
Shares. The capital stock is divided into shares of equal size, usually having a nominal or face value. Shares may be of any denomination within the limits fixed by law; $100.00 par value is customary.

When the stock has been subscribed (and, in some states, paid in) a meeting of the stockholders is held, at which meeting a board of directors is elected to manage the business.

At this meeting each share owned by a stockholder entitles him to one vote. (How many votes should Austin have?) By this method

of voting it is possible for a few stockholders owning a majority of the shares to control the business.

316. Stock Certificates. Each stockholder receives a certificate, signed by the officers of the corporation, stating the number of shares for which he has subscribed and paid.



Capital Stock
$100,000.00

1000 Shares
$100.00 Each

The Eclipse Carriage Company

STOCK CERTIFICATE
No. 4

NUMBER OF SHARES
50

This Certifies that D. L. Robbins

is the owner of Fifty shares of the Capital Stock of The Eclipse Carriage Company, fully paid and non-assessable, transferable only on the books of the Corporation by the holder hereof in person or by Attorney upon surrender of this Certificate properly endorsed

Seal

In Witness Whereof, the said Corporation has caused this Certificate to be signed by its duly authorized officers, and to be sealed with the Seal of the Corporation, at Chicago, Ills. this 1st day of June A D. 1922

William Austin
PRESIDENT

George Fox
TREASURER

Oral Work

Refer to the stock certificate, and answer the following questions:

1. What is the total capital stock of the corporation?
2. Into how many shares is the capital stock divided?
3. What is the par (face) value of each share?
4. How many shares does Robbins own? To how many votes is he entitled?
5. What is the par value of his stock?

317. Dividends. The profits of the corporation which are divided among the stockholders are called **dividends.** The board of directors

has the authority to determine the amount of dividends which shall be declared out of the profits.

318. Surplus. The undivided profits of a corporation are held as **surplus.**

Problems. 1. Let us suppose that the Eclipse Carriage Company's profits at the end of one year amounted to $6,800. What per cent of dividends could the directors declare?

SOLUTION. $6,800 (Profits) ÷ $100,000 (Capital Stock) = 6.8 %.

Therefore, the largest dividend which the directors could declare is 6.8 %. If such a dividend were declared, each stockholder would receive 6.8 % of the par value of his stock.

Austin, owning 500 shares, with a par value of $50,000, would receive a dividend of 6.8 % of the par value of $50,000, or $3,400.

2. Let us suppose that the directors vote to declare a 5 % dividend. How much of the profits would be divided among the stockholders, and how much would remain as surplus?

SOLUTION. 5 % of $100,000 = $5,000, Total Dividend.
$6,800 (Profits) − $5,000 (Dividends) = $1,800, Surplus.

319. Par Value and Market Value of Stock. The par value or nominal value of stock is its face value, shown by the stock certificate.

The market value of a share is the amount for which it can be sold. The market value depends upon several factors, the following being the most important:

a. The value of the property owned by the corporation.

When the Eclipse Carriage Company was organized, each stockholder paid for his stock in cash, and for each share of stock there was $100 in the treasury. Each share, therefore, was worth $100. But let us assume that a surplus of $1,600 has been created. Since there are one thousand shares, this surplus adds $1.60 to the property represented by each share.

b. The dividends paid by the corporation.

Let us suppose that 9 % dividends are declared each year. $9 is 6 % income on $150, and an investor might be willing to pay $150 for a share of this stock.

If only 4 % dividends were paid, an investor would probably not be willing to pay $100 for a share because he would not receive 6 % on his investment.

When stock sells for more than its par value, it is said to be above par, or **at a premium.** Stock selling for $105 per share is quoted at 105.

When stock sells for less than its par value, it is said to be below par, or **at a discount.** Stock selling for $95 is quoted at 95.

320. Stock Without Par Value. The custom of issuing stock with a par value and printing the par value on the certificate, has often resulted in injury to purchasers of the stock. The par value really has very little meaning, because the true value of the stock depends on the value of the property owned by the corporation and the profits earned by it. Many people assume, however, that stock with a par value of $100 must be worth that amount merely because the par value is printed on the certificate.

Several states have authorized corporations to issue common stock without par value. When such stock is issued each certificate must show the total number of shares of no par value stock issued, and the number of shares represented by the certificate. To determine the value of his stock a purchaser first determines the value of the total common stock and multiplies this value by the fractional interest represented by his certificate. Thus, if a certificate represents 10 shares out of a total issue of 1,000 no par value shares, and if the net assets of the corporation are worth $165,000, each share has a book value of $165 and the 10 shares have a book value of $1,650.

If the corporation issues preferred stock as well as no par common stock, the preferred must have a par value. Then the value of the no par common is determined by deducting the value of the preferred stock from the net assets of the company, and dividing the remainder by the number of common shares.

321. Assessments. When money is required to cover losses, or to extend the business, in some companies the stockholders may vote to levy an assessment. The assessment is a per cent of the par value of the stock, and is paid by the stockholders to the treasurer of the corporation. Shares of stock may be non-assessable.

Written Work

1. The stock of a corporation has all been subscribed and is owned as follows:

A, 150 shares; B, 220 shares; C, 80 shares; D, 50 shares.

Par value per share, $100.

What is the capital stock of the corporation?

2. How much will a dividend of 8% yield each stockholder?

3. If the annual profits are $4,500, what per cent dividend can be declared?

4. If a 5% dividend is declared from the profits of $4,500, how much will remain as surplus?

322. Kinds of Stock. There are two principal classes of stock, **common** and **preferred.**

Common Stock entitles its owner to a proportionate share of the net profits of the corporation when dividends are declared by the directors. The rate of dividend is not fixed, but depends upon the success of the business.

Preferred Stock entitles its owner to a fixed per cent of dividend provided the profits are sufficient to pay this dividend. Thus, 5% preferred stock entitles the owner to a 5% dividend before any dividends are declared on the common stock.

Cumulative Preferred Stock. Preferred stock may be cumulative or non-cumulative. If the stock is cumulative, a dividend which is unpaid one year becomes a claim against the corporation, to be paid out of future profits.

Participating Preferred Stock. Unless the preferred stock is expressly declared to be non-participating, the preferred stock participates equally with the common stock in any dividends after both common and preferred stock have received an equal dividend.

That is, after the 6% preferred stock has received its 6% dividend and the common stock has also received 6%, remaining dividends must be shared at the same rate. For example, if the common stock receives an additional 2% dividend, the preferred stock must also receive an additional 2%.

Problem. The Eclipse Carriage Company desires to build and equip a new factory at a cost of $25,000. A meeting of the stockholders is held, and it is decided to issue 5% cumulative non-participating preferred stock, in order to obtain the required funds. Two hundred and fifty shares, with a par value of $100 each, are issued.

At the end of the year, a profit of $9,750 is shown. How may this profit be divided among the holders of the common and preferred stock?

SOLUTION. 5% of $25,000 (Preferred Stock) = $1,250, dividend on preferred stock.

$9,750 − $1,250 = $8,500, remaining for dividends on common stock.

An 8½% dividend may be declared on the common stock.

If an 8½% dividend is declared, no surplus will remain.

Written Work

If an 8½% dividend is declared on the common stock:

1. How many dollars are paid as dividend on each share?

2. How many dollars are paid as dividend on each share of 5% preferred stock?

3. The profits of the second year are only $500, and the directors are able to pay only a 2% dividend on the preferred stock. No dividends are declared on the common stock.

How much does Clark, who owns one share of preferred stock, receive?

4. Comstock owns five shares of preferred stock. How much does he receive?

5. What is the total unpaid dividend on the preferred stock?

The third year a profit of $12,000 is made. Settlement among the shareholders is made as follows:

$12,000	Profits
750	Last year's unpaid dividends on preferred stock
$11,250	Balance
1,250	Dividends on preferred stock for current year
$10,000	Available for dividends on common stock, and surplus.

Clark, the owner of one share of preferred stock, now receives:

3% unpaid dividends of previous year	$3.00
5% dividends of current year	5.00
	$8.00

Dividends up to 10% may be declared on the common stock.

From this illustration you should understand:

a. The claim of the owners of cumulative preferred stock for unpaid dividends in case sufficient profits are made in future years.

b. The fact that preferred stock is usually an investment, but common stock is more of a speculation.

c. In a business of proved success, where the preferred stock is non-participating, the common stock may be a better investment than the preferred stock.

323. Buying Stocks. Stocks are bought and sold at market value which may be the same as par value, but which is usually either more or less than par. The quotation is the statement of market value.

Problem. Mr. Ross purchased 10 shares of the Eclipse Carriage Company's stock from F. L. Robbins at 115. What was the cost of each share? What was the total cost of the stock purchased?

SOLUTION.

\$ 115	market value per share
10	number of shares purchased
\$1,150	cost of ten shares

When this sale of stock is made, Robbins surrenders his stock certificate of fifty shares to the secretary of the corporation, who issues a new certificate to Robbins for forty shares, and a certificate to Ross for ten shares.

324. Dealing in Stock through a Broker. Stock is usually bought and sold through a stock broker. Brokers charge a certain per cent of the par value of the stock which they buy and sell for customers. $\frac{1}{8}$ of one per cent of the par value is a common charge for such services when stock is sold in large blocks.

Problem. 1. What is the cost of 100 shares of stock quoted at $112\frac{1}{2}$, purchased through a broker? Brokerage $\frac{1}{8}\%$.

SOLUTION.

$112\frac{1}{2}$ or \$112.50	market price per share
$\frac{1}{8}$ or .$12\frac{1}{2}$	brokerage added to cost
$112\frac{5}{8}$ or \$112.62$\frac{1}{2}$	cost including brokerage

$100 \times \$112.62\frac{1}{2} = \$11{,}262.50$ cost to purchaser.

2. What is received by the seller of 100 shares of stock quoted at $112\frac{1}{2}$, sold through a broker? Brokerage $\frac{1}{8}\%$.

SOLUTION.

$112\frac{1}{2}$ or \$112.50	price per share
$\frac{1}{8}$ or .$12\frac{1}{2}$	brokerage deducted
$112\frac{3}{8}$ or \$112.37$\frac{1}{2}$	returns per share after deducting brokerage

$100 \times \$112.37\frac{1}{2} = \$11{,}237.50$, returns from sale of 100 shares.

325. Profits from Stocks. Stocks are usually purchased with the hope of making a profit in the following ways:

a. Holding the stock and receiving dividends; or

b. Selling the stock when the market price advances.

Income from Dividends. The per cent of income from dividends depends upon two things:

a. The price paid for the stock.

b. The dividends, which are always expressed in terms of a per cent of the *par value.*

Problem. The par value of the common stock of the Butler Dairy Company is \$100.00. A dividend of 8 % is declared. What per cent of income does the holder of stock receive if the stock cost him $119\frac{7}{8}$, plus brokerage?

SOLUTION.

\$100.00	par value of stock
.08	rate of dividend
\$8.00	dividend per share
$\$119\frac{7}{8}$	market value when purchased
$\frac{1}{8}$	brokerage
\$120	cost per share

\$8.00 (dividend) ÷ \$120 = $6\frac{2}{3}$ % income on investment.

Profit or Loss from Buying and Selling Stock. Problem. Perry buys 200 shares of stock at $107\frac{1}{8}$, and sells at $108\frac{5}{8}$, buying and selling through a broker. What profit does he make?

SOLUTION.

$\$107\frac{1}{8}$	market value when purchased
$\frac{1}{8}$	brokerage added
$\$107\frac{1}{4}$	total cost per share
$\$108\frac{5}{8}$	market value when sold
$\frac{1}{8}$	brokerage deducted
$\$108\frac{1}{2}$	received for each share
$\$108\frac{1}{2}$	received per share
$107\frac{1}{4}$	cost per share
\$ $1\frac{1}{4}$	profit per share
\$1.25	profit per share
200	number of shares
\$250.00	profit on 200 shares.

Written Work

1. What would have been the total loss if Perry had sold at $106\frac{7}{8}$?
2. What would have been the total loss if he had sold at $107\frac{1}{8}$?
3. 7 % preferred stock is purchased at 110, without brokerage. What per cent income does the purchaser receive on his investment?
4. What is the cost of 7 shares of stock, par value 100, purchased through a broker at $109\frac{3}{8}$? Brokerage $\frac{1}{8}$ %.
5. $6\frac{1}{2}$ % dividends are declared on this stock. What per cent income does the purchaser receive on his investment?

The following is a partial list of the quotations of a day's sales by brokers on the New York Stock Exchange. Current quotations can be found in the daily newspapers.

Sales	Description	Open	High	Low	Close
1,200	Am. Smelting pfd.	$104\frac{1}{2}$	105	102	$103\frac{1}{4}$
2,300	Am. Sugar common	$108\frac{7}{8}$	$108\frac{7}{8}$	107	$108\frac{1}{2}$
400	Am. Sugar pfd.	$113\frac{3}{4}$	$113\frac{3}{8}$	$112\frac{1}{8}$	$112\frac{1}{4}$
1,600	Central Leather pfd.	$96\frac{3}{4}$	$97\frac{1}{2}$	$96\frac{3}{4}$	$97\frac{1}{2}$
22,900	C. M. & St. P. common	$105\frac{1}{4}$	$106\frac{7}{8}$	104	$106\frac{5}{8}$
400	C. M. & St. P. pfd.	$141\frac{1}{8}$	$141\frac{1}{4}$	140	141

6. What is received for 400 shares of Am. Smelting pfd. sold through a broker at the opening price in the table? Brokerage $\frac{1}{8}$%.

7. What is the cost of 400 shares of this stock if purchased through a broker? Brokerage $\frac{1}{8}$%.

NOTE. Other problems may be given by the teacher, either from the above table or from the quotations of sales of stock printed in the daily newspapers.

Written Review

Martin, Owen, and Rathbun wish to organize a corporation. They circulate a subscription list, on which they receive subscriptions for the entire authorized capital stock, as follows:

Subscription List

The Roadbed Grader Company

We, the undersigned, hereby subscribe for the number of shares and the amount of the Capital Stock of the Roadbed Grader Company, set opposite our names, agreeing to pay calls upon the said stock as they shall be made by the directors of company.

Date		Number of Shares	Amount	Signatures
1921 August	4	One Hundred Fifty Shares	$ 15000	George R. Martin
	4	One Hundred Twenty Shares	12000	F. G. Owen
	4	One Hundred Ten Shares	11000	D. L. Rathbun
	6	Ninety Shares	9000	D. R. Wilson
	6	Thirty Shares	3000	R. V. Sanders

1. What is the capital stock of the company?

2. Into how many shares is the capital stock divided?

3. What is the par value of each share of stock?

4. How many votes does each stockholder have in the meetings of the stockholders?

5. Which two of the stockholders have votes enough (combined) to control the management of the corporation?

6. The Roadbed Grader Company is duly organized, and a call is made on each of the subscribers for one-half of his subscription, payable on September 1. Martin, Owen, and Sanders pay the call on September 1. How much is received by the treasurer of the company on that date?

7. Rathbun pays his "call" on September 6, and is charged 6% interest on the $5,500 for the 5 days during which the call remained unpaid. How much does Rathbun pay, including call and interest?

8. Wilbur pays his call on September 10, including interest on the delinquent payment. Mr. Wilbur sends a check to the treasurer for $——.

9. Call is made for one-quarter of the subscriptions, payable on September 20. The subscribers pay as follows:

Martin, September 20;
Owen, September 20;
Sanders, September 22;
Wilbur, September 25;
Rathbun, September 27.

How much does each subscriber pay, including interest?

10. Final call for subscriptions is made payable on October 1, at which time each of the subscribers pays the balance of his subscription. How much does each subscriber pay?

11. The net profit for the first year is $4,075.00. A 6% dividend is declared on the stock. The total dividend amounted to $——.

12. What was the amount of the surplus?

13. What was the amount of dividend received by each stockholder?

14. Considering (*a*) the dividends, and (*b*) the surplus, if you owned a share of the stock of the Roadbed Grader Company, would you be willing to sell it at par?

15. The profits for the second year were $4,690.00. What is the

largest per cent of dividend which the directors can declare out of the second year's profits and accumulated surplus?

16. What amount of dividend would each stockholder receive?

17. After the payment of the dividend indicated in question 15, how much surplus remains?

18. Rathbun sells 7 shares of his stock to S. F. Cromer at $114\frac{3}{8}$, without brokerage. How much does Rathbun receive?

19. The Roadbed Grader Company issues \$15,000.00 of 6% non-participating preferred stock, which it sells at 103. How much does the company receive for the stock?

20. The stock is issued in \$100 shares and is sold to the following persons:

F. R. Gridley, 45 shares,
G. C. Moore, 105 shares.

How much does each pay for his stock?

21. The net earnings for the third year were \$6,230.00, which were distributed as follows:

Dividends on preferred stock, \$ ——.
" " common stock, \$4,000.00.

What is the per cent of dividend on the common stock?

22. What is the amount of surplus remaining from this year's profits? What is the total surplus accumulated to date?

23. What per cent of income do the holders of the preferred stock, who purchased it at 103, without brokerage, receive on their investment?

24. F. R. Taylor and J. E. Price engaged in partnership. Taylor invested \$6,000.00 and Price invested \$9,000.00. The business was conducted as a partnership for several years, when it was decided to reorganize as a corporation. A statement of assets and liabilities showed that the net capital of the partnership was \$23,619.50. The corporation was organized with a capital stock of \$25,000.00. The capitalization was how much larger than the actual net resources of the firm?

25. The stock was divided between Taylor and Price in proportion to their original investment. If the stock was issued in \$100 shares, how many shares did each partner receive?

26. Burns, Randall, and Anderson were partners. Burns invested \$8,000, Randall \$9,000, and Anderson \$13,000. They decided to organize as a corporation with a capital stock of \$50,000. \$45,000 of

the stock was distributed among the partners in proportion to their original investments. How many $100 shares did each receive?

27. The remaining stock is sold to Chapman at 98½. How much did Chapman pay for his stock?

28. The profits for the first year were $4,500. What dividend could be declared on the stock?

29. A corporation is organized with $100,000 of common stock and $50,000 of participating 5% preferred stock. Dividends of $13,500 are declared. What is the total amount of dividends paid on the common stock? on the preferred stock? What rate of dividend does the holder of a share of preferred stock receive? What rate of dividend does the holder of a share of common stock receive?

326. Bonds. When an individual borrows money, he usually gives a promissory note.

When a corporation borrows money in large amounts and for long periods of time, it usually issues bonds.

Bonds are usually issued in denominations of $1,000.00, although there are bonds of smaller amounts. There is an increasing tendency to issue bonds in denominations of $500.00 or $100.00, so that they may be sold to small investors.

Bonds are issued by:

Business Corporations,	called Industrial Bonds
U. S. Government,	called Government Bonds
State Governments,	called State Bonds
Cities,	called Municipal Bonds
Counties,	called Municipal Bonds
School Districts,	called Municipal Bonds

Security of Bonds. A first-mortgage bond is secured by a mortgage on the property of the corporation. If the bond is not paid, the property of the corporation may be sold to pay the bondholders.

Second-mortgages and third-mortgage bonds are also secured by mortgages on the property, but the bondholders are paid, if the mortgages are foreclosed, in the numerical order of the mortgages.

Maturity of Bonds. A bond states the time at which the principal is payable. Some bonds state that the corporation reserves the right to redeem the bond at any time after a specified date, either at par or at some specified rate above par.

Interest. Bonds bear interest payable annually, semi-annually, or quarterly, as specified. The method of collecting the interest depends upon whether the bond is

a. A Registered Bond, or
b. A Coupon Bond.

When a person owns a registered bond, his name is recorded, on the books of the corporation. When the interest is due, a check is sent to the bondholder in payment.

A coupon bond is one to which small coupons are attached, there being one coupon for each interest payment. For example: a bond maturing in ten years with interest payable quarterly would have forty coupons. On the date when each interest payment is due, the coupon bearing that date is clipped from the bond and delivered to a bank to collect, or it may be collected directly from the treasurer of the corporation.

Value of Bonds. Bonds, unlike common stock, pay a fixed income. The market value of a bond may be above or below par, depending in general upon the security and the rate of interest.

Bonds are usually known by the name of the issuing corporation or government, the nature of the security, the rate of interest, and the date of maturity. Thus first-mortgage bonds issued by the Eclipse Carriage Company, bearing 5 % interest and payable in 1930, would be called Eclipse Carriage Company, 1st Mortgage 5 % Bonds, 1930.

327. Buying and Selling Bonds. When a bond is sold, the buyer usually pays not only the market value of the bond, but also the interest which has accrued since the last interest day.

Problem. Murdock owned an Eclipse Carriage Company 5 % bond which he wished to sell. The par value of the bond was $1,000.00, and it was quoted at 95. Henderson bought the bond on April 1, at its market value plus accrued interest. Interest is payable semi-annually, June 1 and December 1. How much did Murdock receive?

SOLUTION. Since the bond was sold at 95, the owner received 95 % of its par value.

95 % of $1,000.00 = $950.00.

Interest at 5 % has accrued on the bond from December 1 to April 1, four months.
Interest on $1,000 for 4 months at 5 % = $16.67, accrued interest.

$950.00	market value of bond
16.67	accrued interest
$966.67	selling price

Many banks and trust companies buy an entire series of bonds from corporations and sell them to investors. Prices are quoted by these bond houses in the following ways:

Price, Par and Accrued Interest;
Price, at the Market and Accrued Interest.

328. Dealing in Bonds through a Broker. When bonds are purchased or sold through a broker, brokerage is charged in the same manner as when stocks are transferred. $\frac{1}{8}\%$ of the par value of the bonds is the customary brokerage for large transfers.

Written Work

1. S. & J. 6% coupon bonds are quoted at 104 and accrued interest. Interest payable June 1 and December 1. What is the cost of a $500 S. & J. bond purchased on August 15 at the market and accrued interest? No brokerage.

2. What is the amount of interest which the purchaser receives on December 1st?

3. What would the bond have cost if purchased through a broker? Brokerage $\frac{1}{8}\%$.

4. A bond house quotes Murdock Apartment $5\frac{1}{2}\%$ bonds of $1,000 at par and accrued interest. Interest due March 26 and September 26. What is the cost of a bond purchased May 12? October 19?

5. Laclede Gas Light Company's $1,000 5% bonds were quoted by the Harris Trust and Savings Bank at $101\frac{1}{2}$ and interest. Semi-annual interest payable April 1 and October 1. What is the cost of a bond purchased June 15?

6. Wilmington Power Company's 5% bonds are quoted at $97\frac{1}{2}$ and interest. Interest payable semi-annually January 1 and July 1. The Wilmington Power Company reserves the right to redeem the bonds on January 1, 1927, or on any interest day thereafter at 105 and interest.

What is the cost of a $1,000 bond purchased October 15, 1922, at the quoted price?

If the bond is redeemed on July 1, 1927, what gain is made by the increase in the redemption price over the purchase price?

The following is a partial list of a day's transfers of bonds through Chicago brokers. Brokerage $\frac{1}{8}\%$.

No. Sold	Par Value	Description	Open	High	Low	Close
1	$1000.00	Armour 4½'s	92	92	92	92
67	1,000.00	Chicago City Ry. 5's	100⅜	101½	100¼	100½
16	1,000.00	Com. Edison 5's	101¾	101¾	101⅝	101⅝
214	1,000.00	Peo. Gas 5's	99½	99⅞	99½	99⅞

7. What was the cost of 5 Chicago City Ry. $1,000 bonds, purchased at the opening price in the table?

8. What did the seller receive for these bonds? How much was the brokerage on the sale?

9. What was the gain or loss from purchasing 60 Peo. Gas $1,000 bonds at the opening price and selling them at the closing price? Brokerage ⅛% on purchase and ⅛% on sale.

TABULATIONS TO PROMOTE EFFICIENT MANAGEMENT

CHAPTER XLI

BUYING AND SELLING EXPENSES; NET PROFIT

329. Expenses and Profits. In Chapter XXIII the principles of gross earnings were discussed. You have now studied many of the expenses incurred in conducting a business, and are ready to consider the problems which arise in computing net profit.

Net Profit is the difference between the gross earnings of an enterprise and the total expense of conducting it.

330. Buying Expenses include all the costs of buying goods, such as freight and drayage, customs duties on goods imported, commissions of purchasing agents, insurance, and the cost of keeping goods in warehouses and placing them on the shelves of the salesroom. Buying expenses are added to the original purchase price of goods to determine the total cost. Thus:

Prime Cost of Merchandise is the original or purchase price;

Total Cost of Merchandise is the prime cost plus the buying expenses.

Problem. A retail furniture store purchases goods during the year amounting to $36,450.00. This is the prime cost. The freight, drayage, rent of warehouse, labor of warehouse employees, and other costs of placing the goods in the salesroom are $2,187.00.

What is the total cost of the merchandise purchased?

SOLUTION.

$36,450.00	Prime cost
2,187.00	Buying expenses
$38,637.00	Total cost

331. Marking Cost of Goods. When the cost price is marked on goods, some merchants add a certain per cent to the prime cost to

cover the buying expenses. The records of previous years are used as a basis to determine the per cent to be added.

In the illustration on page 396, the prime cost is \$36,450.00, and the buying expenses \$2,187.00, or 6 % of the prime cost. The furniture dealer might say, "Next year's buying expenses will probably be about the same as they were this year, and I will therefore add 6 % to the prime cost to cover them."

A safer method, however, is to base the computation on the average expenses of several years.

Year	Prime Cost	Buying Expenses
1919	\$ 35,275.00	\$1,840.13
1920	41,391.00	2,069.55
1921	62,387.00	3,618.45
	\$139,053.00	\$7,528.13

\$7,528.13 ÷ \$139,053.00 = 5.41+ %.

For convenience, 5.4 % probably would be adopted as the per cent of buying expenses.

Illustration. An article cost \$8. At what price is it marked to cover buying expenses?

Solution.

\$8 Prime cost
.055 Per cent of buying expenses
\$ .44 Buying expenses

\$8.00 Prime cost
.44 Buying expenses
\$8.44 Total marked cost

Written Work

The prime cost of a merchant's purchases and the buying expenses for several years are given below. Find the per cent of buying expenses for each year and the average for the entire interval.

	Year	Purchases	Buying Expenses
1.	1917	\$ 9,426.90	\$ 643.92
	1918	12,318.26	812.29
	1919	14,126.29	830.12
	1920	11,341.72	690.19
	1921	14,985.29	752.29

	Purchases	Buying Expenses
2. 1920	$39,286.26	$2,326.80
1921	47,562.83	3,146.29
1922	59,286.28	4,210.08

3. The prime cost of merchandise in a certain store is increased 8% to include buying expenses; the selling price is determined by adding 9% to the total cost. Mark the total cost (using "blacksmith" as the key and "g" as a repeater) and the selling price (in figures) of articles, the prime cost of which was as follows:

Stock No.	Prime Cost
A 326	$2.19
A 394	6.25
F 246	.90
A 385	2.78

4. The manager of one of the departments in a large store adds 6% to the prime cost to cover the buying expenses. He receives quotations of prices from manufacturers who offer different discounts. He prepares a table showing the total cost after deducting the discount offered and adding the 6% buying expenses. The following is a similar table which you will complete:

$	.00			.25			.50			.75		
	Less 2% Plus 6%	Less 5% Plus 6%	−7% +6%	−2% +6%	−5% +6%	−7% +6%	−2% +6%	−5% +6%	−7% +6%	−2% +6%	−5% +6%	−7% +6%
0				.2597	.25175	.24645						
1	1.0388	1.007	.9858									
2												
3												
4												
5												
6												
7												
8												
9												
10												

5. By referring to the table, find the total cost, including buying expenses, of an article quoted at $2.25 less 5%; $3 less 2%.

Refer to the table, and find

6. Which is the better price, and how much better: \$1.75 each, less 2%, or \$2 each, less 7%? How much will be saved by purchasing one gross at the cheaper quotation?

332. Selling Expenses. After the goods have been placed on sale, all costs such as rent of store, salaries of clerks, advertising, heat and light, delivery, store supplies, insurance and taxes, depreciation, and bad debts are considered as selling expenses.

333. Finding the per cent of selling expenses. The per cent of selling expenses is a matter of valuable information to a merchant. It is determined by the formula:

Selling Expenses ÷ Net Sales = % of Selling Expenses.

Example. A merchant's net sales during one year are \$124,265; the selling expenses for the same year are \$29,078.01. The selling expenses are what per cent of the net sales?

SOLUTION. \$29,078.01 ÷ \$124,265.00 = 23.4%.

Written Work

The following statistics are reprinted from *System*, and show the result of an investigation made by that magazine to determine *average* or *standard* per cents of selling expenses in various businesses.

Find what per cent each item of expense is of the total sales.

Find what per cent the total selling expenses are of the sales.

Prepare graphs as directed by your teacher.

	DRY GOODS STORE	GENERAL STORE	SHOE STORE
	Sales, \$50,000.00	Sales, \$50,000.00	Sales \$25,000.00
Rent	\$1,550.00	\$1,154.85	\$ 778.13
Salaries	4,800.00	4,067.09	2,786.21
Advertising	750.00	351.48	376.51
Heat and light	200.00	251.04	225.91
Delivery	450.00	954.01	75.30
Supplies	200.00	150.63	100.40
Insurance and taxes	550.00	200.84	301.21
General expense	2,200.00	150.63	1,029.14
Depreciation and shrinkage	700.00	301.27	150.61
Bad debts	150.00	150.63	25.10

	Vehicle Store	Hardware Store	Furniture Store
	Sales, $57,000.00	Sales, $45,000.00	Sales, $100,000.00
Rent	$1,094.63	$1,305.61	$3,507.36
Salaries	5,818.81	5,042.35	8,317.60
Advertising	633.73	405.20	2,906.15
Heat and light	345.67	225.10	801.70
Delivery	518.51	270.13	1,102.33
Supplies	230.45	180.08	501.06
Insurance and taxes	460.90	495.23	1,402.97
General expense	230.45	360.17	1,803.82
Depreciation and shrinkage	403.28	225.10	1,904.03
Bad debts	115.22	180.08	1,202.54

	Clothing Store	Drug Store	Jewelry Store
	Sales, $60,000.00	Sales, $20,000.00	Sales, $30,000.00
Rent	$1,322.24	$ 924.65	$1,080.40
Salaries	5,469.29	2,191.01	3,361.23
Advertising	2,043.47	522.64	1,050.38
Heat and light	180.31	160.81	180.07
Delivery	360.61	80.40	30.01
Supplies	120.20	60.30	270.10
Insurance and taxes	661.12	281.40	540.20
General expense	1,681.84	482.42	630.23
Depreciation and shrinkage	721.22	100.50	360.13
Bad debts	240.41	40.20	90.03

Save your results for use in the problems which follow.

334. Efficient Management to Determine Economies and "Leaks." When a merchant has a reliable standard with which to compare the various per cents of selling expenses in his business, he is able to determine the items of expense which are too large.

Written Work

A dry goods store has sales during one year amounting to $40,000.00. The expenses are as follows:

Rent	$1,200.00	Supplies	$165.00
Salaries	3,240.00	Insurance and taxes	180.00
Advertising	250.00	General expense	155.00
Heat and light	225.00	Depreciation and shrinkage	120.00
Delivery	1,135.00	Bad debts	42.00

1. Each item of expense is what per cent of the sales?
2. The total selling expenses are what per cent of the sales?
3. Accepting the per cents found in the dry goods store problem on page 399 as a standard, determine which expenses are larger than the standard, and which are smaller.
4. Complete the table on the following page.
5. Plot a curve in black ink to show the per cent of gross earnings in each department. Plot a curve on the same paper in red ink to show the per cent of net profit or net loss in each department.

SUGGESTION. Draw a line horizontally across the paper representing 0 %; scale the paper to show per cents of gain above this line, and per cents of loss below the line.

335. Common Omissions in Computing Selling Expenses. Merchants sometimes underestimate their selling expenses by omitting the following:

(*a*) Depreciation.

Store buildings and fixtures depreciate in value, and while such depreciation may not be noticeable, it should be computed annually. Goods also become shopworn or out of style and the depreciation in value should be charged off in the year in which it occurs.

(*b*) Bad debts.

Careful merchants make every effort to collect their accounts, but unless a strictly cash business is conducted, some accounts are almost certain to prove uncollectable. In estimating the probable loss from bad debts, some merchants compute a certain per cent of the sales, while others compute a certain per cent of past due accounts.

Written Work

A merchant invests $5,000.00 in a general store. His sales during a certain year are $17,350.00. He owns his store building valued at $3,000.00. He lists as his selling expenses the items at the top of page 403.

Profits in 12 Departments

Dept.	Stock, Jan. 1, 1922	Purchases, during 1922	Total Cost for 1922	Stock, Dec. 31, 1922	Cost of Goods Sold	Sales, 1922	Gross Profits	%	Expenses	Net Profits	%	Net Loss	%
A	$15,223 40	$118,427 56		$17,327 72		$162,341 72			$26,514 95				
B	21,714 32	168,427 92		25,314 71		184,231 76			25,601 51				
C	26,361 71	176,204 91		25,747 39		197,432 65			21,430 13				
D	13,247 91	112,347 29		15,261 03		120,261 32			29,675 55				
E	20,403 71	150,265 97		20,374 76		174,231 61			14,512 53				
F	27,349 61	160,231 39		28,417 19		174,628 32			16,669 17				
G	25,732 92	237,511 86		27,961 31		284,162 39			13,635 20				
H	21,113 65	107,221 13		22,213 47		135,261 34			15,509 01				
I	23,416 47	121,376 41		24,316 19		142,221 32			13,459 85				
J	30,265 45	164,213 96		29,262 47		174,227 42			11,581 19				
K	18,462 31	174,216 91		19,417 13		202,346 21			9,420 57				
L	15,271 34	121,216 09		16,237 41		161,213 47			14,748 47				
Total													

Per cent of Gross Profit is found by dividing the Gross Profit by the Sales.
Per cent of Net Profit is found by dividing the Net Profit by the Sales.
Per cent of Net Loss is found by dividing the Net Loss by the Sales.

Salaries (one clerk)	$600.00
Advertising	85.00
Heat and light	125.00
Delivery	140.00
Insurance and taxes	95.00
General expenses	110.00

1. What are the total expenses as shown by the list?

2. These expenses are what per cent of the sales?

The following items have been omitted:

7% would be a fair rate for depreciation on the building.

An examination of this merchant's books shows that his annual loss from bad debts is about 1% of his sales.

3. Make a list of the selling expenses of this business, including those which the merchant omitted.

4. What are the total expenses?

5. The total expenses are what per cent of the sales?

6. The cost of the goods sold is $14,520.00.

What is the gross profit?

7. What does the merchant consider to be his net profit after deducting the expenses as stated in his list?

8. After deducting his real expenses as shown by your corrected list, did he have a net gain or a net loss? How much?

9. Find what per cent each item of selling expenses is of the sales.

10. By comparing these per cents with the average or standard per cents, we may find the "leaks" of this business.

Which of these per cents are smaller than the standard?

Which of the per cents are larger than the standard?

Although the general expenses are large, they include supplies for which no separate charges are made.

11. What is the worst "leak" in this business?

336. Reducing the Per Cent of Selling Expenses by Increasing the Gross Sales. It is evident that as the per cent of selling expenses increases, other factors remaining the same, the per cent of profit on the sales decreases. It is therefore desirable to decrease the per cent of selling expenses. The per cent of selling expenses may be reduced by

Reducing the selling expenses, or

Increasing the sales.

Written Work

1. If the sales were \$50,000.00 and the selling expenses \$10,000.00, what was the per cent of selling expenses?

2. If the selling expenses could be reduced to \$8,000.00, without reducing the sales, what would be the per cent of selling expenses?

3. If the selling expenses remained \$10,000.00, and the sales were increased to \$60,000.00, what would be the per cent of selling expenses?

4. If an increase of the sales to \$80,000.00 resulted in an increase of selling expenses to \$12,000.00, what would be the per cent of selling expenses?

Increasing the Sales. The sales may be increased in three ways:
By increasing the selling price; the turnover remaining the same.
By increasing the turnover; the selling price remaining the same.
By increasing the selling price and the turnover.

Increasing the selling price of a commodity may result, however, in decreasing the sales by causing a decreased demand. For this reason the more popular method of increasing sales is to sell at a small profit and to make frequent "turnovers."

The "turnover" means the number of times by which the sales exceed the average inventory.

Thus, if the average value of the stock kept is \$5,000.00 (selling price) and the gross sales for a year are \$5,000.00, there is one turnover. If the gross sales are \$15,000.00, there are three turnovers.

337. Small Profits and Frequent Turnovers. The following illustration shows how a larger profit may be made from several turnovers at a small markup,[1] than by a single turnover at a large markup.

Let us suppose that a merchant's stock inventories at cost prices \$10,000.00 (which includes buying expenses). If the selling expenses are \$5,000.00, the merchant must obtain \$15,000.00 before he begins to make any profit. This means a markup of 50%. In order to make a 5% profit on the cost, he must mark his goods at 55% above cost. An article costing \$1.00 must sell for \$1.55. Thus, a single turnover at 55% markup will net a profit of \$500.00.

Now let us suppose that a 30% markup (selling an article costing \$1.00 for \$1.30) will result in two turnovers without increasing the selling expenses.

[1] By "markup" is meant the per cent added to prime cost and buying expenses to cover selling expenses and profit.

Each turnover of a stock costing \$10,000.00 means gross sales of \$13,000.00; two turnovers make the sales \$26,000.00, and the gross profits \$6,000.00. After deducting the selling expenses of \$5,000.00 a profit of \$1,000.00 remains.

If a markup of 20% results in four turnovers of stock costing \$10,000.00 with selling expenses of \$6,000.00, what is the net profit?

Written Work

1. A certain merchant takes an inventory once a month. The following are the inventories for the year at cost prices.

January	\$27,365.80	July	\$28,346.70
February	25,398.25	August	28,134.62
March	26,285.00	September	26,347.80
April	26,947.30	October	27,228.35
May	27,235.80	November	30,562.95
June	29,165.20	December	31,728.73

What is the average stock carried?

2. Plot a curve on graphically ruled paper to show the monthly inventories.

3. The selling expenses for the year are \$15,342.55, or ——% of the average inventory.

4. If one turnover is made, what must be the per cent of markup in order to make a net profit of 4% on the average inventory?

The sales would be \$——, and the profit would be \$——.

5. If four turnovers are made, what is the cost of the goods sold?

The selling expenses increased to \$18,420; what may be the cause of the increase in the selling expenses?

6. The selling expenses in Problem 5 are what per cent of the cost of the goods sold?

7. In order to make a total profit of 4% of the average inventory during the year, what per cent of profit must be made on each turnover?

8. What is the necessary per cent of markup on each turnover, selling expenses being 16.5%?

9. What would be the selling price of an article which cost \$2?

What is the profit or loss in each of the following?

	Average Inventory	Selling Expenses	Turnovers	Markup
10.	$ 2,000.00	$ 432.00	1	20%
11.	8,340.25	1,365.00	3	15%
12.	12,375.70	3,174.25	4½	13%
13.	27,320.00	12,278.60	9	8%

14. With $12,000 invested in stock, and with selling expenses of $4,000, how much profit is made by

3 turnovers with a markup of 12%?
5 turnovers with a markup of 10%?
8 turnovers with a markup of 6%?

CHAPTER XLII

FINDING THE PROFITABLE DEPARTMENTS

In a business which is divided into several departments, one or more departments may be operating at a loss, while the business as a whole shows a profit. Unless these departments where losses occur are discovered and expenses curtailed, or gross sales or gross profits increased, such departments remain a burden to the business.

In order to find the profit by departments, the following facts must be known for each department:

Cost of Goods Sold;
Gross Sales;
Expenses.

The first two of these facts may be easily determined, since each department may be considered as a separate store, with separate records of purchases, sales, and inventories. But the difficulty comes in determining the expenses — a difficulty due to the fact that the expenses are of two kinds.

338. Local and Overhead Expenses. Local Department Expenses consist of items of expense paid for each department separately, such as salaries of clerks who work in only one department. There is no difficulty in determining the local expenses of each department.

Overhead Expenses consist of all items of expense paid by the store as a whole for the general benefit of all departments. Among these expenses are Rent, Taxes and Insurance, Heat and Light, Janitor and Elevator Service, Salaries of Executives and Office Employees, Delivery Service, Advertising.

339. Prorating Overhead Expenses. All overhead expenses must be divided among the various departments according to some reasonable basis. The proper basis of division depends on the nature of the expense. The method of distribution is called prorating or allocating.

Bases of Prorating Overhead Expenses. The following illustrations show some of the bases of distributing overhead expenses.

a. Rent: Prorated by floor space.

Suppose a store with four departments occupies a building with 4,800 square feet of floor space. If each department occupies 1,200 square feet, each should pay $\frac{1}{4}$ of the rent. But if one department occupies only 400 square feet, it should be charged with only $\frac{4}{48}$ or $\frac{1}{12}$ of the rent. Each department should be charged rent in proportion to the floor space which it occupies. In some concerns the relative values of different floors and locations are taken into consideration.

b. Insurance and Taxes on Stock: Prorated according to the average value of the stock carried in each department.

c. Insurance and Taxes on Building: Prorated by floor space.

d. Heat and Light: Determined by separate meters or prorated by floor space.

e. Expenses of Alterations Department: Charged according to the actual expense of alterations made for each department.

f. Expenses of Returned Goods Department: Prorated among the various departments on the basis of the number of articles returned to each department by customers.

g. Salaries and Expenses of Traveling Salesmen: Prorated according to the sales made by traveling salesmen for each department.

h. Advertising: Prorated according to the space occupied in the advertising mediums.

Oral Work

Discuss the reasons for the basis of prorating adopted in each instance above.

What would be a reasonable basis for prorating the following expenses?

Cost of Delivery Service,
Depreciation on Building.

Method of Prorating.

a. Determine the most reasonable basis;

b. Divide the total expenses by the total number of units of the basis used, to determine the prorate;

c. Multiply the prorate by the number of units of the basis in each department.

Problem. The rent of a store building is $925, to be prorated on the basis of floor space. The space occupied by each department is as follows:

Dept.	Dimensions	Area
I.	15′ × 20′	300 sq. ft.
II.	80′ × 40′	3,200 sq. ft.
III.	25′ × 52′	1,300 sq. ft.
Total		4,800 sq. ft.

What share of the rent should be charged to each department?

Solution. $925.00 ÷ 4800 = $.19270, the prorate, or the rent per square foot.

Dept. I.	$.19270	prorate
	300	sq. ft.
	$57.81	share of rent
Dept. II.	$.19270	prorate
	3200	sq. ft.
	$616.64	share of rent
Dept. III.	$.19270	prorate
	1300	sq. ft.
	$250.51	share of rent

To check the above computations:

$ 57.81	rent, Dept. I
616.64	rent, Dept. II
250.51	rent, Dept. III
$924.96	total rent

The total rent thus computed is four cents less than the actual rent. A discrepancy so small is entirely justifiable in such computations. In order to avoid a larger error, the prorate should include as many significant figures as there are dollars and cents in the expenses. A prorate of .0932 has three significant figures. If the expenses were $36.45, the prorate should have four significant figures; if the expenses were $3,125.00, the prorate should have six significant figures.

Written Work

A blank form similar to the following illustration should be prepared for each of the following problems of prorating overhead expenses.

1. Prorating rent by floor space:

Dept.	Floor Space	Dept.	Floor Space
1.	260 sq. ft.	6.	470 sq. ft.
2.	380 sq. ft.	7.	630 sq. ft.
3.	694 sq. ft.	8.	763 sq. ft.
4.	876 sq. ft.	9.	856 sq. ft.
5.	340 sq. ft.	10.	940 sq. ft.

Total annual rent, $36,000.

Prorating Rent by Floor Space

Total Rent.....
Prorate.....

Department	Floor Space	Prorated Rent	
Totals			

2. Prorating insurance on stock according to the average value of the stock carried in each department:

Dept.	Average Stock	Dept.	Average Stock
1.	$3,600.00	6.	$6,890.00
2.	5,400.00	7.	3,750.00
3.	4,990.00	8.	8,460.00
4.	6,030.00	9.	6,560.00
5.	3,740.00	10.	4,700.00

Cost of insurance on stock, $540.00.

3. Prorating general expenses, heat, light, janitor service, etc., according to floor space.

The total general expenses for the year are $6,346.80. The floor space of the different departments is given in Problem 1.

4. Prorating executive salaries and interest on investment, according to capital invested in each department, as shown by the average stock carried. The total executive salaries and interest on the investment for the year are $12,350.00. The average stock carried in each department is shown in Problem 2.

5. Prorating expenses of returned goods department according to the number of sales returned.

Dept.	No. of Sales Ret'd	Dept.	No. of Sales Ret'd
1.	1,980	5.	1,746
2.	3,764	6.	1,329
3.	Goods purchased from this department cannot be returned.	7.	1,426
		8.	1,276
		9.	1,362
4.	189	10.	1,147

The total expenses of the returned goods department for the year are $375.94.

340. Comparing Profits or Losses by Departments. After the overhead expenses have been prorated among the various departments, and after the sales, purchases, local expenses, and other facts of each department have been determined, the profit or loss and the per cent of profit or loss can be computed, and the departments compared on a tabulated form similar to the one used in the written exercise on page 402.

CHAPTER XLIII

FINDING THE PROFIT OR LOSS ON EACH SALE

341. Factors which Determine Cost of Each Sale. Large business houses keep records to show not only the profit or loss of the entire store and of each department, but also the profit or loss on every sale. In order to do this, a cost book is prepared, showing the cost of every article for sale in the store. The cost shown in the cost book is the total cost; it includes

a. the wholesale cost of the article;
b. a share of the buying expenses;
c. a share of the selling expenses.

Preparing the Cost Book. The three items of cost required in preparing a cost book are found as follows:

a. The wholesale cost of each article is taken from the invoice when the goods are purchased.

b. The share of buying expenses to be added is determined by the following formulas:

[1] Direct Buying Expenses + Prorated Overhead Buying Expenses = Total Buying Expenses.

Total Buying Expenses ÷ Total Cost of Goods Purchased = % of Buying Expenses to be added to the wholesale cost of each article.

Problem. What is the per cent of buying expenses in a department reporting the following:

Salaries of buyers, freight, and other direct buying expenses, $3,500.

Prorated share of overhead buying expenses (warehouse rent, labor, etc.), $2,400.

Cost of purchases for the year, $84,285.50.

[1] Direct buying expenses are the expenses which may be charged directly to the department purchasing the goods.

SOLUTION.

$3,500.00	Direct buying expenses
2,400.00	Share of overhead buying expenses
$5,900.00	Total buying expenses

$5,900 ÷ $84,285.50 = 7 %.

7 % is therefore added to the cost of each article to cover the buying expenses.

c. The share of selling expenses to be added, is computed by the following formulas:

Direct Selling Expenses + Prorated Share of Overhead Selling Expenses = Total Selling Expenses.

Total Selling Expenses ÷ Cost of Goods Sold = % of Selling Expenses included in Cost.

Problem. The department furnishing the facts for the preceding illustration reported the following:

Salaries of salesmen, and other direct selling expenses, $5,160;
Prorated share of overhead selling expenses, $7,340;
Cost of goods sold, $96,153.85.
What per cent should be added to the cost of goods to cover the selling expenses?

SOLUTION.

$5,160.00	Direct selling expenses
7,340.00	Overhead selling expenses
$12,500.00	Total selling expenses

$12,500 ÷ $96,153.85 = 13 %.

13 % would, therefore, be added to the cost of each article in this department to cover the selling expenses.

342. Combining the Buying and Selling Expenses. After the per cent of buying expenses and the per cent of selling expenses have been determined, the two per cents are added, and the resulting per cent of the cost is added to the cost of each article to cover all expenses. Although the per cents are merely estimates, determined from the business of former years, firms of long experience use per cents which the business of the past shows to be approximately correct.

Written Work

Complete the following table showing the per cent of buying expenses, the per cent of selling expenses, and the total per cent to be added to the cost of each article purchased.

Dept.	Direct Buying Expenses Dollars	Share of Overhead Buying Expenses Dollars	Total Buying Expenses Dollars	Purchases, 1921 Dollars	Per Cent of Buying Expenses	Inventory, Jan. 1, 1921 Dollars	Purchases, 1921 Dollars
I	3,500 00	2,400 00	5,900 00	84,285 50	7 %	27,624 85	84,285 50
II	2,112 19	1,354 25		69,328 72		16,582 88	
III	1,565 58	1,327 42		48,216 73		4,532 42	
IV	3,402 51	3,928 36		91,635 82		12,013 16	
V	926 39	712 46		23,412 17		5,969 48	
VI	1,426 12	951 33		39,624 17		2,023 72	

Total Cost of Mdse., 1921 Dollars	Inventory Dec. 31, 1921 Dollars	Cost of Goods Sold, 1921 Dollars	Direct Selling Expenses Dollars	Share of Overhead Selling Expenses Dollars	Total Selling Expenses Dollars	Per Cent of Total Expenses	Total Per Cent
111,910 35	15,756 50	96,153 85	5,160 00	7,340 00	12,500 00	13 %	20 %
	14,217 23		6,236 48	5,951 56			
	9,532 80		3,829 16	2,653 29			
	14,216 23		10,560 26	7,326 29			
	4,162 39		1,332 21	1,946 29			
	4,328 63		3,929 23	2,788 24			

Save your results for future use:

343. The cost book is ruled as in the illustration below:

Article No.	Dept. I	Dept. II	Dept. III	Dept. IV	Dept. V	Dept. VI
1	$4 68					
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						

The numbers of the departments are stated at the head of the blank. The numbers of the articles in the departments are stated at the left. Each article is given a stock number when it is purchased.

The first figure of the stock number indicates the department; the remaining figures of the stock number indicate the number of the article in the department. For example, the third article purchased for Dept. I is given the number 13, meaning department 1, number 3.

When an article is purchased, the cost clerk adds to the wholesale cost a share of the buying and selling expenses of the department, and enters this total in the proper place in the cost book.

Written Work

You are now to prepare a cost book for articles purchased for six different departments. You will be told the prime cost of each article purchased. The per cent of the prime cost required to cover buying and selling expenses in each department was found in the preceding exercise. Multiply the prime cost by the proper per cent to determine the share of expenses; add this to the prime cost, and enter the total in the cost book.

Problem. The prime cost of the first article purchased for Dept. I is $3.90. What cost should be entered in the cost book?

SOLUTION.

$3.90	Prime cost
.20	To cover buying and selling expenses in Dept. I
.78	Expenses to be added
$3.90	Prime cost
.78	Expenses
$4.68	Total cost to buy and sell

Prepare a cost book for the following purchases, using the per cents of buying and selling expenses for each department found in the exercise on page 414.

Prime cost of purchases: (The numbers of the articles are stated first; then the prime costs.) The items are given in the order of their purchase from left to right.

31,	$22.18;	21,	$16.42;	51,	$ 2.40;	11,	$ 3.90;	22,	$36.27;
12,	4.25;	23,	45.32;	32,	4.23;	61,	12.90;	52,	3.90;
13,	9.18;	41,	12.46;	62,	18.75;	24,	27.19;	33,	19.27;
42,	9.13;	53,	2.85;	63,	15.30;	25,	36.14;	34,	2.30;

43, 2.56; 14, 2.32; 54, 6.23; 44, 5.80; 26, 27.23; 64, 23.95; 15, 13.50; 35, 4.67; 16, 17.60; 36, 2.13; 45, 4.50; 55, 2.19; 56, 9.29; 17, 13.47; 37, 4.23; 18, 9.28; 57, 4.80; 58, 2.65; 65, 26.80; 19, 4.62; 66, 13.86.

Save your results for future use.

344. Work of the Profit Clerk. In stores where a cost book is kept, it is the duty of the profit clerk to compute the profit or loss on each sale. The profit clerk is provided with a copy of the cost book, and a duplicate of each invoice of goods sold.

He finds the selling price on the invoice, and the cost from the cost book; by subtraction, he is able to compute the profit or loss on each sale.

Written Work

Use the cost book prepared in the preceding written exercises to find the profit or loss on each of the following sales:

1. Invoice No. 2345.

		Selling Price
6	No. 17	$16.45.
2	No. 35	5.80.
8	No. 56	12.00.

2. Invoice No. 2346.

		Selling Price
5	No. 52	$ 4.75.
6	No. 63	19.00.
12	No. 32	5.00.

3. Invoice No. 2347.

		Selling Price
12	No. 58	$ 3.25.
10	No. 13	11.00.
8	No. 36	2.70.
24	No. 41	16.00.

Less a cash discount of 1 %.

4. Invoice No. 2348.

		Selling Price
1	No. 61	$16.30.
15	No. 63	19.00.
12	No. 44	8.00
20	No. 45	6.00.

5. Invoice No. 2349.

		Selling Price
10	No. 26	$33.50.
5	No. 22	50.00.
18	No. 21	22.50.
25	No. 53	3.25.

6. Invoice No. 2350.

		Selling Price
20	No. 15	$16.50.
20	No. 13	12.00.
30	No. 12	5.00.

CHAPTER XLIV

FACTORY COSTS

DURING recent years an increasing interest has been shown by manufacturers in the subject of cost records and accounting. While the system of factory cost records is complex, the fundamental principles are simple.

345. Elements of Cost. Factories usually divide their expenses, or "costs," into four classes, namely, direct labor, direct material, indirect expenses or manufacturing expenses, and selling expenses.

Direct Labor is labor employed directly in the production of an article. This item is found from the pay roll and the time tickets of workmen.

Direct Material costs include all outlays of materials used in the making of an article, entering into and becoming a part of it.

Indirect Expenses, often called **burden** or **overhead expenses,** or **manufacturing expenses** consist of all the other expenses incurred in the manufacture (but not the sale) of the product.

Indirect expenses include such items as rent, taxes on factory, insurance, interest on factory and equipment (some accountants and efficiency experts object to including interest as an expense), heat, light and power, repairs, depreciation. Indirect labor includes salaries of factory superintendents and foremen; wages of all men employed in the factory at other than direct labor on the product, such as janitors, watchmen, and messengers.

Selling Expenses include all expenses incurred in marketing the product, such as advertising, salaries, and expenses of salesmen.

The various costs are grouped as follows:

Labor + Material = Prime Cost.
Prime Cost + Indirect Expenses = Cost to Make (factory cost).
Factory Cost + Selling Expenses = Total Cost.
Selling Price − Total Cost = Profit.

Problem. What are the costs and the profit of a factory, the records of which show the following:

Labor,	$33,286.45
Materials,	18,973.98
Indirect expenses	15,471.25
Selling expenses,	4,726.35
Sales,	92,487.50

SOLUTION.

Labor	$33,286.45
Materials	18,973.98
Prime cost	$52,260.43
Prime cost	$52,260.43
Indirect expenses,	15,471.25
Factory cost	$67,731.68
Sales	$92,487.50
Factory cost	67,731.68
Gross profit on sales	$24,755.82
Gross profit on sales	$24,755.82
Selling expenses	4,726.35
Net profit on sales	$20,029.47

346. Finding the Profit or Loss on each Article. A manufacturer can determine with little difficulty his total costs and his profit, but it is more difficult to determine the exact cost of manufacturing any particular article, and the profit or loss arising from its sale. However, it is very important for the manufacturer to be able to do this, because, while the business as a whole may make a profit, many individual articles may be sold at a loss. It is highly desirable to know what all articles cost.

To find the factory cost of an article, the manufacturer must know:

a. The cost of the labor spent upon it;

b. The cost of the materials entering into it;

c. The share of the indirect expenses or burden which should be charged to it.

The records which show these facts are called "cost records."

347. Cost Records Illustrated. Printing offices usually have well-established cost systems, and we can illustrate the method by following a job through such an office. An order for 5,000 letter heads is sent to the printing office. A cost card is made out in the office, similar to the following illustration:

JOB TICKET

Job. No. 369

Date, September 4th 1922

Job 5000 Letter Heads

For A. S. Winston

Stock Jonathan Bond

Type Old English

Ink Black

Promised September 10th 1922

COSTS

Stock $_____

Labor $_____

Burden $_____

Total Cost $_____

Selling Price $_____

Profit $_____

Finding the Material Cost. When the paper is cut, its cost is computed, and entered on the card.

Finding the Labor Cost. Each workman who is engaged on the job fills out a time ticket showing the number of hours which he spent on it.

TIME TICKET

Date 9/11/22

Workman Harvey Brown

Job. No. 369

Operation Composition

Began 1.00 Rate $.28

Finished 2.15

Time 1:15 Amount $.35

348. Finding the Indirect Expenses. This part of the cost record is the most difficult to obtain. Before any job can be charged with a share of the burden, two things must be determined. They are:

a. The total indirect expenses, overhead or burden.

b. The best method of distributing the expenses among the various articles manufactured.

Computing the Total Indirect Expenses. At the beginning of the year, an estimate, or budget, of probable expenses is prepared. It is similar to the following:

Interest on plant and equipment	$ 500.00	
Insurance	75.00	
Heat, light, and power	640.00	
Repairs	125.00	
Depreciation	300.00	
Indirect labor	1,620.40	
	52)$3,260.40	Yearly burden.
	$62.70	Weekly burden.

These items can be foretold with comparative accuracy by referring to the records of the previous years. Many other items may be included, but those stated will illustrate the method.

We have now determined that the overhead expenses for the week are $62.70, and this amount must be distributed over all the jobs completed or worked on during the week.

Various methods for distributing overhead expenses are discussed on page 421.

Written Work

1. What is the weekly burden in a factory with the following annual overhead expenses?

Rent, $420.00; Interest on machinery and other equipment, $90.00; Insurance on equipment, $12.00; Heat and light, $285.00; Power, $600.00; Repairs, $150.00; Depreciation, $25.00; Miscellaneous, $380.00.

2. Find the total burden and the weekly burden in a factory reporting the following:

Building cost, $8,000.00; Equipment cost, $16,500.00.

Interest on invested capital at 5½%.

Taxes: Building valued by assessor at $7,500.00, taxed on ⅓ of its value: Equipment assessed at $5,000.00; Tax rate, $2.68.

Insurance: Building insured at 80% of cost; Equipment insured by a $12,000.00 policy. Rate on building and contents, $1.30.

Depreciation: 4% on building; 12½% on machinery.

Miscellaneous, $1,865.00.

DISTRIBUTING THE OVERHEAD EXPENSES AMONG THE VARIOUS JOBS

Several methods are in use for distributing the burden. The following are the most common:

a. The Direct Labor Cost Method;
b. The Direct Labor Hours Method;
c. The Direct Labor and Materials Cost Method;
d. The Department Machine Rate Method.

349. The Direct Labor Cost Method. When this method is used, the indirect expenses are distributed among the various jobs in the same ratio as the cost of the labor put on each of the jobs.

The computation is made as follows:

Divide the weekly burden by the total labor cost for the week to find what per cent the burden is of the labor cost.

Multiply the labor cost of each job by this per cent to find the burden for each job.

Problem. In the printing office with a weekly burden of \$62.70, the pay roll for the week ending September 17 was \$370.00. The labor cost of job No. 635 was \$18.50. What share of the weekly burden should be charged to job No. 635?

SOLUTION.

Labor cost for week	\$370.00
Weekly burden	\$ 62.70

The burden is, therefore, $\frac{62.70}{370.00}$ of the labor, or about 17 %.

17 % of the labor cost of each job is added as burden.
17 % of \$18.50 = \$3.15, the burden on job No. 635.

Written Work

1. If the annual burden of a factory is \$3,280, what is the weekly burden?

2. The pay roll of this factory for a week is \$735.60. What per cent of the labor cost of each job should be added to cover the burden?

3. The labor cost of job No. 1729 was \$27.69. What amount of burden should be included in the "cost to make"?

4. Annual burden, \$2,150.
Pay roll, week ending May 11, \$296.48.
Labor cost of job No. 234, \$64.96. What is the burden?
Labor cost of job No. 235, \$18.26. What is the burden?

5. Annual burden, $4,620.
Labor costs per job, for week ending August 26.

Job No. 1,325	$ 27.69
1,326	34.19
1,327	96.41
1,328	117.12
1,329	26.14
1,330	115.86
1,331	76.90
Total pay roll	$494.31

What charge for burden should be included in the cost of each job? (Find per cent of burden approximate to the nearest even per cent.)

6. The cost of material for each job was as follows:

1,325	$19.80	1,326	$21.75	1,327	$48.69
1,328	$59.74	1,329	$ 4.90	1,330	$48.75
1,331	$27.96				

What was the total cost to make each job?

7. The selling expenses have been found, by experience, to be 21 % of the total cost to make. What is the cost to make and sell each job?

8. The selling price is determined by adding 28 % to the cost to make. What is the selling price of each job, and what is the profit?

350. The Direct Labor Hours Method. This method is similar to the direct labor cost method, with the exception that the hours of labor, instead of the cost of labor, are made the basis of distribution.

Problem. In the factory with a weekly burden of $62.70, the total number of hours worked by all direct laborers during the week ending October 17 was 482. What burden should be charged to job number 173, on which 32 hours of direct labor were spent?

SOLUTION.

Total burden for week, $62.70
Total direct labor hours 482
$62.70 ÷ 482 = $.13, hourly burden.

32 Number of hours spent on job number 173
$.13 Hourly burden
$4.16 Burden charged to job number 173

Written Work

1. The following jobs were begun and completed during the week ending September 27.

Job No.	Hours of Labor	Labor Cost	Material Cost
2,789	93	$28.15	$14.20
2,790	104	36.18	15.55
2,791	32	10.16	6.12
2,796	48	15.28	8.19

The total number of hours of direct labor on all jobs during the week was 623, and the annual burden was $2,591.68.

What was the hourly burden rate for the week ending September 27?

What amount of burden should be charged to each job, and what was the total cost to make each job?

2. The following table shows the figures for jobs on which work was done during the week ending September 27, and which were completed during the week ending October 4.

Job No.	Hours of Labor		Labor Cost	Material Cost
	Week Ending Sept. 27	Week Ending Oct. 4		
2,792	19	43	$19.40	$ 8.70
2,793	23	69	28.55	13.25
2,794	9	46	17.90	6.45
2,795	12	52	19.35	7.90
2,797	8	95	35.98	14.62

The total number of hours of direct labor for the week ending October 4 was 712. What was the hourly burden rate for work done during the week ending October 4?

What is the amount of burden to be added to the cost of each job in the table above for work done during the week ending September 27?

What is the amount of burden to be added to the cost of each job in the table above for the work done during the week ending October 4?

What is the total cost of each job?

3. 15% of the factory cost is added to the cost of each job as an estimate to cover selling expenses. What is the cost to make and sell each job, 2,789 to 2,797 inclusive?

4. What profit or loss was realized by selling these jobs at the following prices?

Job No.	Selling Price	Job No.	Selling Price
2,789	$63.00	2,793	$63.00
2,790	80.00	2,794	30.00
2,791	20.00	2,795	40.00
2,792	45.00	2,796	38.50

5. Manufacturers who do not operate a cost system frequently add a certain per cent of the prime cost (material and labor) to cover burden, selling expenses, and profit.

What was the prime cost of each job in Problems 1 and 2?

If 33 % had been added to the prime cost to determine selling price, what would have been the selling price of each job?

6. The actual cost to make and sell each job was computed by the cost system in Problem 3. Compare these costs with the selling prices determined in Problem 5, and find what profit or loss would have been realized from the sale of each job.

351. Direct Labor and Materials Cost Method. Some manufacturers prefer to distribute the burden among the various products in proportion to the cost of both the labor and the material entering into them. When this method is employed, the process is as follows:

Total labor cost per week (or other unit of time)	$525.80
Total materials cost for the week	264.70
Total labor and materials cost	$790.50

$62.70 (weekly burden) ÷ $790.50 = 7.9 %. Per cent of labor and materials cost to be added to each job as burden.

If the job ticket, or cost sheet, shows that $35.00 worth of material and $53.20 worth of labor have been put into a job, the burden would be computed thus:

$53.20	Labor Cost
35.00	Materials Cost
$88.20	Labor and Materials Cost

7.9 % of $88.20 = $6.97, Burden.

$53.20	Labor Cost
35.00	Materials Cost
6.97	Burden
$95.17	Cost to Make

Written Work

1. If the weekly burden of a factory is \$6,340, and the cost of the materials and labor put into the product of a week is \$923.70, what per cent of the labor and materials cost of each job must be added to cover burden? Approximate result to the nearest half per cent.

2. Labor cost on job No. 884, \$16.90; materials cost, \$13.25. What is the burden, determined by labor and materials cost method, using the per cent found in the first problem?

3. Complete the following table.

Annual Burden	Weekly Burden	Week's Labor Cost	Week's Materials Cost	% Burden	Labor Cost One Job	Materials Cost One Job	Burden Per Job	Cost to Make Each Job
\$3,145 90		\$215 60	\$412 90	Approximate to nearest tenth of a per cent	\$32 15	\$ 9 75		
2,268 25		186 00	45 25		12 15	4 70		
5,496 00		276 25	396 90		19 25	22 70		
12,962 00		893 56	592 90		63 00	49 75		

352. Department Machine Rate Method. The preceding methods do not take into consideration the fact that more burden should be charged for some operations than for others.

For example, let us suppose that a factory produces two kinds of articles. One is made on a very expensive machine, occupying a large room, and requiring a great deal of power. The machine is delicate, it requires considerable repairing, and will wear out in a few years. The depreciation, therefore, is large. The other article is made at a bench by hand, and with cheap tools.

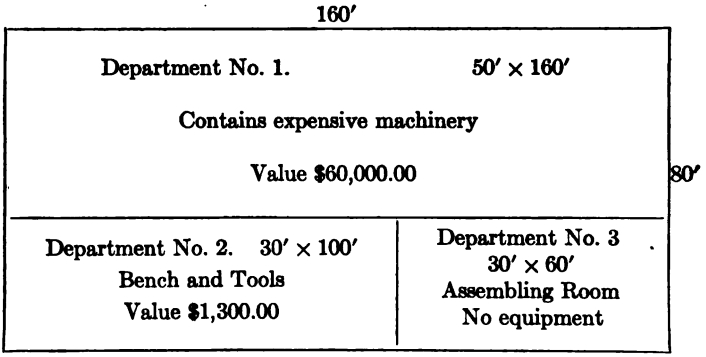
Suppose the same amount of material and labor were required to make each of these articles. If any of the methods explained in the previous sections were employed, the same amount of burden would be charged to each article. But it is evident that the expense of manufacturing the former is much greater than that of the latter. The items which make up the expense are listed below.

Overhead Expenses of Making Article No. 1	Overhead Expenses of Making Article No. 2
Interest on Expensive Machinery.	Interest on Cheap Bench and Tools.
Rent of Large Room.	Rent of Small Room.

Cost of Power.	
Repairs on Machinery.	Repairs (very little).
Depreciation of Machinery.	Depreciation (very little).
Heat and Light.	Heat and Light.
Incidentals.	Incidentals.

The machine rate method is intended to distribute the burden in accordance with the real expenses of manufacture. The factory is divided into departments or "factors," similar processes with about the same expenses being grouped together. The burden is then divided among all departments, as shown by the following illustration.

Suppose that a factory has three departments, as shown by the following floor plan:



The burden would be divided among the departments thus:

Items	Dept. No. 1	Dept. No. 2	Dept. No. 3	Total
Rent (floor space 160 × 80)	$1,000.00	$375.00	$225.00	$1,600.00
Interest	3,000.00	65.00		3,065.00
Insurance	480.00	10.00	45.00	535.00
Heat and Light	275.00	130.00	60.00	465.00
Power	500.00			500.00
Depreciation	1,200.00	13.00		1,213.00
Supervision	720.00	250.00	175.00	1,145.00
Incidentals	245.00	130.00	85.00	460.00
Total	$7,420.00	$973.00	$590.00	$8,983.00
Per week	$ 142.69	$ 18.71	$ 11.35	$ 172.75

METHODS OF DISTRIBUTING COSTS AMONG DEPARTMENTS

Rent. The rent is prorated among the three departments according to floor space; thus,

Annual rent of building, $1,600. Total floor space, 12,800 sq. ft.
$1,600 ÷ 12,800 = $.12½, rent per square foot.
Floor space Dept. 1 is 8,000 sq. ft.
8,000 × $.12½ = $1,000, Rent Dept. 1.
Floor space Dept. 2 is 3,000 sq. ft.
3,000 × $.12½ = $375, Rent Dept. 2.
Floor space Dept. 3 is 1,800 sq. ft.
1,800 × $.12½ = $225, Rent Dept. 3.

Interest. 5% interest on $60,000 = $3,000, Int. Dept. 1.
5% interest on $1,300 = $65, Int. Dept. 2.

Insurance. Insurance on the building is prorated among the departments in proportion to floor space.

Insurance on equipment is determined by the value of the equipment.

Heat and Light. Heat is prorated according to floor space.

Light may be prorated by floor space, or a separate meter may show the cost of the light used in each department.

Depreciation. Depreciation on the building is prorated by floor space.

Depreciation on equipment depends upon the value of the equipment.

Finding the Hourly Burden Rate for Each Department. To apply this method of distributing the burden among the various jobs, the total number of hours of labor spent in each department for a week, or other period, must be known. Then the weekly burden in each department is divided by the number of hours of labor each week in the department, to determine the hourly burden rate.

Problem. Let us suppose the total hours of labor in each department for a certain week were as follows:

Dept. No. 1, 317; Dept. No. 2, 104; Dept. No. 3, 95.

What was the hourly burden rate in each department?

SOLUTION. Dept. 1. $142.69 (weekly burden) ÷ 317 = $.45, hourly rate.
Dept. 2. $18.72 (weekly burden) ÷ 104 = $.18, hourly rate.
Dept. 3. $11.35 (weekly burden) ÷ 95 = $.12, hourly burden.

Finding the Overhead Expenses and the Total Cost, when the Machine or Department Rate is Used.

Problem. The labor cost of an article (made during the week the hourly burden rates of which were determined above) is \$13.40; materials cost, \$11.80. The labor hours in the different departments were as follows:

Dept. 1, 28 hours; Dept. 2, 5 hours; Dept. 3, 8 hours.

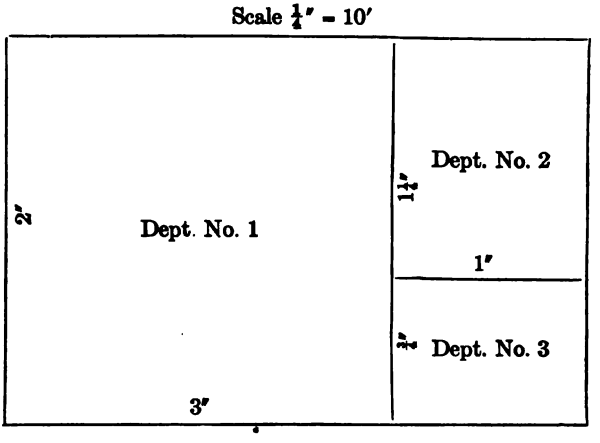
What were the overhead expenses charged to this job, and what was the total cost?

SOLUTION.

Labor cost		\$13.40
Materials cost		11.80
Burden		
Dept. 1; 28 hours at \$.45	\$12.60	
Dept. 2; 5 hours at .18	.90	
Dept. 3; 8 hours at .12	.96	
Total burden		14.46
Total cost to make		\$39.66

Written Work

1. Prepare an estimate of the annual burden of the factory occupying a building with the following floor plan:



Interest on Building. The building cost \$15,000.00. 5½% interest on this cost is prorated among the departments in proportion to the square feet of floor space occupied by each.

Insurance on Building. The building is insured at $\frac{3}{4}$ of its cost. Rate, $1.65. The total cost of the premium is prorated by floor space.

Taxes on Building. The building is assessed at $\frac{1}{3}$ of its cost, and taxed at $2.90. Tax prorated by floor space.

Depreciation of Building. 4% annual depreciation on the original value of the building is prorated by floor space.

Interest on Equipment. Dept. No. 1 contains machinery valued at $18,000.00.

Dept. No. 2 contains benches and tools valued at $2,000.00.

Dept. No. 3 is an assembly room and contains no machinery.

Each department is charged 5½% interest on the value of its equipment.

Insurance on Equipment. A $15,000.00, three-year, insurance policy covering the equipment, cost $3.30 per hundred. One-third of the cost of this insurance is charged annually as an expense, and is prorated among the departments in proportion to the value of the equipment

Taxes on Equipment. The taxes on the equipment, $116.00, are prorated according to the value of the equipment. The tax rate is the same as on the building.

Depreciation. 8% depreciation is charged on the machinery in Dept. No. 1; 4% depreciation on equipment in Dept. No. 2.

Heat and Light. The records show that the heat and light bills for the three preceding years were: $625.00; $680.00; $630.00. Find the average annual expense and assume that this will be the expense for the coming year. Prorate by floor space.

Power. The average annual cost of power to run the machinery is $1,460.00. Power is required in Dept. 1 only.

Supervision. The salaries of foremen are charged to the departments as follows:

	Dept. No. 1	$865.00
	Dept. No. 2	530.00
	Dept. No. 3	680.00
Incidentals.	Dept. No. 1	$325.90
	Dept. No. 2	280.00
	Dept. No. 3	175.00

2. What is the weekly burden in the factory and in each department?

3. Since the number of hours of labor differs each week, the hourly burden will also differ each week. Complete the following table.

TABLE OF HOURLY BURDEN RATES

WEEK ENDING	DEPARTMENT NO. 1		DEPARTMENT NO. 2		DEPARTMENT NO. 3	
	Labor Hours	Hourly Burden	Labor Hours	Hourly Burden	Labor Hours	Hourly Burden
April 4	460		244		148	
April 11	495		285		163	
April 18	480		260		157	
April 25	395		203		130	
May 2	409		221		142	

Approximate results to the nearest cent.

4. The following table shows the number of labor hours in each of three departments spent on job No. 493, and also the labor cost of the job.

LABOR HOURS AND COST — Order No. 493

DATE		WORKMAN No.	HOURS			RATE	WAGES
			Department 1	Department 2	Department 3		
April	2	16	9			.36	$3 24
	3	16	7			.36	2 52
		23	9			.29	2 61
	4	16	5			.36	1 80
			30				
	6	23	6			.29	1 74
	7	28		9		.24	2 16
	10	28		6		.24	1 44
		21	8			.25	2 00
	11	28		7		.24	1 68
			14	22			
	13	16	4			.36	1 44
	14	28		5		.24	1 20
	15	21	8			.25	2 00
	16	31			7	.38	2 66
			12	5	7		$26 49

The materials cost of job No. 493 was $13.75.

The burden was computed as follows:

The total number of hours of labor per week in each department was determined; the number of hours of labor in each department was multiplied by the hourly burden rate for the respective weeks. (The rates used in this illustration are not the same as those in the completed table on page 430.)

COST SHEET — Order No. 493

WEEK ENDING	DEP'T. No.	HOURS	RATE	AMOUNT	SUMMARY	
April 4	1	30	26.9	$ 8 07	Labor	$26 49
April 11	1	14	28.4	3 98	Materials	13 75
	2	22	10.6	2 33	Burden	19 28
18	1	12	27.8	3 34	Total	$59 52
	2	5	11.3	57		
	3	7	14.2	99		
				$19 28		

The labor cost is what per cent of the total cost?

The materials cost is what per cent of the total cost?

The burden is what per cent of the total cost?

5. Rule a cost sheet and find the total cost of order No. 494. Use the weekly burden rates found in the table on page 430.

MATERIALS COST, $18.83

LABOR — Order No. 494

DATE	WORKMAN No.	HOURS			RATE
		Dept. I	Dept. II	Dept. III	
April 3	27	6			.35
April 4	27	9			.35
April 4	42		4		.32
April 6	27	8			.35
April 6	42		9		.32
April 8	27	5			.35
April 8	42		3		.32
April 9	27	6			.35
April 9	42		7		.32

Date	Workman No.	Hours			Rate
		Dept. I	Dept. II.	Dept. III	
April 13	27	4			.35
April 13	42		8		.32
April 15	27	2			.35
April 15	42		9		.32
April 15	61			2	.39
April 16	42		2		.32
April 16	61			7	.39

6. Labor cost is what per cent of total cost to make?

Materials cost is what per cent of total cost to make?

Burden cost is what per cent of total cost to make?

7. If 6% is added to the cost to make, to cover selling expenses, what is the cost to make and sell?

8. What must the selling price be to make a profit of $16\frac{2}{3}$% on the cost to make?

CHAPTER XLV

TABULATIONS FOR THE SALES MANAGER

BUSINESS men who are conducting large enterprises realize the importance of having carefully prepared statistics showing the results of their past activities, in order to judge the wisdom of their policies and to assist them in planning for the future.

The facts relating to the operations of the business are recorded in the books of account, and these records furnish the information necessary to make up various reports for the executives.

Since profits depend upon sales it is desirable to have comprehensive reports in regard to the sales, so that the sales manager may have all possible facts to guide him in determining his policies.

353. Duties of the Sales Manager. The sales manager is expected to increase the volume of business. In order to accomplish this, he must know the sales of each clerk and of each department. When he has this information, he is in a position to compare the efficiency of his clerks. The facts which he will be able to obtain from such tabulations as are illustrated in this chapter will enable him to manage his work more efficiently.

354. Individual Daily Sales Sheet. On the following form there are entered the sales of one day in one department of a store.

If a clerk on some preceding day sold a customer an article which the customer returned, the clerk's profit to the store is thereby decreased. Returned sales are, therefore, entered in the second column, and deducted from the gross sales to determine net sales.

In the last column is entered the number of checks, or sales tickets, made out by each clerk during the day. Record is made of the checks in order to determine the number of sales made by each clerk. The daily sales may have been small, but the clerk may have served a large number of customers.

Of what value would this blank be to the sales manager?

Written Work

Prepare a daily sales sheet from the following:

INDIVIDUAL DAILY SALES SHEET

Section No. 45 = Dress Goods Date 5/2/22

CLERK'S NUMBER	GROSS SALES	RETURNED GOODS	NET SALES	CHECKS
1	287 96			113
2	314 39			128
3	136 32	4 92		84
4	315 26	12 25		115
5	503 16			121
6	263 50	7 25		103
7	98 30	2 80		67
8	193 50			123
9	215 67	3 96		89
10	309 50	15 75		131
11	235 80	26 15		98
12	96 45			35
13	103 28	5 60		65
14	128 95			82
15	202 75	3 10		96
16	127 30			60
Total				

Present these statistics graphically.

355. Salesman's Cumulative Tabulation. This blank is used to record the sales of a traveling salesman who sells the goods of several departments. Each month his sales are entered by departments in the proper column. After the January and February sales are entered, the amounts for the two months are cumulated; that is, added. As each month's sales are returned they are cumulated with the preceding total. Thus, at any time during the year, this blank shows the following facts:

The salesman's sales per month in each department;
His total sales to date in each department;
His total sales each month in all departments;
His total sales in all departments.

Written Work

Prepare a cumulative sales report for Salesman No. 32.

Find the total sales per month.

Cumulate the sales as indicated on the form on page 436.

356. Monthly Percentage Comparison of Salesmen. Sales managers consider it very important to have comparisons of their different salesmen. They wish to know not only the amount of the sales made by each, but also what fractional part of the total sales was sold by each. The most convenient way to state these fractions is by percentage.

Written Work

Prepare a form similar to the one indicated in the following illustration:

MONTHLY COMPARISON OF SALESMEN

SALESMAN'S No.	FIRST WEEK	SECOND WEEK	THIRD WEEK	FOURTH WEEK	FIFTH WEEK	TOTAL	PER CENT
I	$320.70	$230.49	$563.40	$834.70	$123.70		
II	470.65	870.43	933.60	274.80	85.70		
III	376.80	543.70	625.80	735.43	137.89		
IV	339.40	763.35	347.84	347.96	99.89		
V	276.80	563.90	743.90	876.90	470.96		
VI	723.96	439.65	375.63	648.93	275.93		
VII	374.99	687.93	684.96	532.89	99.64		
VIII	268.37	536.96	648.97	374.88	63.96		
IX	469.89	746.93	387.93	479.69	364.84		
X	479.89	489.89	683.89	874.96	216.80		
XI	376.86	375.94	632.61	276.93	39.96		
XII	546.64	789.94	346.49	832.24	162.27		
XIII	345.49	328.84	769.94	962.63	137.90		
XIV	476.63	374.84	265.89	649.84	202.03		
XV	763.47	239.84	646.63	389.84	62.99		
Totals							

Enter the sales in the proper columns.

Find the ~~total sales~~ made each week by all salesmen. These totals should be entered at the foot of the columns.

Find the total sales made by each salesman during the month. This will require horizontal addition, and the totals should appear in the "Total" column at the right.

Salesman's Cumulative Tabulation of Sales by Departments

Salesman No. 32

Dept. No.	Jan.	Feb.	Cum.	Mar.	Cum.	Apr.	Cum.	May	Cum.	June	Cum.
I	$ 478 60	$ 392 85		$ 429 65		$ 356 00		$ 400 25		$ 351 35	
II	832 40	926 43		845 36		875 63		867 94		925 33	
III	927 39	874 32		898 99		933 40		950 42		895 48	
IV	1,246 89	1,362 94		1,123 25		1,150 99		1,234 56		1,345 67	
V	1,023 47	1,127 47		1,134 57		1,126 49		1,324 59		1,254 32	
VI	247 89	365 85		350 25		375 26		260 50		275 67	
VII	1,468 32	1,232 74		1,334 53		1,450 12		1,327 68		1,227 25	
VIII	784 39	836 39		825 36		794 35		821 33		777 77	
IX	462 37	529 83		531 33		535 35		464 64		525 25	
X	1,894 32	1,926 39		1,899 00		1,876 34		1,879 32		1,923 76	
XI	1,362 37	1,423 92		1,300 25		1,234 76		1,501 78		1,434 76	
XII	2,009 93	937 41		1,567 32		1,655 38		1,323 58		1,736 87	
XIII	3,462 87	2,634 77		3,047 52		3,327 89		2,933 89		3,000 00	
XIV	347 92	629 89		432 45		567 89		389 23		497 86	
XV	826 27	924 32		859 76		1,023 76		991 02		1,021 03	

Find the grand total sales for the month. (As a means of checking your work, find the grand total in two ways: (*a*) by adding the totals at the foot of the blank, and (*b*) by comparing this result with the total of the "Total" column at the right.)

Find what per cent of the grand total sales was made by each salesman. Enter these per cents in the column at the right.

Prepare a graph.

357. Comparisons with Previous Year. Business men are interested in the growth of their sales, profits, etc., and make frequent use of comparative tables which bring out clearly the increase or decrease in the business. In addition to recording each salesman's sales by weeks, months, and cumulatively, and in addition to comparing the records of different salesmen to determine their comparative efficiency as shown by their per cent of sales, the office often prepares a record of each salesman's sales for the current year as compared with his sales of the preceding year, to show the amount of increase or decrease, and also the per cent of increase or decrease.

Written Work

Enter the following information on a properly ruled form. Find the yearly totals, the increase or decrease in dollars, and also in per cents.

CHART SHOWING INCREASE OR DECREASE OF SALES

NAME _____

MONTH	SALES LAST YEAR	SALES THIS YEAR	INCREASE	PER CENT INCREASE	DECREASE	PER CENT DECREASE
January	$346 29	$427 95				
February	457 75	515 86				
March	385 86	395 57				
April	416 87	402 75				
May	295 84	312 75				
June	212 67	235 84				
July	416 47	493 75				
August	328 57	319 56				
September	425 65	483 75				
October	513 86	603 52				
November	627 94	673 49				
December	743 85	623 57				
Total						

Present part or all of these statistics graphically.

358. Salesman's Record of Comparative Sales and Profits by Departments. The following form compares the profits of one year with the profits of the preceding year. This traveling salesman sells goods in ten departments. The profits from his sales are computed by the profit clerk, and his total profits by departments are entered on the record.

Written Work

Prepare a record of comparative sales and profits for the sales of E. H. Barlow.

Find the total sales and the total profits for each year.

The profits were what per cent of the sales in each department?

After the per cent of profit in each department for the two years has been determined, the increase or decrease in the per cent of profit can be found by subtraction. Enter increases in black ink, and decreases in red ink.

Also prepare one or more graphs using these statistics.

SALESMAN'S RECORD OF COMPARATIVE SALES BY DEPARTMENTS
SALESMAN'S NAME — E. H. BARLOW

DEPT.	1921			1922			PER CENT DECREASE INCREASE
	Sales	Profits	Per Cent	Sales	Profits	Per Cent	
I	$12,162.40	$1,456.80		$13,162.90	$1,562.80		
II	3,147.90	629.60		3,349.80	640.90		
III	14,127.65	1,732.86		15,216.90	1,927.90		
IV	3,287.96	362.80		3,347.80	367.90		
V	5,372.47	437.89		5,629.90	469.90		
VI	6,392.49	326.80		5,146.79	294.90		
VII	15,271.34	1,376.45		15,423.66	1,389.90		
VIII	896.29	46.86		1,989.60	66.80		
IX	1,247.89	97.62		1,462.96	103.60		
X	13,467.47	1,263.43		14,362.62	1,346.90		
Total							

359. Daily Classification of Sales by Departments. The tabulation described in this section shows the daily sales in each department,

the total daily sales in the entire store, and the total sales of each department for the week.

WEEKLY CLASSIFICATION OF SALES BY DEPARTMENTS

DEPT.	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	TOTAL	PER CENT
I								
II								
III								
IV								
V								
VI								
VII								
VIII								
IX								
X								
Total								
Per Cent								

The per cent column at the bottom shows what per cent of the week's total sales was made each day.

The per cent column at the right shows what per cent of the week's total sales was made in each department.

Written Work

Prepare a form similar to the illustration. Enter the following facts on the form.

DEPT.	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
I	$1,213.85	$1,194.83	$ 949.37	$1,384.39	$1,029.47	$1,593.48
II	1,023.75	1,094.82	1,127.49	1,284.29	974.92	1,326.83
III	1,125.94	1,183.27	1,248.48	1,494.84	827.84	1,426.46
IV	923.85	1,184.37	1,395.47	1,029.84	831.15	1,285.31
V	2,539.75	1,823.57	1,531.84	1,925.29	2,041.73	2,848.57
VI	1,925.85	1,482.16	1,632.83	1,451.61	1,348.41	2,154.38
VII	1,594.85	1,238.57	1,285.94	923.74	1,128.48	1,328.94
VIII	1,923.84	1,528.49	1,228.47	1,395.47	1,463.94	1,927.47
IX	2,594.48	2,049.47	1,829.46	1,739.84	1,832.47	2,885.81
X	1,523.82	1,449.82	1,327.46	1,554.53	1,232.84	1,723.49

Find the total sales for each day.

Find the total sales for the week in each department.

Find the total sales made by the store during the week.

On the bottom line, show what per cent of the week's sales was made each day.

In the column at the right, show what per cent of the week's sales was made in each department.

Plot a curve on graphically ruled paper, showing the total sales made in the store each day of the week.

From this form and the graph, the sales manager can determine which days had the lowest sales and he will probably offer extra bargains on those days in order to attract more customers. He will also be able to locate the departments with the smallest per cent of sales, and will take measures to increase the trade in those departments.

360. Yearly Sales by Months in All Departments. The form illustrated in this exercise shows the total sales per month in each department of a store. It furnishes the basis for various comparisons; for illustration, the gross sales by departments can be compared either by months or for the entire year. A graph could easily be prepared from this table showing the variation of monthly sales in each department.

Written Work

Prepare a table from the following information. Find the annual sales in each department; find the total sales each month for the entire store.

	Dept. I	Dept. II	Dept. III	Dept. IV	Dept. V	Total
January . . .	$1,238.95	$3,728.94	$5,238.64	$2,927.46	$6,223.58	
February . . .	1,136.95	3,927.47	5,329.47	2,842.14	6,128.47	
March	1,145.98	4,023.45	5,183.92	2,812.51	5,923.74	
April	1,032.84	3,925.36	5,023.94	2,642.23	5,414.42	
May	947.83	3,723.75	4,823.46	2,523.95	5,324.94	
June	923.45	3,582.45	4,213.27	2,348.41	5,942.38	
July	1,123.76	3,327.74	3,928.48	2,493.48	5,283.85	
August	946.73	3,527.94	3,215.13	2,493.15	5,328.47	
September . .	982.67	3,628.94	3,921.14	2,532.57	5,623.95	
October . . .	1,095.85	3,825.93	4,428.48	2,724.71	5,923.42	
November . .	1,123.86	3,927.85	4,913.73	3,124.93	6,113.81	
December. . .	1,357.94	4,129.54	5,492.14	3,295.58	6,395.73	

Plot a curve on graphically ruled paper, showing the variations in monthly sales in Department I.

361. Sales and Returned Goods by Departments. The form illustrated on this page shows the annual sales in each department, the goods returned by customers to each department, the net sales, and the per cent of sales returned to each department.

Written Work

Prepare a form similar to the model.

Enter the following facts:

Dept. I, Sales, $27,493.75; Returns, $823.85. Dept. II, Sales, $54,294.74; Returns, $924.74. Dept. III, Sales, $82,583.84; Returns, $1,239.36. Dept. IV, Sales, $107,239.75; Returns, $1,257.34. Dept. V, Sales, $85,294.63; Returns, $923.75. Dept. VI, Sales, $112,437.85; Returns, $723.74. Dept. VII, Sales, $82,374.50; Returns, $1,328.85.

Dept.	Sales		Returned Goods		Net Sales		Per Cent of Sales Returned
Total . .							

Find the net sales for each department, the per cent of sales returned to each department, the total sales for the year, the total amount of goods returned to all departments during the year, the net sales for the year, and the per cent of sales returned to the store during the year.

Show by a curve plotted on graphically ruled paper, the per cent of sales returned to each department.

362. Sales Classified by Days and Departments. The form on page 442 shows the daily sales for one week in twenty-five selling sections. (Each department may be subdivided into several selling sec-

tions; for example, the men's clothing department into sections for suits, overcoats, hats, etc.)

Written Work

Prepare a classification of sales by days and by departments.

Enter the amounts in the proper column; find the total sales for each day, and for each section. Also find the grand total sales. Be sure that this grand total checks all additions.

SALES CLASSIFIED BY DAYS AND BY DEPARTMENTS

DEPT.	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	TOTAL
1	$423.60	$187.47	$279.19	$789.01	$203.94	$101.97	
2	543.54	632.65	443.22	339.75	365.93	234.56	
3	100.10	999.99	203.82	901.09	584.85	475.86	
4	594.32	567.98	213.45	191.12	218.12	345.09	
5	555.33	224.60	100.00	201.35	857.02	905.43	
6	504.60	543.52	421.05	164.30	305.34	465.89	
7	102.93	621.19	178.92	630.95	304.89	985.64	
8	567.23	324.54	858.87	664.88	234.98	876.32	
9	908.23	765.54	510.25	654.21	894.32	564.32	
10	809.32	432.66	141.15	234.65	124.56	784.32	
11	239.08	111.03	400.00	191.31	123.45	621.31	
12	178.92	543.98	753.60	555.33	135.62	152.02	
13	164.30	324.98	827.26	578.98	113.56	105.15	
14	594.32	234.65	252.42	421.05	211.31	202.01	
15	339.75	432.66	323.23	213.45	135.62	615.20	
16	911.26	234.56	789.02	512.12	218.21	201.51	
17	345.67	890.23	456.78	518.29	141.19	520.20	
18	902.34	567.89	589.13	234.13	720.95	111.11	
19	979.15	860.17	762.95	729.84	789.01	33.33	
20	519.79	790.11	825.85	678.11	101.97	754.46	
21	200.20	996.57	908.23	456.34	656.54	644.57	
22	987.87	100.98	819.38	517.83	671.78	135.76	
23	112.34	567.67	450.78	189.58	150.85	200.43	
24	109.32	528.49	246.18	180.81	917.99	774.82	
25	933.47	467.83	239.08	652.29	493.86	496.73	
Total							

363. Daily Sales Compared with Sales of Corresponding Day of Preceding Year. One of the most common methods of noting the growth of a business is to compare the sales of each day with the sales of the corresponding day of the preceding year. It is assumed that

trade conditions will be about the same, year after year, at the same time of the year. By comparing the sales of the first Monday in October, 1922, with the sales of the first Monday in October, 1921, a basis of comparison is furnished to show increase or decrease in business. It is customary to enter on the blank the weather conditions for the two days, since the weather would affect the volume of sales

Written Work

Prepare a blank similar to the model.
Enter the following facts on the form.

DAILY SALES REPORT BY SELLING SECTIONS

AUG. 2, 1922

SECTION NO.	THIS YEAR	LAST YEAR	INCREASE	PER CENT	DECREASE	PER CENT
1	$ 619.20	$603.60				
2	85.70	90.21				
3	214.00	196.80				
4	716.26	654.30				
5	426.80	397.80				
6	562.70	571.32				
7	627.80	503.60				
8	462.80	403.40				
9	1,037.90	862.40				
10	729.60	642.80				
11	327.96	417.84				
12	526.94	327.64				
13	827.83	941.62				
14	987.60	864.95				
15	479.90	532.60				
16	362.85	325.90				
17	762.35	629.89				
18	236.80	329.40				
19	246.89	639.90				
20	427.86	547.77				
21	572.96	563.84				
22	294.84	289.77				
23	276.89	286.89				
24	347.80	329.99				
25	532.87	531.39				
26	274.79	239.80				
Total						

Find the increase or decrease, and the per cent of increase or decrease, in the sales for each selling section, and for the entire department.

364. Form Showing the Per Cent of the Stock on Hand at the Beginning of the Month Sold During the Month. The following form shows what per cent of the stock on hand at the beginning of the month is turned during the month. An inventory is taken at the beginning of the month, the sales of the month are recorded, and the per cent of the inventory sold is computed.

Problem. The inventory at the beginning of January showed 540 suits on hand. During the month 360 suits were sold. What per cent of the stock was sold?

SOLUTION. $\frac{360}{540}$, the fraction of the stock sold.
$360 \div 540 = 66\frac{2}{3}\%$.

Written Work

Prepare a blank similar to the model.
Enter the following data.

MONTH	1921			1922			
	STOCK	SALES	PER CENT OF STOCK SOLD	STOCK	SALES	PER CENT OF STOCK SOLD	PER CENT INCREASE
Jan.	$1,864	$1,631	87.5	$2,163	$1,731	80.0	
Feb.	1,632	1,437		1,973	1,562		
March	1,689	1,307		1,865	1,437		
April	1,432	787		1,437	986		
May	1,063	836		1,237	884		
June	1,773	824		1,147	835		
July	1,089	829		1,312	846		
Aug.	1,002	786		1,562	897		
Sept.	1,347	965		1,575	979		
Oct.	1,689	1,087		1,734	1,137		
Nov.	1,947	1,476		2,074	1,562		
Dec.	2,137	1,694		2,263	1,734		
Total							

Complete the form by showing what per cent of the stock on hand at the beginning of the month was sold during the month.

365. Department Record of Sales, Profits, etc. The following table shows one of the more elaborate forms of store records.

Written Work

Rule a form similar to the illustration. Use the following data and complete the blank. Your ability to prepare this exercise without further explanation will be a good test of your efficiency. Prorate the salaries, commissions, and other expenses, according to sales.

DEPARTMENT SALES, GROSS PROFITS, RETURNED GOODS, NET PROFITS, PER CENT OF NET PROFITS, AND COMPARISON WITH LAST YEAR

DEPT.	SALES	GROSS PROFITS	PER CENT	RETURNED GOODS	PROFIT	PER CENT	PRORATED SHARE OF SALESMAN'S SALARY AND EXPENSES	NET PROFITS	PER CENT NET PROFITS	PER CENT PROFIT LAST YEAR	PER CENT INCREASE	PER CENT DECREASE
1	$ 3,465.80	$ 832.46		$134.60	$27.30					7		
2	2,247.95	629.22		94.97	21.13					12		
3	1,276.96	169.94								4		
4	974.44	202.41								15		
5	326.37	164.69								21		
6	16,374.39	2,234.62		99.84	11.20					10		
7	22,365.24	4,162.94		232.62	49.20					14		
8	16,264.64	3,148.88		194.39	41.29					10		
9	22,239.39	3,366.86		265.24	42.30					12		
10	1,646.62	624.30		42.00	19.00					14		
11	9,347.27	1,689.90		164.69	16.24					8		
12	6,362.94	2,144.64		196.62	62.21					18		
13	976.46	202.03		12.13	2.63					14		
14	3,274.74	968.66		47.31	11.21					12		
15	9,639.99	1,244.19		111.12	12.16					10		
16	12,244.84	2,646.29		199.96	35.19					13		
17	8,348.98	1,294.68		96.25	12.12					11		
18	16,264.42	3,000.99		47.73	10.09					13		
19	18,297.78	2,346.98		169.89	18.94					12		
20	1,362.63	423.37		21.31	7.96					11		

Salesman's total salary and commissions $2,461.00
Salesman's total expenses 1,992.00

MISCELLANEOUS

CHAPTER XLVI

CONSIGNMENTS AND COMMISSIONS

GRAIN, live stock, and many other articles of produce and manufacture are usually marketed through commission merchants or brokers.

366. Illustration and Explanation of Terms. Mr. Williams, who lives in a small town in Minnesota, sends a car of grain to Owen and Bartlett, commission merchants of Chicago. Owen and Bartlett are engaged to sell the grain, pay all charges, deduct their commission, and remit the proceeds to Williams.

Williams is called the **principal**; Owen and Bartlett, the **agents.**

Williams calls the grain a **shipment**; Owen and Bartlett call it a **consignment.** The statement of the transaction furnished to Williams is called an **account sales.** When a commission merchant is engaged to buy produce, the statement of the transaction furnished to his principal is called an **account purchase.**

367. Selling Consignments of Grain. The city produce exchanges provide a ready market for grain. The commission merchants and brokers are members of these exchanges, and act as the agents of farmers and elevator companies in the sale of the grain. The farmer or elevator owner consigns his cars of grain to his broker, who sells the grain, has it unloaded at the elevator designated by the purchaser, receives cash in payment, and accounts to his shipper for the full amount received, minus his commission and the charges which he has paid.

Commission is the charge made by the broker for handling the sale. The following are Chicago rates:

$\frac{1}{2}$ cent per bushel on corn and oats.

1 cent per bushel on wheat, rye, and barley.

Inspection. A nominal fee (usually 35 cents) is charged by the state grain inspection bureau for determining the grade or quality of the grain.

Weighing. A fee is charged by the board of trade weighing bureau for determining the number of pounds of grain in the car.

SALES BY OWEN & BARTLETT

COMMISSION MERCHANTS AND RECEIVERS

CHICAGO

No. _____ FOR ACCOUNT OF Sar Elev Co Danville

DATE OF SALE		CAR NO	KIND OF GRAIN	BUSHELS	LBS.	PRICE	TERMS	AMOUNT
Aug	18	82724	No 2 Yellow Corn	1225	14	80	S.O	980 20
Pro. No								
		CHARGES						
		Freight, 5½			37	74		
		Extra Charges, Demurrage			1	00		
		Interest 15 days @ 6 %			2	00		
		Switching,						
		Weighing,				30		
		Inspection,				36		
		Commission,			6	13		47 52
		Net proceeds to your credit,						932 68
		E. & O. E			Draft			800 00
		CHICAGO, 9/5/21 191				Balance		132 68
Bill Book, Folio					OWEN & BARTLETT			
					By			

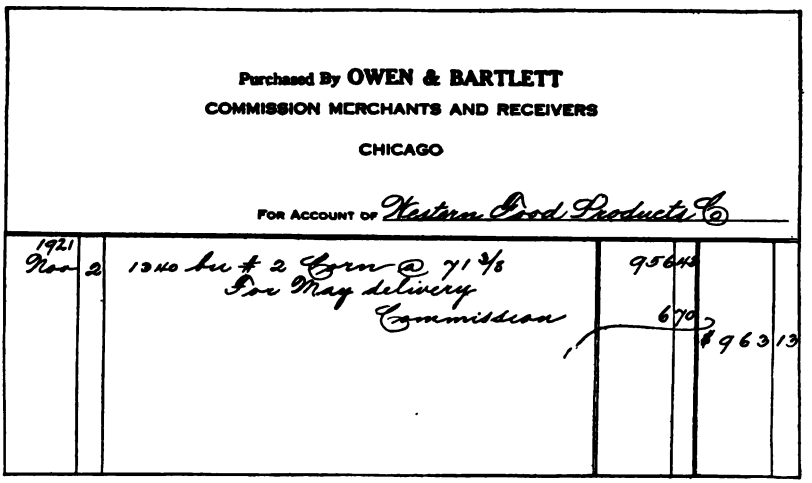
AN ACCOUNT SALES

Drafts. The consignor frequently draws a demand draft on his broker in part payment. The draft does not usually exceed 80 % of the estimated value of the grain. The shipper deposits the draft, with bill of lading attached, with his local bank. The local bank sends the draft to its correspondent bank in the city where the broker is located. The city bank presents the draft to the broker, receives payment, and turns over the bill of lading. When the broker renders an account sales of the grain, he deducts the amount of the draft, plus interest at

the ruling rate from the date the draft was paid until the date on which he received the money from the sale of the grain.

368. Buying Grain. Brokers buy grain as the agents of millers, refiners, and other manufacturers using grain as a raw material. Purchases are made both for immediate and for future delivery.

In the produce exchanges there is a quotation on grain purchased for immediate delivery, and also on contracts calling for the delivery


Purchased By OWEN & BARTLETT
COMMISSION MERCHANTS AND RECEIVERS
CHICAGO
For Account of Western Food Products Co
1921 May 2 1340 bu # 2 Corn @ 71 3/8 For May delivery 956.45
Commission 6.70
963.13

AN ACCOUNT PURCHASE

of grain at some definite future date. These future prices are affected by the prospects of large or small crops, by the demand for grain abroad, by the money market, and by many other factors.

If the purchase is made for immediate delivery, grain is purchased by the broker for his principal, at the market price, and delivery is made as soon as possible. If the grain is purchased for future delivery, it is delivered at the time specified when the contract was made, and charged at the price prevailing for future orders on the day of purchase.

Charges for Buying Grain. The only charge made by brokers for buying grain is commission. The commission on a purchase is the same as on a sale.

369. Speculating in Futures. Speculators often instruct a broker to buy grain for future delivery, hoping that when the day of delivery arrives, the market price will have risen and a gain may be realized.

On the other hand, a speculator may instruct his broker to sell grain for future delivery, hoping that when the day of delivery arrives, the market price will have dropped. In such a case he can buy the grain at the market price, and deliver it to the purchaser at the contract price, or the purchaser may pay him the difference between the market price and the contract price.

The following account purchase and sale shows a transaction in which a speculator contracted for the purchase of September grain in July, and sold it in September, realizing a profit of $1760.00, after paying his commission.

Account Purchase and Sale of 10 Thousand Bushels Sept Corn

By OWEN & BARTLETT For Account and Risk of

A. G. Mason

Chicago, 9/5 1921

July 7	Bought 10 Thousand Bus. @ 63½		6350		
	Commission		15		6365
Sept 5	Sold 10 Thousand Bus. @ 81¼				8125
	Balance to your Credit				$ 1760 00
	E. & O. E.				OWEN & BARTLETT
					By

TABLE OF GRAIN WEIGHTS PER BUSHEL

Corn	56 pounds
Wheat	60 pounds
Oats	32 pounds
Barley	48 pounds

Written Work

1. What is the value of 71,980 pounds of corn at 62 cents per bushel?

2. What is the value of 89,720 pounds of wheat at $1.32 per bushel?

3. What is the value of 54,350 pounds of oats at 42 cents per bushel?

4. What is the value of 67,320 pounds of barley at 71 cents per bushel?

5. If a farmer sells the elevator company in his town 514 bushels and 28 pounds of corn at 62½ cents per bushel, how much does the farmer receive?

6. If an elevator company buys

514 bu. 28 lb. of corn at 58½ cents,
312 bu. 14 lb. of corn at 58¾ cents,
410 bu. 14 lb. of corn at 59 cents,

what does the grain cost the elevator company?

7. If the elevator company sells this grain through a Chicago broker for 65½ cents, and pays

6½ cents per hundred pounds, freight,
½ cent per bushel commission,
25 cents for weighing,
35 cents for inspection,

how much profit does the elevator company realize on the transaction?

8. The Sedan Elevator Company shipped you, as its broker, 1145 bushels and 40 pounds of wheat, which you sold at $1.40. Charges were as follows:

Commission, 1 cent per bushel;
Freight, 8½ cents per hundred pounds;
Weighing, 30 cents;
Inspection, 35 cents.

The elevator company drew a draft for $700, which you paid eleven days before receiving your payment for the grain, interest at 6%.

Prepare an account sales for the transaction.

9. A milling concern bought 96,740 bushels, 20 pounds of wheat through a broker at $1.40. They paid freight at the rate of 11½ cents per hundred pounds, a commission of one cent per bushel, and switching charges of $34. How much did the grain cost them, delivered at their mills?

10. The manager of a breakfast food company purchased on April 25, 75,000 bushels of September corn at $61\frac{1}{2}$ cents. On September 5, the price of September corn was $79\frac{1}{4}$ cents. How much did this company save by taking advantage of the lower price of corn in April? Do not consider interest.

CHAPTER XLVII

LIFE INSURANCE

INSURANCE against loss due to sickness, accident, or death is called **Personal Insurance.**

There are several classes of Personal Insurance of which the following are the most common: Accident Insurance, Health Insurance, Liability Insurance, and Life Insurance. Only Life Insurance will be considered in this chapter.

In consideration of certain payments, called **Premiums**, a Life Insurance Company agrees to pay, at the death of the insured, or at some other time stipulated in the contract, called the **Policy**, a stated sum of money to the person designated by the insured. The person to whom payment is to be made is called the **Beneficiary.** There are many kinds of policies, and policies of the same name often differ in certain details.

While it will be impossible to present the subject of Life Insurance in great detail in this chapter, the fundamental principles of the various types of policies will be discussed.

370. Principal Kinds of Life Insurance Policies. The principal kinds of policies are:

a. Ordinary Life policy.
b. Limited Life policy.
c. Endowment policy.
d. Term policy.

These differ in three important particulars.

a. The number of premiums paid by the insured.
b. The amount of each premium.
c. The time when payment is made by the company.

COMPARISON OF SEVERAL KINDS OF POLICIES

KIND OF POLICY	NUMBER OF YEARS PREMIUMS ARE PAID	TIME WHEN PAYMENT IS MADE BY THE COMPANY
Ordinary Life	During life of insured [In some companies, if dividends are allowed to accumulate with the company, this period may be shortened.]	At death of insured
* Limited Life —		
20-payment life . . .	20 years	At death of insured
10-payment life . . .	10 years	At death of insured
* Endowment Policy —		
20-year endowment .	20 years	At death of insured payment made to beneficiary; or at expiration of 20 years payment made to insured if still living
10-year endowment .	10 years	At death of insured or at expiration of 10 years
20-payment, 30-year endowment . . .	20 years	At death of insured or at expiration of 30 years
* Term Policy —		
20 years	20 years	At death of insured if he dies within 20 years. If he lives beyond this term, no payment is made

* The policies listed under this heading are illustrative only. Policies are issued for other periods than those mentioned.

371. Insurance Rates. Insurance rates are usually quoted at a certain price per $1000 of insurance. The rate depends upon the age of the insured when he " takes out the policy," and upon the kind of policy. The premiums are payable in advance.

The desirability of insuring at as early an age as practicable, is shown by a study of the comparative premiums at different ages.

The following table shows the premium rates per thousand on various kinds of policies for various ages.

ANNUAL PREMIUM ON DIFFERENT KINDS OF INSURANCE PER $1000.00

AGE	ORDINARY LIFE	20-PAY. LIFE	10-PAY. LIFE	10-YEAR ENDOWMENT	20-YEAR ENDOWMENT
15	$ 17.40	$27.34	$ 44.62	$100.60	$47.79
20	19.21	29.39	47.85	101.57	48.48
25	21.49	31.83	51.67	102.73	49.33
30	24.38	34.76	56.18	104.14	50.43
35	28.11	38.34	61.53	105.87	51.91
40	33.01	42.79	67.90	108.07	54.06
45	39.55	48.52	75.57	111.03	57.34
50	48.48	56.17	84.99	115.28	62.55
55	60.72	66.69	96.66	121.48	70.81
60	77.69	81.60	111.47	130.76	83.82
65	101.48		131.13	145.08	

Written Work

From the preceding table find the annual premium required for:

1. An ordinary life policy for $4000, taken by a man 20 years of age.

2. A ten-payment life policy for $3000, taken by a man 25 years of age.

3. A ten-year endowment policy for $5000, taken by a man 35 years of age.

4. A twenty-year endowment policy for $5000, taken by a man 25 years of age.

5. A man 25 years of age took out a twenty-year endowment policy for $5000, and died after making eight payments. How much less would the combined premiums have been on an ordinary life policy?

6. A man on his 30th birthday took out a twenty-payment life policy for $3000, and 5 years later he took out an ordinary life policy for $5000. He died after making fifteen payments on his first policy. By how much did the insurance received by his beneficiary exceed the premiums paid (not considering accumulations)?

372. Dividends. Policies written on the "participating plan" provide that a portion of the annual profits of the company shall be shared by the holders of the policies. These annual payments of a share of the profits are called "dividends." Since the profits of the

company vary from year to year, no specific amount of dividend is guaranteed to the policyholder. Policyholders usually receive no dividend the first year. Dividends may, at the option of the insured, be

a. Paid to him in cash;

b. Applied to the payment of his premium;

c. Left with the company and allowed to accumulate at compound interest;

d. Left with the company to increase the amount of insurance carried;

e. Left with the company in order to decrease the number of payments of premiums.

The following table shows the effect of the dividends in reducing the annual premiums.

ANNUAL CASH DIVIDENDS AND NET COST OF INSURANCE ON POLICIES OF $1000 — AGE 35

	Ordinary Life Policy Annual Premium, $27.30		20-Premium Life Policy Annual Premium, $36.00		20-Year Endowment Policy Annual Premium, $50.80	
	Dividend at End of Year	Cost for Each Year	Dividend at End of Year	Cost for Each Year	Dividend at End of Year	Cost for Each Year
1	$3.00	$24.30	$3.30	$32.70	$3.80	$47.00
2	3.10	24.20	3.45	32.55	4.05	46.75
3	3.20	24.10	3.60	32.40	4.35	46.45
4	4.10	23.20	4.40	31.60	4.95	45.85
5	4.25	23.05	4.60	31.40	5.25	45.55
6	4.70	22.60	5.30	30.70	6.60	44.20
7	5.25	22.05	6.05	29.95	7.70	43.10
8	5.80	21.50	6.75	29.25	8.65	42.15
9	6.30	21.00	7.30	28.70	9.30	41.50
10	6.50	20.80	7.55	28.45	9.65	41.15

373. Lapses. If the annual premium is not paid when due, the policy lapses. However, most companies allow one month's grace, during which time the overdue premium may be paid with interest. If death occurs during this period of grace, the amount of the overdue premium and the accrued interest are deducted in making settlement with the beneficiary.

If the premium remains unpaid after the expiration of the period of grace, the policy may be reinstated, but the policyholder must furnish satisfactory evidence of health and pay all back premiums with accrued interest.

374. Cash Surrender, Loan, and Paid-up Insurance. After a policy has been in force for a certain number of years (usually two or three), many companies offer the insured the following privileges:

a. Borrowing money from the company;

b. Surrendering the policy for a cash payment;

c. Receiving a " paid-up " policy which gives a fixed amount of insurance during the remainder of life without further payment of premiums.

d. Being insured for the face of the policy for a fixed number of years and months.

For example, after an ordinary life policy for $1000.00, taken at the age of 35, in a certain company, has been in force 8 years, the insured can —

a. Borrow $108.00 from the company on the security of the policy.

b. Surrender the policy and receive $108.00 in cash.

c. Cease the payment of premiums and be insured for the remainder of his life for $223.00.

d. Cease the payment of premiums and be insured for $1000.00 for 10 years and 9 months.

375. Modes of Settlement. Upon proof of the death of the insured (or, in case of an endowment policy, at the expiration of the endowment period), the policy becomes payable by the company. Settlement may be made in a variety of ways, the most common of which are:

a. Payment of cash to the beneficiary;

b. Annual payment of interest during the life of the beneficiary, and payment of the face of the policy at the death of the beneficiary;

c. Payment of equal annual installments for the number of years specified to the beneficiary;

d. Payment of equal annual installments for a period of twenty years, and for as many years thereafter as the beneficiary shall live. The amount of each annual payment depends upon the age of the beneficiary at the death of the insured.

Oral Work

1. Explain the reason for the difference in the premiums of the different kinds of policies.
2. Why is it often wise to take insurance at an early age?
3. What is participating insurance?

Written Work

Refer to the table of annual premiums on different kinds of insurance and find the annual premium on the following:

1. $2000, ordinary life policy, taken at the age of 35.
2. $5000, 20-year endowment policy, taken at the age of 35.
3. $3000, 10-payment life policy, taken at the age of 35.
4. $6000, 20-payment life policy, taken at the age of 35.
5. $10,000, 10-year endowment policy, taken at the age of 35.
6. On his 25th birthday Mr. Anderson took out a 20-payment life policy for $3000; on his 30th birthday he took out a 20-year endowment policy for $5000; on his 35th birthday he took out an ordinary life policy for $7000. He died at the age of 38 years 7 months. How much more did his beneficiary receive (dividends excepted) than he paid the company? (Use the table on page 455.)

CHAPTER XLVIII

FINAL WRITTEN REVIEW

1. A manufacturer sells an article to a wholesaler at a price 20 % above cost; the wholesaler sells it to a retailer at a price 15 % above the amount he paid for it. The retailer adds a profit of $11.32 and sells it for $130.00. What was the cost to the manufacturer?

2. A manufacturer sold an article to gain 33⅓ % on the selling price; the wholesaler sold to make 25 % on sales; the retailer sold to make 25 % on sales. The retail selling price was $160.00. What was the manufacturing cost?

3. You purchase an invoice of goods for $30.00 and find that 10 % of the goods are unsalable. You price the remaining goods to sell so that you will make 10 % on the invoice cost. What is the selling price? the rate of profit on cost, and the rate of profit on sales?

4. Mr. Hawkins purchases a house for $10,000.00 and rents it for a year at $85.00 per month. Taxes, insurance, and incidental expenses are $205.00 for the year and he estimates the depreciation at 3 % per annum. What net rate of return does he receive on his investment?

5. A merchant purchases a commodity for $3.00 and sells it for $4.00. In order to meet competition he is forced to reduce the selling price to $3.75. What per cent must he increase his sales to make the same gross profit as before?

6. I mark up a certain kind of merchandise 30 % on cost, but am unable to sell it at that price, therefore I reduce the selling price 10 %. After reducing the selling price, what rate of profit will I make on cost and what rate on selling price?

7. A business man marks up his goods the first year 40 % on cost and turns his stock three times during the year. What per cent profit does he make on the capital invested in stock? The second year he marks his goods 30 % above cost and turns his stock five times. What per cent profit does he make on the capital invested in stock?

8. An article cost $8.00 to manufacture and is sold for $12.00. Because of increases in the cost of materials, labor and manufacturing expenses, the cost to manufacture goes up to $10.00 and the selling

price is increased to \$14.00. What per cent of profit is made before the increase in cost, and what per cent is made after the increase?

9. At what price must the article sell to make the same rate of profit on sales as before?

10. I offered to sell goods at a profit of 25 % but was unable to make the sale. In order to dispose of the goods I sold them for \$1,800.00, making a profit of 20 %. What was the cost of the goods, and at what price did I originally offer them?

11. Two concerns offer you the same goods; the first quotes a price of \$340.00 less trade discounts of 15 % and 10 %; the second quotes the same price, with discounts of 10 %, 10 %, and 5 %. Which is the better offer and how much?

12. What difference is there in the following discount series?

15 %, 10 %, and 5 %
10 %, 15 %, and 5 %
5 %, 10 %, and 15 %

13. What is the difference between a discount series of 10 %, 10 % and 5 % and a single discount of 25 %? What is the difference on an invoice with a list price of \$1,500.00?

14. R. S. Dutton marks goods at an increase of 30 % on cost and allows a discount of 10 % from the list price. What rate of profit does he make on sales?

15. If a merchant buys goods at 20 %, 25 %, and 10 % off the list price and sells them at 15 % off the list price, what rate of profit does he make on sales?

16. If a merchant sells goods at 20 % and 10 % off from the list price, what per cent must he add to the cost of the goods in order to make 20 % on the selling price?

17. At the end of February the Second National Bank sends a statement to D. B. Warren showing a balance of \$376.85. Warren examines his returned checks and finds that the following checks issued in February have not been returned:

No. 2,019	\$6.50
No. 2,023	\$7.55

A deposit of \$93.68 made on February 28 does not appear on the bank's statement. What balance should be shown on Warren's check stub at the end of February in order to have the stub and the bank statement reconcile?

18. What is the cost of transferring $215.60 by postal money order?

19. What is the cost of transferring the same amount by express money order?

20. What is the value of £300 at today's exchange rate? (Use check rate.)

21. What is the value of 1,000 francs at today's exchange rate?

22. How many pounds can be purchased for $650.00 at today's rate?

23. How many francs can be purchased for the same amount at today's rate?

24. I have a claim against J. H. Gunther for $850.00, which is in the hands of an attorney who charges 2½% commission for collections. Gunther goes into bankruptcy with liabilities of $14,600.00. His assets realize $12,400.00 and there are expenses connected with the bankruptcy proceedings totaling $350.00. How much do I receive in final settlement?

25. A manufacturer sells washing machines through an agent to whom he pays 15% of the selling price, and makes a gross profit of 25% on sales. He decides to increase the selling price 10% and allow the agent 20% on sales. What is his per cent of profit on sales?

26. I owe $409.50 on a promissory note which has run 5 months and 10 days and is due today. The note bears 5½% interest. What total payment will be required to pay the note?

27. Bailey and Wharton allow 1% discount if bills are paid within 10 days from the date of invoice; bills are due 30 days after date and interest at 6% is charged thereafter. If I buy goods on March 17 for $621.80 what amount will settle the bill on March 23? on April 12? on June 23?

28. On April 1 I bought a house for $6,500.00 paying $1,500.00 in cash, giving a 6% note for the balance, and agreeing to make payments of $75.00 per month thereon. After making twelve payments on the note, what will I owe if interest is computed

(*a*) by the long end method?
(*b*) by the merchants' rule?
(*c*) by the United States rule?

29. A loan of $15,000.00 at 4½% made on May 16 is called on July 23. How much interest is payable?

30. The Western Manufacturing Company has several bond issues as follows:

$30,000.00 first mortgage bonds bearing 6%;
$20,000.00 second mortgage bonds bearing 6½%;
$ 5,000.00 equipment bonds bearing 5¼%;
$20,000.00 collateral trust bonds bearing 6¾%.

It has an opportunity to refund these bonds, calling them all in and issuing a new series of first mortgage bonds bearing 6¼%. Will it be wise to do so? How much will be gained or lost in interest annually?

31. Hartley and Knight went into partnership with investments of $20,000.00 and $25,000.00 respectively. The profits for the first year were $4,890.00 and they were divided equally. Hartley had drawn $1,000.00 during the year and Knight had drawn $1,200.00. What rate of income did each partner earn on his investment?

32. Dalton loans $764.00 on March 18 at 6%. The note is due on August 18. How much interest is due if computed by the customary commercial method? How much is the accurate interest?

33. Accountants in valuing inventories follow the rule "cost or market, whichever is lower." At December 31 the inventory of a certain business is worth $21,346.00 if valued at cost and $20,830.00 if valued at market prices. To what extent are the profits for the year affected by the decline in market values? If the purchases for the year were $104,650.75 and the sales $108,458.50, what was the gross profit on sales and the rate of gross profit?

34. The interest on a $630.00 note for 85 days is $10.41. What is the rate?

35. A bank discounts a $600.00 sixty day note and credits the borrower's account with proceeds of $593.50. At what rate was the note discounted?

36. The interest on a certain loan of $750.00 at 6% is $5.63. How many days did the note run?

37. What is the periodic interest on a loan of $2,000.00 at 6% for 2 years, if interest is due quarterly and no interest is paid until the maturity of the note?

38. The interest for 75 days at 6% on an unknown principal is $16.88. What is the principal?

39. What is the bank discount on a non-interest bearing note discounted at 7% fifty days before maturity?

40. You held a note signed by G. L. Kenton dated July 15. The note bears 5% interest and is due in 90 days. On August 20 you dis-

counted the note at the bank at 7 %. State the discount period and the proceeds.

41. Refer to the stock quotations in the daily paper today and a week ago today. Select some stock which is quoted in both issues of the paper. Take the lowest price quoted a week ago and the highest price quoted today and compute the profit or loss which would have been made by purchasing 100 shares a week ago and selling them today, with $\frac{1}{8}$ % brokerage on both purchase and sale.

42. An investor buys 40 shares of stock at $83\frac{1}{2}$ plus brokerage. An annual dividend of 6 % is received. The stock is \$100 par. What is the amount of the dividend? What rate of income is earned on the investment?

43. A hospital has endowment funds which it is not allowed to invest in securities paying less than 6 % on the investment. The trustees of the fund are offered stock which bears only 5 % dividends. What is the highest price which can be paid for the stock to net the required rate of income?

44. I place in my broker's hands 120 shares of Toledo Dairy Company stock with instructions to sell and invest the proceeds in Interior Produce stock. He sells the Toledo stock at $112\frac{1}{2}$ and buys Interior at $95\frac{1}{4}$, charging brokerage on sale and purchase. If both classes of stock are \$100 par, how many shares of Interior will I receive, and how much uninvested cash will be returned to me by the broker?

45. A manufacturer decides to use the differential piece work method of paying wages. After careful study he fixes his rates as follows:

30 cents each for 20 articles if made in a day;

a reduction of 5 % in the rate for each article less than 20 made per day; that is, if 19 are made, the rate is 95 % of \$.30; if 18 are made, the rate is 90 % of \$.30 each.

an increase of 10 % in the rate for each article in excess of 20, the first 20 being paid for at the standard rate; that is, if 23 are made in a day, the first 20 are paid for at \$.30 each. The 21st article is paid for at the rate of 110 % of \$.30; the 22d article at 120 % of \$.30; the 23d article at 130 % of \$.30.

Draw up a wage scale showing the amount of daily wages to be paid for the work on 10 to 30 pieces inclusive.

46. Draw up a coin sheet for the payment of a week's payroll of the above factory:

Operative's Number	Name	Number Produced					
		M	T	W	T	F	S
1	D. H. Mather	21	23	22	24	20	19
2	Oscar Hart	27	26	24	29	25	23
3	Harold Barclay	18	19	20	22	17	21
4	George Benton	22	22	29	28	24	25

47. Compute the annual depreciation, straight line method, on an asset costing $15,000.00 with an estimated life of 8 years and an estimated residual value of $3,000.00.

48. Prepare a table showing the annual depreciation for each of the eight years using the sum of the years' digits method.

49. An asset costing $5,000.00 is depreciated by the diminishing value method using a rate of 15%. State the annual depreciation for the first five years.

50. The owner of a brick residence takes an insurance policy of $8,000.00 on March 15 at a rate of 48 cents. He also takes a policy of $4,000.00 on the contents of the residence, the rate being 10% higher. Both are one year policies. What is the cost of the insurance?

51. If the policies are cancelled by the insurance company on June 10, what portion of the premium is returned?

52. What was the agent's commission on these policies at 20%?

53. M has insured his residence for $7,500.00 under a policy containing the 80% co-insurance clause. The building is burned and at the time of the fire it is estimated that its sound value was $10,500.00 How much insurance will M recover?

54. Property in a certain state is assessed at $\frac{2}{5}$ of its real value; the rate in one town in this state is $2.60. The assessor considers a piece of real estate worth $18,500.00. What is the amount of the property tax?

55. Referring to the tariff rates in the chapter on customs duties compute the duty on an importation of $45,000.00 worth of silk ribbons.

56. Compute the turnover from the following statistics: Inventory, January 1, $12,460.85; purchases for the year, $78,935.84; sales for the year, $83,542.69; inventory, December 31, $11,984.67.

57. What is the average profit per turnover?

58. What is the rate of gross profit per turnover based on the average investment in stock?

59. Reduce $750.00 to English money at the par of exchange. Reduce the same number of dollars to English money at the cable rate quoted in today's paper.

60. What is the compound interest at 5% on $940.00 for 2 years, 3 months, 15 days, interest compounded semi-annually?

61. The statistics of a certain business show that 4% should be added to the cost of merchandise purchased to cover buying expenses. An invoice costing $28.75 wholesale is marked to sell for $35.00. What is the gross profit, and the rate of profit on sales?

62. A man deposits $15.00 monthly in a savings bank which pays 4% annually. Interest is computed quarterly on all amounts which have been on deposit for a month or more. How much will his balance be at the end of two years if no withdrawals are made?

63. Keep a record of the way you spend your time for twenty-four hours, and prepare a graph showing the time spent in sleep, at recitations, in studying, in recreation, at outside work, at meals, etc. Prepare this graph on the basis of hours and minutes.

64. Use the same data to prepare a graph showing the per cent of time devoted to each.

65. For this problem divide the class into groups of about five or six members each. Each individual except one in each group represents a bank which will be known by his name, as Smith National Bank. Let each individual bank prepare a statement purporting to represent the total checks which it has paid during the preceding day for each other bank in the group; these payments shall not be more than $10,000.00 for each other bank in the group. In addition to the statement, each banker should prepare a slip the size of a check for each bank for which it has made payment; on this slip write the name of the bank and the amount of checks paid. These slips will represent the checks paid by the bank. One member of each group should be appointed as clearing house manager and the bankers should turn over their statements and checks to the manager. Each banker should also retain a copy of his statement. The clearing house manager should then prepare a manager's sheet to determine the debit balances due from some banks and the credit balances due to other banks. The manager should also turn over to each banker the checks paid for him by the other banks, and each banker should determine whether he has a debit or a credit balance. Compare the balances computed by the manager with the various balances computed by the bankers. The manager should then require each banker with a debit balance to give him a bank draft for the amount of the balance and he should endorse these drafts to be deposited in some city bank to the credit of the clearing house. The bankers may draw their drafts on a bank in any city where

they are assumed to have deposits. The clearing house manager should now draw checks on the bank where the clearing house makes its deposit, these checks to be payable to the banks with credit balances.

66. A invested \$7,500.00 in a business on January 1, 1921. On June 1, he invested an additional \$1,500.00. He withdrew \$125.00 at the end of each month. At December 31, 1921, a statement of assets and liabilities showed that the net worth of the business was \$11,763.50. What was the profit for the year?

67. From the following facts determine the net profit for the year and the net worth at the end of the year: Net worth, January 1, \$16,375.00; merchandise inventory, January 1, \$6,275.10; inventory, December 31, \$6,840.26; purchases, \$46,210.80; net sales, \$63,710.41; miscellaneous income, \$275.30; expenses, \$5,310.20; additional investment, August 1, \$2,000.00; drawings, \$175.00 per month.

68. X,Y, and Z are partners, with capitals on January 1, of \$6,000.00, \$12,000.00 and \$15,000.00 respectively. The profits for the year are \$12,632.19. X has drawn \$100.00 per month; Y, \$125.00 per month; and Z, \$150.00 per month. Prepare statements of their capitals at the end of the year under the following assumptions as to the division of profits:

(*a*) Equally.
(*b*) X, $\frac{1}{8}$; Y, $\frac{3}{8}$; Z, $\frac{1}{2}$.
(*c*) In the capital ratio.
(*d*) Interest on capitals at 6% and the balance equally.

69. B is insolvent. An estimate shows that his assets will probably realize \$16,340.65. He owes \$165.00 taxes and \$2,575.00 wages which must be paid in full. Z holds B's note for \$5,000.00 on which there is \$65.00 accrued interest; as security Z holds bonds belonging to B which are worth \$3,500.00 which amount is included in the \$16,340.65. There are other liabilities in the form of notes totaling \$6,300.00 and accounts totaling \$11,275.80. What estimated amount will the various creditors receive?

70. What is the cost of 500 shares of stock purchased for $118\frac{3}{8}$ with brokerage of $\frac{1}{8}$? A 5% dividend is what per cent income to the owner of the stock? If the stock is sold for $121\frac{7}{8}$, with brokerage of $\frac{1}{8}$, what is the profit?

71. Which is the better investment from the standpoint of earnings: stock paying 5% dividends bought at $125\frac{3}{8}$ plus brokerage, or stock paying 6% dividends bought at $138\frac{3}{4}$ without brokerage?

72. A corporation has issued $200,000.00 non-participating 6 % preferred stock and $150,000.00 common stock. Its surplus at January 1 was $22,115.61 and its profits for the year have been $25,631.87. What is the largest whole per cent dividend which can be declared on the common stock? What rate of income would this give a stockholder who had purchased his stock at 112?

73. C is in the retail grocery business. His inventory at January 1 was $12,617.50, and at the close of the year it was $13,161.50. His net purchases for the year were $87,620.50 and his net sales $109,315.60. What was the turnover? the gross profit? the average rate of mark-up? and the rate of gross profit?

74. The total overhead manufacturing expense in a factory during a certain week was $861.50. The direct labor payroll for the week was $1,365.80. Overhead is distributed by the direct labor cost method. What was the total cost of an article manufactured during the week, the material cost being $210.95 and the labor cost $197.80?

APPENDIX A

FARM RECORDS

The Cash Book. One of the most important records for a farmer is the cash book, in which he can tabulate the various sources of his income and the amounts of his various expenditures.

The following are three of the forms which may be used for this purpose:

A

1922			Receipts	Expenditures
Oct.	2	Balance	123 75	
	2	15 bu. apples @ .70	10 50	
	2	12 doz. eggs @ .30	3 60	
	3	Groceries		5 95
	5	Lumber		26 25

B

Receipts						Expenditures					
1922						1922					
Oct.	2	Balance			123 75	Oct.	3	Groceries	5	95	
	2	15 bu. apples @ .70	10	50			5	Lumber	26	25	
	2	12 doz. eggs @ .30	3	60							

C

Receipts

1922		Item	Dairy	Poultry	Crops	General
May	1	2 cows to Owen	125 00			
	2	10 doz. eggs @ .28		2 80		
	3	4 T. hay			60 00	

EXPENDITURES

1922		ITEM	DAIRY	POULTRY	CROPS	HOUSEHOLD	GENERAL
May	1	500 # bran	8 00				
	2	10 bu. seed oats			6 00		
	3	Groceries				3 60	
	4	Oyster shell		1 50			
	5	Cement					8 00

Written Work

Rule a form for a cash book similar to the last illustration.

Include an extra column in both "Receipts" and "Expenditures" labeled "Stock" and enter the following facts:

March 1, Balance, $280.00.
1, Sold for cash 1 carload hogs, $960.00.
1, Bought 2 cows, $45.00 and $58.00
2, Bought 1 set double work harness, $21.00.
3, Bought 1 work horse, $120.00.
4, Sold 30 doz. eggs at 45 cents.
5, Sold 8 T. timothy hay at $14.00 per ton.
6, Sold 1 load oats to elevator, 95 bu., at 36 cents.
6, Bought groceries, $7.95.
8, Sold one second-hand riding plow, $15.00.
9, Bought 20 sacks cement at $.50.
10, Bought 90 bu. seed oats at 48 cents.
11, Bought 500# middling for hogs at $30.00 per ton.
12, Bought 2 riding plows at $55.00 each.
15, Received milk check for 30 da., $226.47.
16, Bought groceries, $6.83.
17, Bought wheat screening for chickens, $2.50.
20, Sold 2 steer calves at $9.00 and $12.00.
22, Sold 18 young roosters, 57# at 14¢ per pound.
24, Rented a three-horse team to a neighbor for 2½ da. at $3.75 per day.
26, Bought 10 T. rock phosphate at $3.00 per ton.
28, Sold 28 doz. eggs at 37 cents.
28, Bought 1 pair shoes, $3.85; suit of clothes, $18.00.

Find the total receipts and expenditures for each column, and the balance at the end of the month.

Farm Profits. The annual increase or decrease of the farmer's wealth may be determined by taking an inventory each year, and comparing it with the inventory of the preceding year. The inventory should be classified to show the capital invested in land, building, live stock, machinery, and other property. An inventory is reproduced on pages 474 and 475.

What is Farm Profit? The annual increase shown by the two inventories is not the farmer's net gain for the year. The real annual profit is found by the following method:

Increase shown by inventories, plus living supplied by farm to family, plus interest paid on indebtedness, minus interest on capital invested, minus wages of farmer and family, equals actual net profit.

Problem. Let us suppose the inventories show an increase for the year of $2,000. The farm has also supplied a residence and provisions for the farmer's family. If the house rent is estimated at $120 and the supplies are valued at $900, the total, $1,020, should be credited to the farm. Adding $80 interest paid on the farm mortgage, we have the following:

$2,000	Increase of inventory
1,020	Household expenses
80	Interest paid
$3,100	Produced by farm

The farmer has $10,000 invested in the farm. If this money were loaned at 5% interest, it would produce $500 interest annually. $500 interest must therefore be subtracted.

If the farmer had worked for some one else, he might have earned wages of, say, $700. This amount must also be subtracted in order to determine real profit.

Thus, we have

$3,100 − $500 (interest) − $700 (wages) = $1,900, profit.

Real profit may be determined by the preparation of a statement similar to the following:

Item	March 1, 1921	March 1, 1922
Resources		
Real estate	$15,000	$15,000
Live stock	3,160	3,590
Machinery and tools	530	575
Feed and supplies	860	735
Cash	140	275
Bills receivable		50
Accounts receivable	93	126
Total resources	19,783	20,351
Liabilities		
Accounts payable	135 60	215 20
Mortgage	1,000	1,000
Total liabilities	1,135 60	1,215 20
Net worth	18,647 40	19,135 80
Increase in net worth		488 40
Add		
Interest paid on mortgage 5 % on $1,000		50
Personal and household expenses paid in cash		750
Supplies furnished by farm for household		363
Rent of farmhouse		120
Gross farm gain		1,771 40
Deduct		
Interest on net investment at 5 %	932 37	
Labor of owner and family (estimated)	600	
Total deduction		1,532 37
True net gain		239 03

Written Work

From the following facts prepare a statement similar to the preceding illustration.

Condition of farmer's affairs on March 1, 1922.

He owns a farm of 160 acres, worth $225 per acre, on which he has given a mortgage of $6,000, bearing 5 % interest. Cash on hand and in bank, $345.85. His live stock is worth $5,280; his poultry, $286; his machinery, $1,250; unsold crops, $900. He has just sold a car of hogs to a local stock buyer, who owes him $975 for them. He also has

on hand fertilizer, lumber, and other supplies worth $387.50. He owes sundry accounts, amounting to $236.25.

During the year between March 1, 1921 and March 1, 1922, he paid the interest on his mortgage, and also paid $1,000 on the principal of the mortgage. He paid by cash and by butter, eggs, and other produce, for household expenses, $596.50. He estimated the rent of his house at $15 per month; the produce of the farm consumed by the family, $645; and his wages, $900. He paid taxes and insurance, $275. Compute interest on net investment at the beginning of the year at 5%.

On March 1, 1922, he had, cash $467.20; live stock, $6,140; poultry, $270; machinery, same as previous year, less 10% depreciation, plus $165 worth of new machinery; crops on hand, $650; miscellaneous supplies, $295. He owes personal accounts amounting to $193.70 and the unpaid balance of the mortgage.

Finding the Profits by Crops. In order to determine the profit from any particular crop, the farmer must know:

a. The value of the crop produced;

b. The cost of producing the crop, including seed, fertilizer, labor, and a share of the taxes, interest, and general expenses.

Determining the Expenses. The cost of seed and fertilizer may be determined with comparative ease, because measurable quantities are put on each field.

Taxes, interest, and general expenses are prorated among the different crops in the ratio of the land occupied.

Problem. A farmer who has 40 acres of hay, 60 acres of corn, and 30 acres of oats under cultivation, finds that his total interest, taxes, and expenses are $1,200. How should this be prorated?

SOLUTION.	$1,200 ÷ 130 = $9.23	Charge per acre.
	40 × $9.23 = $369.20	Share for hay crop.
	60 × $9.23 = $553.80	Share for corn crop.
	30 × $9.23 = $276.90	Share for oat crop.

Labor of men and horses must be determined by records of the actual labor spent on each crop.

Finding the Cost of Man Labor and Horse Labor per Hour. At the end of a month or other period of time, the total number of

man hours and horse hours can be determined from the records. The cost per hour may be found by the method illustrated below:

Cost of man labor per year

Wages,	$625.00
Board,	200.00
Total,	$825.00

Hours worked during year, 2,500.

$825.00 ÷ 2,500 = $.33 labor cost per man hour.

Cost of a horse hour is determined in a similar way.

Total value of 2 horses, $400.00	
Int. on $600.00 at 6%,	$36.00
Feed and expense,	450.00
Total cost per year,	$486.00

Total number of horse labor hours, 6,000.

$486.00 ÷ 6,000 = 8.1 cents, labor cost per horse hour.

After the labor cost records have been made, the profit or loss from the crops may be determined.

The labor records show the number of horse hours and man hours spent on each crop. After determining the hourly rate, it is necessary to multiply as follows:

The number of man hours by the rate per man hour; the number of horse hours by the rate per horse hour; to determine the total labor cost of each.

The costs and returns of each crop may then be summarized, and the gain computed, as illustrated in the table on page 473.

Written Work.

1. Find the labor cost to be charged against a corn crop, in the production of which 136 man hours and 312 horse hours of labor were expended. The total number of hours of man labor for the year was 4,824, and the annual cost was $1206; total number of hours of horse labor, 4,312; total cost of horse labor for the year, $388.08.

2. Use the following facts to prepare a table similar to the model on page 473.

CORN CROP ON SOUTH 40

ITEM		40 ACRES	PER ACRE
Seed		$24.00	.60
Fertilizer		70.00	1.75
Taxes		28.30	.7075
Interest		450.00	11.25
Expenses		40.00	1.00
Man labor,		109.01	2.725
Horse labor,		228.60	5.715
Total cost		949.91	23.7475
Yield, 2552 bu. corn at $.48	$1,224.96		
Stalks, 75¢ per acre	30.00		
Total returns		1254.96	31.374
		949.91	23.7475
Gain		305.05	7.6265

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CORN CROP ON 20 ACRES

Yield: 10 bushels of seed corn worth $2.50 per bushel.
Corn crop harvested, 67 bushels per acre, worth 54 cents per bushel.
Stalks were estimated at a value of 60 cents per acre.

Costs: $3\frac{1}{2}$ bushels of seed at $2.60 per bushel.
Fertilizer, $38.00.
Taxes: The farm was worth $175.00 per acre, and was taxed on $\frac{1}{5}$ the real value at a rate of $1.65. The farm contained 160 acres, and the taxes were prorated among the various fields in proportion to their acreage.
Interest: 6% on the value of the land.
Expenses: $31.25.
Labor: 560 man hours at 22 cents.
1,348 horse hours at 9 cents.

Sample farm inventory: Farm of ——

Property	April 1, 1921				April 1, 1922			
	No.	Rate	Valuation		No.	Rate	Valuation	
REAL ESTATE								
Farm of 180 acres (155 tillable), including buildings (dwelling $1,600, barns $1,800, other buildings $600), fences, and other improvements				$13,500.00				$13,500.00
LIVE STOCK								
Dairy cattle:								
Cows, dry and in milk	24	$50.00	$1,200.00		26	$50.00	$1,300.00	
Bull	1	.....	50.00		1	.....	45.00	
Calves	6	14.00	84.00		8	15.00	120.00	
Two-year-olds	4	28.00	112.00		6	20.00	120.00	
Total value of dairy cattle				1,446.00				1,585.00
Hogs:								
Brood sows	2	22.00	44.00		2	21.00	42.00	
Pigs	8	4.00	32.00		6	3.00	18.00	
Total value of hogs				76.00				60.00
Horses:								
Horse, Jim, 7 years old	1	.....	120.00		1	.....	100.00	
Team, Nell and Bess, 5 and 6 years old	1	.....	325.00		1	.....	325.00	
Team, Jack and Prince, 6 and 7 years old	1	.....	300.00		1	.....	300.00	
Colt, 1 year old	1	.....	75 00		1	.....	135.00	
Total value of horses				820.00				860.00
Poultry:								
Hens	160	.60	96.00		125	.60	75.00	
Roosters	5	1.00	5.00		4	1.00	4.00	
Turkeys	2	3.00	6.00		3	3.00	9.00	
Total value of poultry				107.00				88.00
Total value of live stock				2,449.00				2,593.00
MACHINERY AND TOOLS								
Grain binder	1	.....	90.00		1	.....	82.00	
Sulky plow	2	45.00	90.00		2	41.00	82.00	
Disk harrow	2	28.00	56.00		2	25.00	50.00	
Mower	1	.....	35.00		1	.....	30.00	
Hay rake	1	.....	20.00		1	.....	19.00	
(List all items of farm machines, wagons, harness, and small tools.)								
Total investment in machinery and tools (not all listed here)				475.00				461.00

Sample farm inventory: Farm of —— (Continued)

Property	April 1, 1921				April 1, 1922			
	No.	Rate	Valuation		No.	Rate	Valuation	
FEED AND SUPPLIES								
Farm products:								
Corn.....bushels	80	.60	48.00		125	.60	75.00	
Oats..... "	200	.42	84.00		90	.50	45.00	
Potatoes..... "	40	.75	30.00		80	.60	48.00	
Hay, timothy.... tons	10	16.00	160.00		20	15.00	300.00	
Hay, mixed..... "	5	12.00	60.00		4	12.00	48.00	
Silage..... "	40	4.00	160.00		40	4.00	160.00	
Bran..... "	0½	.....	15.00		.....	.....	.....	.....
Mixed feed..... "	1	.....	31.00		2½	30.00	75.00	
Seed oats.....bushels	30	.80	24.00		35	.80	28.00	
Seed potatoes.... "	45	.80	36.00		50	1.00	50.00	
Seed corn..... "	3	2.00	6.00		3	2.00	6.00	
Cement..... sacks	4	.50	2.00		.....	.....	.....	
Twine.....pounds	20	.10	2.00		10	.10	1.00	
Total value of feed and supplies.....	.....	.....	.....	658.00	.....	.....	.....	836.00
BILLS RECEIVABLE								
J. A. Brown, hay... tons	2	13.00	26.00					
R. S. Jones, potatoes, bushels	40	.50	20.00					
Total.....	.....	.....	.....	46.00				
CASH								
On hand.....	.....	.....	90.00		.....	.....	210.00	
In bank.....	.....	.....	580.00		.....	.....	1,938.00	
Total.....	.....	.....	.....	670.00	.....	.....	.....	2,148.00
BILLS PAYABLE								
Farm mortgage	.....	.....	.....	2,000.00	.....	.....	.....	1,500.00
SUMMARY								
Real estate.....	.....	.....	13,500.00		.....	.....	13,500.00	
Live stock.....	.....	.....	2,449.00		.....	.....	2,593.00	
Machinery and tools.....	.....	.....	475.00		.....	.....	461.00	
Feed and supplies.....	.....	.....	658.00		.....	.....	836.00	
Bills receivable.....	.....	.....	46.00		.....	.....	.....	
Cash on hand and in bank.	.....	.....	670.00		.....	.....	2,148.00	
Total investment.....	.....	.....	.....	17,798.00	.....	.....	.....	19,538.00
Bills Payable.....	.....	.....	.....	2,000.00	.....	.....	.....	1,500.00
Net Worth.....	.....	.....	.....	15,798.00	.....	.....	.....	18,038.00
Increase in inventory $2,240.								

APPENDIX B

AVERAGING ACCOUNTS

Average Date. Merchants make sales under definite credit terms which fix the dates on which payments are due. Thus, a customer may be charged with several invoices, each of which is due at a different date. The average date of an account is the date when the entire balance of the account may be paid in accordance with the credit terms and without any interest.

For example, George Smith may have the following account in his ledger with William Dawson:

William Dawson

June 1	Terms 30 days	$100.00
June 11	Terms 30 days	100.00

Dawson could comply with the credit terms by paying the first invoice on July 1st and the second on July 11th, or he could pay the entire balance of $200 on July 6th, because, while he would be paying $100 five days after it became due, this late payment would be exactly offset by the payment of the second $100 five days before it became due. July 6th is, therefore, the average date of the account.

Simple Average. When an account contains debits only or credits only, the procedure of determining the average date is called *simple average.*

Procedure of Simple Average. The average date of an account which contains debits or credits only, may be determined as follows:

(*a*) Assume that all items in the account are paid at a date prior to the maturity of any item in the account. This assumed date is called the *focal date.* The most convenient focal date is the last day of the month preceding the first date on which any item is due.

(*b*) Determine the number of months and the number of days between the focal date and the maturity of each item. This is the length of time which each item is assumed to be prepaid.

(*c*) Multiply each item in the account by the number of months and by the number of days between the focal date and the maturity date of the item. This is necessary because the items may be of varying amounts; the multiplication reduces them all to the same basis, namely the prepayment of one dollar. For example, the payment of $100 fifteen days before it is due is equivalent to the payment of $1 fifteen hundred days before it is due.

(*d*) Add the products of months and reduce to a basis of days by multiplying by 30.

(*e*) Add the products of days, including those obtained by reducing months to days. Thus, all assumed prepayments are reduced to a basis of a single prepayment of $1 for a total number of days.

(*f*) Divide this total number of days by the balance of the account. The reason for this step follows: if the total number of days determined in step (*e*) is 4800,

the prepayment of all items on the focal date is equivalent to a $1 payment made 4,800 days before it is due. Assume that the balance of the account is $100. Then

a prepayment of $ 1.00 4,800 days before it is due

is equivalent to

a prepayment of $100.00 48 days before it is due.

(*g*) Reduce the days, determined in step (*f*), to months and days on the basis of 30 days to the month. In the illustration just given, 48 days are equivalent to 1 month and 18 days.

(*h*) This result is the length of time the entire account would be prepaid if the balance were paid on the focal date. Therefore, in the illustration, the average date is 1 month and 18 days forward from the focal date.

Problem. Using this method, the aver ge date of the account with William Dawson, in the first paragraph of this chapter, would be determined as follows:

The first step is to determine the focal date. The first item is due July 1st; the second item is due July 11th. Therefore June 30th will be used as the focal date. The remaining steps are shown in the following computation:

		Time from Focal Date		Products	
Debits	*Date Due*	*Months*	*Days*	*Months*	*Days*
$100.00	July 1	0	1	0	100.00
100.00	" 11	0	11	0	1,100.00
200.00					1,200.00

1,200 ÷ 200 = 6. Therefore, the average date is 6 days forward from June 30th, the focal date. Hence the average date is July 6th.

Problem. Determine the average date of the following account:

D. E. Watson

Sept. 5	Terms 30 days	500.00
" 17	Terms Cash	200.00
Oct. 9	Terms 30 days	150.65

The second item was sold on cash terms, and was therefore due on September 17th. Hence August 31st will be used as the focal date.

		Time from Focal Date		Products	
Debits	*Date Due*	*Months*	*Days*	*Months*	*Days*
$500.00	Oct. 5	1	5	500.00	2,500.00
200.00	Sept. 17	0	17	0	3,400.00
150.65	Nov. 8	2	8	301.30	1,205.20
				801.30 × 30 =	24,039.00
850.65					31,144.20

Then 31,144.20 ÷ 850.65 = 36.6. Hence the average date is 37 days, or 1 month and seven days, forward from August 31st. The average date is October 7th.

Problem. Find the average date of the following account:

James White

August	3	Terms Cash	600.00
"	12	" 2/10; N/30	500.00
Sept.	7	" 2/10; N/30	400.00

SOLUTION. Focal date, July 31.

			Time from Focal Date		Products	
Debits	*Date of Entry*	*Date Due*	*Months*	*Days*	*Months*	*Days*
600.00	Aug. 3	Aug. 3	0	3	0	1,800.00
500.00	" 12	Sept. 11	1	11	500.00	5,500.00
400.00	Sept. 7	Oct. 7	2	7	800.00	2,800.00
$1,500.00					1,300.00	10,100.00
					1300 × 30 =	39,000.00
						49,100.00

49,100 ÷ 1,500 = $32\frac{1}{15}$ days, or 1 month and 2 days. Hence the average date is 1 month and 2 days forward from July 31, or September 2.

Utility of Average Date. The average date of an account is often desired in order to compute the interest which the debtor should pay to the creditor in order to make an equitable settlement when payment is made after the average date.

To illustrate: In the last preceding illustration, the items were due at various dates, but the entire $1,500.00 could be paid on September 2, the average date, and the debtor would not owe the creditor any interest. But if the terms of sale called for interest on each item after its due date, and if the debtor did not pay the $1,500.00 until September 30, he should pay interest on $1,500.00 for 28 days.

Written Work

1. Find the average date of the following account:

J. R. Waters

April 10	Terms Cash	350.00
" 20	1/10; N/30	210.00
May 5	1/10; N/60	300.00

The account was paid on June 25th with interest at 6% from the average date. What was the amount of the payment?

2. Find the average date of the following account:

Wharton & Leahy

June 10	1/10; N/30	215.60
" 18	Terms Cash	365.10
July 7	2/10; N/60	710.65

On July 20th, Wharton & Leahy gave a note in payment of the account. The note was dated on the average date and was due 60 days thereafter. What was the

date of the note, when was it due, and how much should have been paid at its maturity including interest at 6%?

3. Find the average date of the following account.

Harry McGee

March 10	1/10; N/30	75.35
" 25	1/20; N/30	69.50
April 17	Terms Cash	98.30

On June 20th McGee gave a note in settlement. The note was dated June 20th and was made out for the balance of the account plus interest at 6% from the average date until June 20th. What was the face of the note?

The note was due 60 days after date with interest at 6% on the face. How much was paid at maturity?

Compound Average. When an account contains both debits and credits, the process of determining the average date is called *compound average.* The credits, as well as the debits, enter into the average at the dates when they have a cash value equal to their face. For instance, a note bearing interest is worth its face on the day it is made, while a non-interest bearing note is not worth its face until maturity.

Compound averaging is performed as follows:

Steps (*a*), (*b*), (*c*) and (*d*) are taken as in simple average, both for debits and for credits.

(*e*) Compute the balance of the account.

(*f*) Determine the difference between the sum of the debit products and the sum of the credit products.

(*g*) Divide this difference by the balance of the account, and reduce the quotient to months and days.

(*h*) If the balance of the account and the balance of the products are both debits or both credits, the average date is forward from the focal date. If either is a debit and the other a credit, the average date is backward from the focal date.

Problem. Find the average date of the following account:

D. L. Burton

May 7	Terms Cash	750.00	June 2	Int.-bearing note — 1 mo.	1,500.00
" 19	2/10; N/30	1,200.00	" 10	Cash	100.00

SOLUTION. Focal date, April 30.

			Time from Focal Date		Products	
Date	*Date in Ave.*	*Amt.*	*Mos.*	*Days*	*Mos.*	*Days*
Debits:						
May 7	May 7	750.00	0	7	0	5,250
May 19	June 18	1,200.00	1	18	1,200	21,600
		1,950.00			1,200	26,850
					1,200 × 30	36,000
						62,850

Credits:						
June 2	June 2	1,500.00	1	2	1,500	3,000
June 10	June 10	100.00	1	10	100	1,000
		1,600.00			1,600	4,000
					1,600 × 30 =	48,000
						52,000
Balances		350.00				10,850

10,850 ÷ 350 = 31 days, or 1 month and 1 day. Hence the average date is June 1.

Problem. To illustrate how the average date may be backward from the focal date, assume the same account, except that the note does not bear interest.

SOLUTION. The debits would be the same as before, totaling:

			Time from Focal Date		Products	
Date	*Date in Ave.*	*Amt.*	*Mos.*	*Days*	*Mos.*	*Days*
Debits:		1,950.00				62,850
Credits:						
June 2	July 2	1,500.00	2	2	3,000	3,000
" 10	June 10	100.00	1	10	100	1,000
		1,600.00			3,100	4,000
					3,100 × 30 =	93,000
Debit balance of account		350.00				97,000
Credit balance of products						34,150

34,150 ÷ 350 = 98, or 3 months and 8 days. Hence the average date is 3 months and 8 days backward from April 30, or January 23. The reason is this: the creditor is obliged to wait a month to collect the $1,500 note and he obtains no interest. Therefore, he should receive a considerable amount of interest on the $350.00 balance of the account, which is accomplished by giving it a due date of January 23.

Written Work

Compute the average dates of the following account:

1.

Jan. 15	2/10; N/30	500.00
" 27	1/10; N/30	600.00

2.

March 8	Cash	600.00

3.

Aug. 10	Terms Cash	700.00
" 19	" "	650.00
" 27	" "	320.00
Sept. 9	1/10; N/30	75.00

A

March 2	Cash	700.00

B

Feb. 6	2/10; N/60	700.00
" 17	Terms Cash	900.00

C

Sept. 1	Cash	200.00
" 10	Int. bearing Note	500.00

APPENDIX C

COMPOUND INTEREST AND ANNUITIES

Annuities. An ordinary annuity is a series of equal payments made at the end of equal periods of time.

Amount of Ordinary Annuity. The amount of an ordinary annuity is the sum of all of the payments plus compound interest thereon until the date of the last payment.

The amount of an ordinary annuity of $1 is computed by dividing the compound interest on $1 for the given number of periods at the given rate, by the interest rate.

Illustration. What is the amount of an annuity of $1.00 for four periods at 3%?

SOLUTION. The compound interest on $1 for 4 periods at 3% is .125509. Then .125509 ÷ .03 = 4.18363, the amount of an ordinary annuity of $1.

Illustration. What is the amount of an annuity of $100.00 for four periods at 3%?

SOLUTION.

Amount of annuity of $1 (as above)	$4.18363
Multiply by	100
Amount of annuity of $100	418.36

The accumulation of this annuity may be tabulated as follows:

Accumulation of Amount of Annuity

End of first period:	
Payment	$100.00
End of second period:	
Interest on 100 00 @3 %	3.00
Payment	100.00
Total	203.00
End of third period:	
Interest on 203.00 @ 3 %	6.09
Payment	100.00
Total	309.09
End of fourth period:	
Interest on 309.09 @ 3 %	9.27
Payment	100.00
Total (as above)	418.36

Present Value of Ordinary Annuity. The present value of an ordinary annuity is the sum which, invested at compound interest, will provide for the withdrawal of

equal sums at the end of each of a stated number of periods, the last withdrawal exhausting the investment.

The present value of an ordinary annuity of $1.00 is computed by dividing the compound discount on $1.00 for the given number of periods at the given rate, by the interest rate.

Problem. What is the present value of an ordinary annuity of $1.00 for four periods at 3%?

SOLUTION. The compound discount on$ 1 for four periods at 3% is $.111513. Then $.111513 ÷ .03 = $3.7171, the present value of an annuity of $1.

Problem. What is the present value of an annuity of $100.00 for four periods at 3%?

SOLUTION.

Present value of annuity of $1 (as above)	$3.7171
Multiply by	100
Present value of annuity of $100	$371.71

This means that if $371.71 is invested at 3% interest per period, it will be possible to withdraw $100.00 at the end of each of four periods, the last withdrawal exhausting the investment, as shown in the following tabulation:

Reduction of Present Value of Annuity

Beginning of first period:	
Investment of present value	$371.71
End of first period:	
Add interest on $371.71 @ 3%	11.15
Total	382.86
Deduct withdrawal	100.00
Balance	282.86
End of second period:	
Add interest on $282.86 @ 3%	8.49
Total	291.35
Deduct withdrawal	100.00
Balance	191.35
End of third period:	
Add interest on $191.35 @ 3%	5.74
Total	197.09
Deduct withdrawal	100.00
Balance	97.09
End of fourth period:	
Add interest on $97.09 @ 3%	2.91
Total	100.00
Deduct withdrawal	100.00
Balance	0.00

Sinking Funds. A sinking fund is a fund of assets accumulated at compound interest to pay a debt at maturity. The equal contributions to be made to the fund are computed by dividing the total desired fund by the amount of an annuity of $1 for the stated number of periods in which contributions are to be made at the rate which it is assumed that the sinking fund will earn. That is, if contributions are to be made at the end of each of five years, and it is expected that the investments in the fund will earn 4%, the annual contributions to be made to produce a fund of $10,000.00 will be computed by dividing $10,000.00 by the amount of an annuity of $1 for five periods at 4%.

Illustration. What equal contributions made at the end of each of five years will produce a fund of $10,000.00 on a 4% basis?

SOLUTION.

Amount of $1 for 5 periods at 4% (see page 274)	$1.216653
Then the compound interest is	.216653

And $.216653 ÷ .04 = $5.416325 the amount of an annuity of $1
And $10,000.00 ÷ $5.416325 = $1,846.27 the annual contribution.

Table of Accumulation of Sinking Fund

End of Year	*Interest Earned*	*Contribution Made*	*Total Fund*
1		$1,846.27	$ 1,846.27
2	$ 73.85	1,846.27	3,766.39
3	150.66	1,846.27	5,763.32
4	230.53	1,846.27	7,840.12
5	313.61	1,846.27	10,000.00
	$768.65	$9,231.35	

Rent of Annuity. The periodical payments of an annuity are called rents. The rents which may be withdrawn by making a stated investment, are computed by dividing the stated investment by the present value of an annuity of $1 for the agreed number of periods at the agreed rate.

Illustration. A man has $10,000.00 which he desires to invest in a five year annuity at 4%. What rents can he withdraw at the end of each of five years?

SOLUTION. Amount of $1 at 4% for five periods (see page 274) $1.216653

Then $1 ÷ $1.216653 = $.821927 Present value of $1
And $1 − $.821927 = $.178073 Compound discount
And $.178073 ÷ .04 = $4.451825 Present value of annuity of $1
$10,000.00 ÷ $4.451825 = $2,246.27 Rents which can be withdrawn.

Table of Reduction of Annuity

End of Year	*Interest Added*	*Rent Withdrawn*	*Balance*
			$10,000.00
1	$ 400.00	$ 2,246.27	8,153.73
2	326.15	2,246.27	6,233.61
3	249.34	2,246.27	4,236.68
4	169.47	2,246.27	2,159.88
5	86.40	2,246.27	.01
	$1,231.36	$11,231.35	

The balance of one cent is unavoidable unless $2,246.28 is withdrawn the fifth year.

Equal Installments in Payment of a Debt. Chapter XXVI contained a discussion of installment payments with long-end interest and short-end interest. When long-end interest is taken, the total payment, including interest and an application on the principal, decreases each period. With short-end interest the total payment increases each period.

If equal installments, including principal and interest, are made, the installments are computed by the method just explained for the computation of rents of an annuity.

Problem. A owes Z $10,000.00 which he agrees to repay in five equal annual installments, each of which shall include interest at 4 % on the unpaid balance of the principal, the remainder of the payment to apply on the principal. What is the annual installment?

SOLUTION. The computation would be identical with the solution of the example of "Rents of an Annuity," the annual installment being $2,246.27. The reduction of the debt could be tabulated in the manner shown above, or as follows:

Table of Reduction of Debt in Equal Installments

Original debt		$10,000.00
End of first year:		
Payment	$2,246.27	
Less interest on $10,000.00 @ 4 %	400.00	
Balance applied on principal		1,846.27
Unpaid principal		8,153.73
End of second year:		
Payment	2,246.27	
Less interest on $8,153.73 @ 4 %	326.15	
Balance applied to principal		1,920.12
Unpaid principal		6,233.61

End of third year:		
Payment	2,246.27	
Less interest on $6,233.61 @ 4%	249.34	
Balance applied to principal		1,996.93
Unpaid principal		4,236.68
End of fourth year:		
Payment	2,246.27	
Less interest on $4,236.68 @ 4%	169.47	
Balance applied to principal		2,076.80
Unpaid principal		2,159.88
End of fifth year:		
Payment	2,246.28	
Less interest on $2,159,88 @ 4%	86.40	
Balance applied to principal		2,159.88
Unpaid principal		0.00

Written Work

1. What is the amount of an annuity of $500.00 per annum for ten years at 5%, compounded annually? Set up a table showing the accumulation of the annuity. What is the present value of an annuity of $500.00 per annum for ten years at 5%, compounded annually? Set up a table showing the reduction of the annuity.

2. A deposits $100.00 in a bank on his child's first birthday and on each birthday thereafter including the twenty-first. Assuming that the bank adds 4% interest annually on the dates of deposits, how much will there be on deposit at the twenty-first birthday?

3. What annual contributions will produce a sinking fund of $25,000.00 in ten years if 5% interest is compounded annually on the fund? Set up a table of accumulation.

4. B invests $25,000.00 in a ten year annuity on a 5% basis. What annual rents will be paid to him? Set up a table of reduction.

5. C owes $5,000.00 which he agrees to pay off in five equal annual installments which shall include 6% interest on unpaid balances of the debt. What annual payments does he make? Set up a table of reduction.

APPENDIX D

DENOMINATE NUMBERS

SURVEYORS' LONG MEASURE

7.92 inches	= 1 link (lk.)
25 links	= 1 rod
4 rods, or 100 links	= 1 chain (ch.)
80 chains	= 1 mile

SURVEYORS' SQUARE MEASURE

625 square links	= 1 square rod
10 square rods	= 1 square chain
16 square chains	= 1 acre
640 acres	= 1 square mile
36 square miles	= 1 township

The following units of measure are used by sailors:

6 feet	= 1 fathom (Used for measuring depths at sea.)
120 fathoms	= 1 cable length (Used for measuring depths at sea.)
About 1.15 statute miles	= 1 knot, or 1 nautical mile, or 6080.27 ft.

CIRCULAR OR ANGULAR MEASURE

60 seconds (60″)	= 1 minute (′)
60 minutes	= 1 degree (1°)
90 degrees	= 1 right angle
360 degrees	= 1 circumference

TROY WEIGHT

24 grains (gr.)	= 1 pennyweight (pwt. or dwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)

APOTHECARIES' WEIGHT

20 grains (gr.) = 1 scruple (sc. or ℈)
3 scruples = 1 dram (dr. or ʒ)
8 drams = 1 ounce (oz. or ℥)
12 ounces = 1 pound (lb.)

Apothecaries' weight is used by physicians and druggists. Troy weight is used in the measurement of precious metals.

COMPARISON OF TROY AND AVOIRDUPOIS WEIGHTS

1 pound Troy = 5760 grains
1 pound avoirdupois = 7000 grains
1 ounce Troy = 437½ grains
1 ounce avoirdupois = 480 grains

The term *carat* has two meanings:

In weighing precious stones, a carat usually means 3.2 Troy grains.

In expressing the purity of gold, 24 carats means pure gold; 18 carats means $\frac{18}{24}$ pure gold, and $\frac{6}{24}$ alloy.

PAPER MEASURE

24 sheets = 1 quire (qr.)
20 quires = 1 ream (rm.)

Although a ream contains 480 sheets, 500 sheets are usually sold as a ream.

STANDARD UNITS OF WEIGHT

1 barrel flour	weighs 196 pounds
1 barrel salt	weighs 280 pounds
1 barrel pork or beef	weighs 200 pounds
1 keg of nails	weighs 100 pounds

STANDARD BUSHELS IN MANY STATES

1 bushel shelled corn	weighs 56 pounds
1 bushel ear corn	weighs 70 pounds
1 bushel wheat	weighs 60 pounds
1 bushel barley	weighs 48 pounds
1 bushel rye	weighs 56 pounds
1 bushel oats	weighs 32 pounds

ABBREVIATIONS COMMONLY USED IN BUSINESS

(The singular form is commonly used for both the singular and plural.)

A. . . . . acre
acct. or a/c. account
agt. . . . . agent
amt. . . . . amount
ans. . . . . answer
Apr. . . . April
Aug. . . . August
av. . . . . average
bg. . . . . bag; bags
bal. . . . . balance
bbl. or brl. . barrel
bdl. . . . . bundle
bkt. . . . . basket
bl. . . . . bale
B/L . . . . bill of lading
bot. . . . . bought
bu. . . . . bushel
bx. . . . . box
C . . . . . one hundred
cd. . . . . cord; card
cg. . . . . centigram
ch. . . . . chain; chest
chg. . . . . charge
c.i.f. . . . . carriage and insurance free
ck. . . . . check
cm. . . . . centimeter
cml. . . . . commercial
c/o . . . . care of
Co. . . . . company; county
c.o.d. . . . collect on delivery
coll. . . . . collection
com. . . . . commission
consg't . . . consignment
cr. . . . . creditor; credit; crate
cs. . . . . case
csk. . . . . cask
c., ct. . . . cent; centime
cu. ft. . . . cubic foot
cu. in. . . . cubic inch
cu. yd. . . . cubic yard
cwt. . . . . hundredweight
d. . . . . . pence
da. . . . . day
Dec. . . . . December
dep't . . . department
dft. . . . . draft
disc. . . . . discount
do. . . . . ditto (the same)
doz.; dz. . . dozen; dozens
Dr. . . . . debit; debtor; doctor
E. . . . . East
ea. . . . . each
e.g. . . . . for example
e.o.e. . . . errors and omissions excepted
etc. . . . . and so forth
ex. . . . . example; express
exch. . . . exchange
exp. . . . . expense
f. . . . . franc
far. . . . . farthing
Feb. . . . February
fir. . . . . firkins
f.o.b. . . . free on board
frt. . . . . freight
ft. . . . . foot
gal. . . . . gallon
gi. . . . . gill
gr. . . . . grain
gro. . . . . gross
gr. gro. . . great gross
guar. . . . guaranty; guarantee
hf. . . . . half
hf. cht. . . half chest
hhd. . . . hogshead
hr. . . . . hour
i.e. . . . . that is
in. . . . . inch; inches
ins. . . . . insurance
inst. . . . . instant; the present month
int. . . . . interest
I; inv. . . invoice
inv't . . . inventory
Jan. . . . . January
kg. . . . . keg
km. . . . . kilometer
l. . . . . link
lb. . . . . pound
l.p. . . . . list price
ltd. . . . . limited

ABBREVIATIONS COMMONLY USED IN BUSINESS — (*continued*)

M . . . . one thousand
m. . . . . mill; meter
Mar. . . . March
mdse. . . . merchandise
Messrs. . . *Messieurs;* Gentlemen; Sirs
mi. . . . . mile
min. . . . . minute
mo. . . . . month
mortg. . . . mortgage
Mr. . . . . Mister
Mrs. . . . . Mistress
N. . . . . North
no. . . . . number
Nov. . . . November
Oct. . . . . October
O.K. . . . all correct
oz. . . . . ounce
p. . . . . page
pay't . . . payment
pc. . . . . piece
pd. . . . . paid
per . . . . by; by the
per cent . . per centum; by the hundred
pfd. . . . . preferred
pk. . . . . peck; pecks
pkg. . . . . package
pp. . . . . pages
pr. . . . . pair
prox. . . . the following month
pt. . . . . pint
pwt. . . . . pennyweight
qr. . . . . quire
qt. . . . . quart
rd. . . . . rod
rec'd . . . received
rec't . . . receipt
rm. . . . . ream
Rm. (or M.) Reichmark; Mark
s. . . . . shilling; shillings
S. . . . . south; sales
sec. . . . . second
sec'y . . . secretary
Sept. . . . September
set. . . . . settlement
ship. . . . shipment
shipt. . . . shipped
sig. . . . . signed; signature
sq. ch. . . . square chain
sq. ft. . . . square foot
sq. mi. . . square mile
sq. rd. . . . square rd.
sq. yd. . . square yard
stk. . . . . stock
sund. . . . sundries
T. . . . . ton
tb. . . . . tub
Tp.; Twp. township; townships
tr.; trans. . transfer
treas. . . . treasurer; treasury
ult. . . . . last month
via . . . . by way of
viz. . . . . namely; to wit
vol. . . . . volume
wk. . . . . week
wt. . . . . weight; weigh
yd. . . . . yard
yr. . . . . year

TABLE OF SYMBOLS

a/c . . account
a/s . . account sales
+ . . . addition
( ) . . aggregation
& . . . and
... . . . and so on
@ . . . at; each; to
′ . . . foot; feet; minutes
c/o . . . . . care of
¢ . . . . . cent
√ . . . . . check mark; correct
° . . . . . degree
÷ . . . . . division
$ . . . . . dollar; dollars
= . . . . . equal; equals
o/d . . . . . on demand

TABLE OF SYMBOLS — (*continued*)

$^1, ^2, ^3$	fourths (written as exponents, thus, $3^1 = 3\frac{1}{4}$)	%	per cent
>	greater than	£	pounds sterling
C	hundred	:	ratio
″	inch; inches; seconds	∵	since
<	less than	−	subtraction
×	multiplication	∴	therefore
♯	if written before figures, means number; if written after figures, means pounds	M	thousand

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